Equilibrium Social Distancing

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Research

- **Non-pharmaceutical interventions:**
- *Equilibrium Social Distancing*
- *Social Distancing with Asymptomatic Infection: Beliefs, Fatalism and Testing*
- **Pharmaceutical interventions:**
- *On the Management of Population Immunity* w. R. Rowthorn

Social distancing...

- The Great Plague of Milan (1639)

Literature on behavior in SI, SIS and SIR models

- Reluga (2010)
- Rowthorn and Toxvaerd (2012)
- Fenichel et al. (2011), Fenichel (2013)
- Chen et al. (2011), Chen (2012)
- Gersovitz and Hammer (2004)
- Chen and Toxvaerd (2014)
- Greenwood et al. (2017, 2019)
- Toxvaerd (2019)
- New wave of literature in both macro and micro, many presented in this workshop

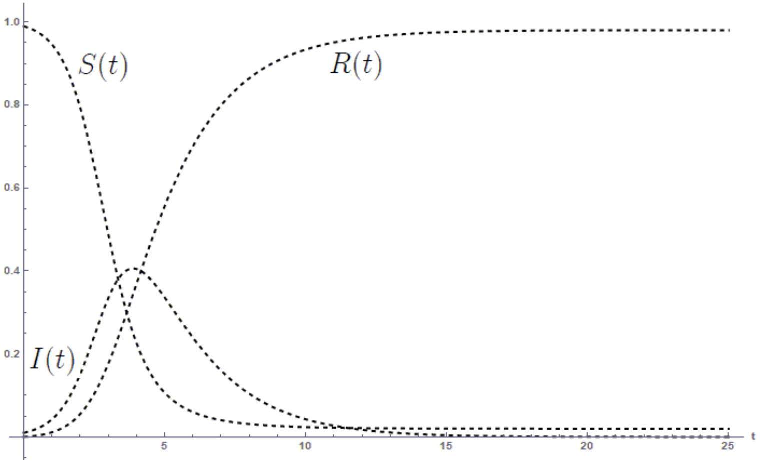
The SIR model

- The SIR model equations:

$$\begin{aligned}
\dot{S}(t) &= -\beta I(t) S(t) \\
\dot{I}(t) &= I(t)\left[\beta S(t) - \gamma\right] \\
\dot{R}(t) &= \gamma I(t) \\
S(t) &= 1 - I(t) - R(t), \quad S(0) = S_0 > \gamma/\beta
\end{aligned}$$

The SIR model

- The SIR model paths:



The economic model

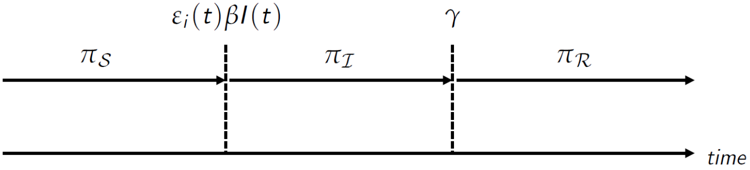
- Preferences over health states:

$$\pi_{\mathcal{S}} \geq \pi_{\mathcal{R}} \geq \pi_{\mathcal{I}}$$

- Choose exposure $\varepsilon_i(t) \in [0, 1] \rightarrow$ inf. at rate $\varepsilon_i(t)\beta I(t)$
- Cost of social distancing is $(1 - \varepsilon_i(t))c$
- People discount future at rate ρ

The economic model

- Timeline of payoffs and transitions:



- Transition times follow Poisson distributions

The individual's decision problem

- Population game formulation of Reluga and Galvani (2011)
- Each individual solves this problem:

$$\max_{\varepsilon_i(t)\in[0,1]} \int_0^\infty e^{-\rho t} \left\{(1-p_i(t))[\pi_{\mathcal{S}} - (1-\varepsilon_i(t))c] + p_i(t)\rho V_{\mathcal{I}}\right\} dt$$

$$s.t. \quad \dot{p}_i(t) = \varepsilon_i(t)\beta I(t)(1-p_i(t)), \quad p_i(0) = 0$$

- Here $p_i(t) \in [0,1]$ denotes probability of infection residence in infectious state
- NPV of transitioning into the infected state is

$$V_{\mathcal{I}} = \frac{1}{\rho+\gamma}\left[\pi_{\mathcal{I}} + \gamma\frac{\pi_{\mathcal{R}}}{\rho}\right]$$

- Get $\pi_{\mathcal{I}}$ while infected, $\pi_{\mathcal{R}}$ while recovered (and zero if deceased)

The individual's decision problem

- Aggregate infection evolves according to

$$\dot{I}(t) = I(t)\left[\boldsymbol{\varepsilon}(t)\beta S(t) - \gamma\right] = 0$$

- Here aggregate exposure is

$$\boldsymbol{\varepsilon}(t) \equiv \int_{i \in \mathcal{S}(t)} S(t)^{-1} \varepsilon_i(t) di$$

- If $\varepsilon_i(t) = 1$ for all $i \in \mathcal{S}(t)$, $\boldsymbol{\varepsilon}(t) = 1$ so like in epi model

The individual's decision problem

- The best response is

$$\varepsilon_i(t) = \begin{cases} 0 & \textit{for} \quad I(t) > I^* \\ \varepsilon & \textit{for} \quad I(t) = I^* \\ 1 & \textit{for} \quad I(t) < I^* \end{cases}$$

- Critical threshold given by

$$I^* \equiv \frac{\rho c}{\beta \left(\pi_{\mathcal{S}} - \frac{\rho \pi_{\mathcal{I}} + \gamma \pi_{\mathcal{R}}}{\rho + \gamma} - c \right)}$$

- With homogeneous pop., equilibrium involves mixed strategies

Heterogeneous population

- With het. pop., individuals draw costs $c_i \sim F[0, \overline{c}]$
- Now have type-specific thresholds

$$I^*(c_i) \equiv \frac{\rho c_i}{\beta\left(\pi_{\mathcal{S}} - \frac{\gamma \pi_{\mathcal{I}} + \rho \pi_{\mathcal{R}}}{\rho + \gamma} - c_i\right)}$$

- Since $I^*(c_i)$ increasing in c_i, higher cost individuals
  - More willing to tolerate risk of infection
  - Wait longer before scaling back exposure

Heterogeneous population

- At prevalence $I(t)$, all types w. $I^*(c_i) \leq I(t)$ socially distance
- So infection equation becomes

$$\dot{I}(t) = [1 - F_t(I(t))] \, \beta I(t) S(t) - \gamma I(t)$$

- As $I(t)$ increases, more aggregate social distancing, which curbs further increase
- In equilibrium, lower cost types socially distance first and higher cost types free-ride
- High cost types more exposed, and over-represented amongst infected/recovered
- Thus lower cost types more likely to benefit from herd immunity
- There is an *intertemporal quid pro quo*

Heterogeneous population

- To see quid pro quo, define **total exposure** as

$$\pi_i \equiv \int_0^\infty \beta \varepsilon_i^*(t) I(t) dt = \int_0^\infty \beta \max\{I^*(c_i), I(t)\} dt$$

- Two things to observe:
- As others socially distance, path $I(t)$ depressed $\longrightarrow$ total exposure lower
- Since $I^*(c_i)$ increasing in c_i, so is total exposure π_i
- So high cost individuals likelier to become infected

Equilibrium social distancing

- Because equilibrium decisions depend on biological and preference parameters, can compare effects of changes
- Comparative statics for het. pop. case are:

	β	γ	c	ρ	$\pi_{\mathcal{S}}$	$\pi_{\mathcal{I}}$	$\pi_{\mathcal{R}}$
I^*	$-$	$+$	$+$	$+/-$	$-$	$+$	$+$
$\bar{I}$	$+$	$-$	0	0	0	0	0

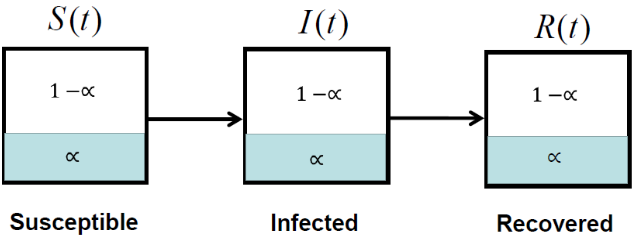
- Note that results wrt β and γ opposite in two models!
- More infectious disease leads to more social distancing in equilibrium
- This helps dampen the epidemic
- Biological model would have predicted the opposite to be true

Asymptomatic infection

- Special features of COVID-19 is high asymptomatic ratio
- Estimates vary widely, but most are about a third
- Many people get infected and recover without knowing they were infected
- How does this change equilibrium behavior?

Asymptomatic infection

- Model as before, but fraction $\alpha \in [0, 1]$ never show symptoms
- Everyone understands aggregate dynamics
- But individual who has never had symptoms cannot distinguish b/w being susceptible or asymptomatically infected or recovered



Asymptomatic infection

- Aggregate measures give by

	Susceptible	Infected	Recovered
Symp.\Classes	$\mathcal{S}(t)$	$\mathcal{I}(t)$	$\mathcal{R}(t)$
Asymptomatic	$\alpha S(t)$	$\alpha I(t)$	$\alpha R(t)$
Symptomatic	$(1-\alpha)S(t)$	$(1-\alpha)I(t)$	$(1-\alpha)R(t)$

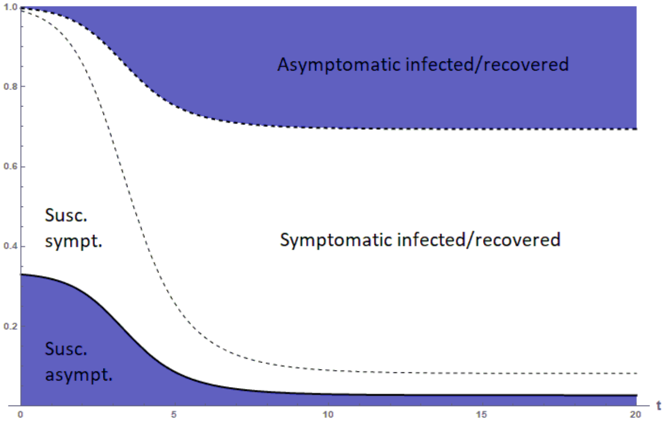
- Measures common knowledge and determine beliefs of people
- As individuals homogeneous, probabilities of being in each class same as pop. frequencies

Asymptomatic infection

- How does this change individual's problem?
- Someone who knows he/she is infected or recovered does not socially distance
- So must determine prob. of being (symptomatically) susceptible conditional on not having shown symptoms in past
- Will outline how beliefs are formed

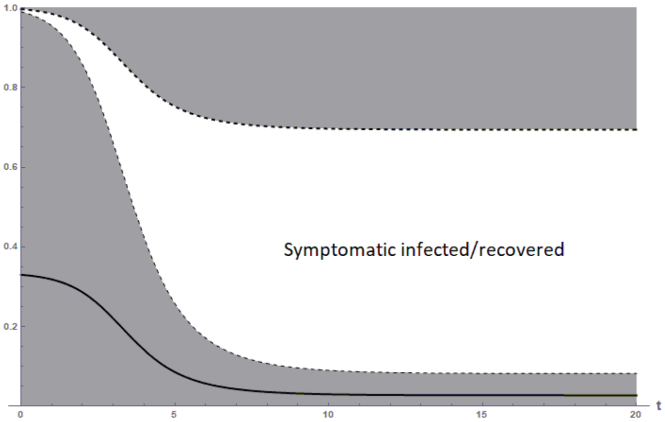
Asymptomatic infection

- Assume that $\alpha = 1/3$
- Classes evolve as follows across epidemic



Asymptomatic infection

- Evolution of asymptomatic versus symptomatic classes



- Initially, lack of symptoms most likely b/c of susceptibility
- But increasingly, more likely b/c of asymptomaticity

Asymptomatic infection

- Probability of being susceptible is

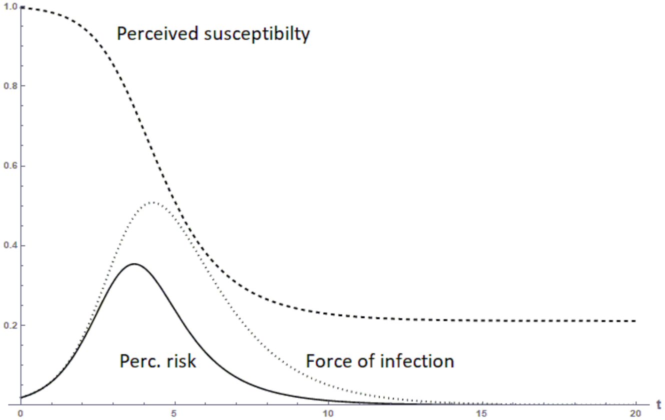
$$\sigma_i(t) = \frac{S(t)}{S(t) + \alpha(I(t) + R(t))}$$

- This probability mon. decreasing
- Increasingly likely that individual was "lucky", was infected and recovered w/o knowing it
- Recall that risk faced by individual is now

$$(1-\alpha)\sigma_i(t)\beta I(t)\left[(1-\alpha) + \alpha\varepsilon(t)\right]$$

Asymptomatic infection

- Evolution of susceptibility, risk and force of infection across epidemic



- Important to notice that perceived risk is non-monotone!
- There is **rational fatalism** (Philipson and Posner, 1993)

Asymptomatic infection

- Critical threshold now becomes time-dependent
- Note that as $\sigma_i(t)$ decreases, $I_i^*(t)$ increases over time
- This is intuitive: as time passes, people w/o symptoms attach decreasing prob. to being susceptible
- Thus increasingly tolerant of risks from exposure
- This is reflected in higher critical level $I^*(t)$
- **There is less social distancing in equilibrium when infection is asymptomatic**
- In equilibrium, more infection but because of less distancing
- Asympt. setting nests perfect info case ($\alpha = 0$) and biological model ($\alpha = 1$)

Testing

- Can distinguish between different types of tests:
- An i-test may detect whether infection currently present
- An r-test may detect whether antibodies present - immunity?
- Let k-test have precision $q_k > 1/2$, $k = i, r$
- Outcomes then

	$m_k(t) = 1$	$m_k(t) = 0$
$\theta_k(t) = 1$	q_k	$1 - q_k$
$\theta_k(t) = 0$	$1 - q_k$	q_k

- **Pretest probs of being in health states are pop. frequencies**
- So inference from tests depends on when taken!
- Value of information/test also changes across stages of epidemic

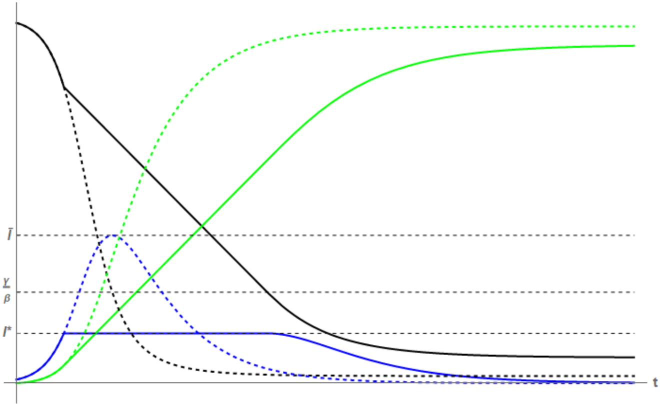
Testing

- Tests for immunity likely socially beneficial
  - Their behavior doesn't influence infection but costly social distancing reduced
- Tests for infection have ambiguous effect
- Some who test positive may choose higher exposure
- Thus different tests lead to different post-test beliefs and hence different post-test changes in behavior
- Difference between private and social value of tests

Back

Equilibrium social distancing

- Overall view of epidemic under equilibrium social distancing:



The individual's decision problem

- The best response given by switching function

$$\eta(t)\beta I(t) + c = 0$$

- Change over time is

$$\frac{d}{dt}\left[\eta(t)\beta I(t) + c\right] = \beta\left[\dot{\eta}(t)I(t) + \eta(t)\dot{I}(t)\right]$$

where

$$\dot{\eta}(t) = \eta(t)[\rho + \varepsilon_i(t)\beta I(t)] + \left[\pi_{\mathcal{S}} - \tfrac{\rho\pi_{\mathcal{I}} + \gamma\pi_{\mathcal{R}}}{\rho+\gamma} - (1 - \varepsilon_i(t))c\right]$$

$$\dot{I}(t) = I(t)\left[\boldsymbol{\varepsilon}(t)\beta S(t) - \gamma\right]$$

$$\boldsymbol{\varepsilon}(t) \equiv \int_{i\in\mathcal{S}(t)} S(t)^{-1}\varepsilon_i(t)di$$

- Best responses change over time as function of aggr. system
- Contrast to myopic decision making