Supplemental Appendix to

Minimum Wages, Efficiency and Welfare

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The indexing of this ***Supplemental Appendix*** follows on from the ***Online Appendix***. Section E contains additional details on calibration of preference parameters. Section F provides a pedagogical step-by-step solution of a simplified version of the model and the algorithm for solving the minimum wage economy. Section G contains mathematical derivations for the full quantitative model with household heterogeneity. Section I contains derivations of the solution of the model under the tax and transfer system in Section 6.

E Disciplining preference parameters

This Section details how we use recent evidence from Golosov, Graber, Mogstad, and Novgorodsky (2021) to discipline preference parameters σ and φ.

Background. Consider a budget constraint, where b_i is unearned income and $\mathcal{T}$ gives taxes and transfers which depend on pre-tax labor income y_i:

$$c_i = y_i - \mathcal{T}\left(y_i\right) + b_i$$

Totally differentiating with respect to b_i:

$$\frac{dc_i}{db_i} = \frac{dy_i}{db_i} - \frac{d\mathcal{T}_i}{db_i} + 1 \quad \text{, which we can write} \quad MPC_i = MPE_i - MPT_i + 1$$

Table 4.1 of Golosov, Graber, Mogstad, and Novgorodsky (2021, henceforth GGMN) gives estimates of the marginal propensity to consume (MPC) and marginal propensity to earn (MPE) for different income groups, where lottery winnings are used as an instrument for the endogenous variable b_i. For example, results are of the type: *An extra dollar in unearned income leads to a $MPE = -0.52$ cent reduction in labor earnings.* We show how their results can be used to discipline preference parameters (φ, σ) in a simple labor supply setting that is consistent with our model.

Derivation. Consider the following individual problem, where preferences are as in the main text, and $y = wn$, where w is taken as given:

$$u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{1}{\overline{\varphi}^{1/\varphi}} \frac{n^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \quad \text{subject to} \quad c = wn + \mathcal{T}(wn) + b \tag{E1}$$

Optimality conditions for c and n give labor supply, which can be expressed in terms of earnings:

$$y = \overline{\varphi} c^{-\varphi\sigma} w^{\varphi+1} \left(1 - \mathcal{T}'(y)\right)^{\varphi}$$

Totally differentiating with respect to b

$$\frac{dy}{db} = -\varphi\sigma \frac{dc}{db}\left(\frac{y}{c}\right) - \varphi\left(\frac{\mathcal{T}''(y)\, y}{1-\mathcal{T}'(y)}\right)\frac{dy}{db}.$$

Now suppose that post-tax labor earnings were of the form used in Heathcote, Storesletten, and Violante (2020, henceforth HSV): $y - \mathcal{T}(y) = \lambda y^{1-\tau}$. In this case, the elasticity term is simply the progressivity of taxes, τ.

$$\frac{dy}{db} = -\varphi\sigma \frac{dc}{db}\left(\frac{y}{c}\right) - \varphi\tau\frac{dy}{db}.$$

Using the definitions of MPC, MPE, the average propensity to consume $APC = c/y$, and after rearranging, we have a closed-form relationship between σ and φ, given data on $\{MPC, MPE, APC, \tau\}$:

$$\varphi = -\frac{1}{\sigma \frac{MPC}{MPE}\frac{1}{APC} - \tau}. \tag{E2}$$

If we let $\sigma = 1$ and $\tau = 0$, it is straightforward to observe that a lower MPC and higher MPE in absolute terms (as will be the case for richer households), requires a higher φ.

$$\varphi = \frac{|MPE|}{MPC} APC.$$

Data. We use BLS data to compute APC for non-high-school, high-school, and college completion households. We map these into the four quartiles of income groups in GGMN Table 4.1 as given in the following table. We take a value of $\tau = 0.181$ from HSV.

	All	**Group**		
BLS category GGMN category		Non-High School Q1	High school Q2-Q3	Completed college Q4
APC (BLS)	0.69	0.73	0.71	0.67
MPE (GGMN)	-0.5227	-0.3080	-0.5549	-0.6735
MPC (GGMN)	0.5836	0.7315	0.5429	0.4990

Table E1: Data used in calibrating preference parameters

Results. Using equation (E2), we can then determine φ given σ. Figure E1 plots $\varphi(\sigma)$ for $\sigma \in [1,2]$. As a benchmark, with log preferences, and when calibrated to the whole sample values, $\varphi(1) = 0.65$. For low income (Q1) households $\varphi(1) = 0.32$, for high

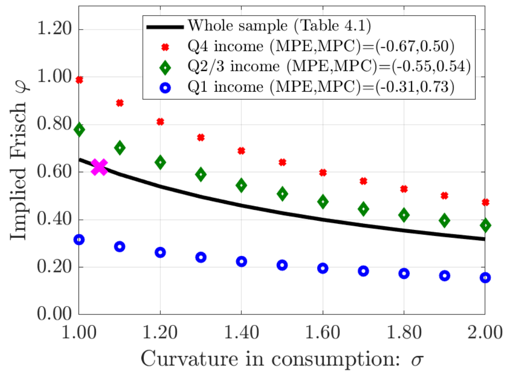


Figure E1: Implied parameters

Notes: Given a value for the coefficient of relative risk aversion σ, this figure plots the Frisch elasticity of labor supply φ required for the optimality conditions of the simple labor supply model E1 to be consistent with (i) empirical measures of the marginal propensity to earn and marginal propensity to consume following changes in unearned income from Golosov, Graber, Mogstad, and Novgorodsky (2021), (ii) estimates of the average propensity to consume from the BLS, (iii) estimates of the progressivity of post-tax labor income to pre-tax-and-transfer income from Heathcote, Storesletten, and Violante (2020).

income households $\varphi(1) = 0.987$. High income (Q4) households have higher MPE's, and their MPC is lower, reducing $|MPC/MPE|$, and requiring a higher φ. The pink cross corresponds to $(\sigma, \varphi) = (1.05, 0.62)$, which are the values used in the baseline calibration of our model (see Table 1).

F Pedagogical example & algorithm

The aim of this section is to clearly lay out the algorithm for solving the minimum wage equilibrium, and to present a full solution of a simplified model, which may be pedagogically useful relative to the extensive derivations in Appendix D. The algorithm for the minimum wage equilibrium is nested in the broader solution to the equilibrium of the model described in Appendix G.

For ease of exposition, we lay out the minimum wage problem (i) ignoring capital, (ii) consider an economy with a single type of household, (iii) to simplify exposition we also consider GHH preferences, which are not used in the main text, (iv) as well as a static environment, (v) set the coefficient on labor in utility $\overline{\varphi} = 1$. We derive conditions for this simplified economy and then present the algorithm.

F.1 Pedagogical example

Consider the household problem with the rationing constraint $n_{ij} \leq \overline{n}_{ij}$. For ease of interpretation we attach multiplier $\zeta_{ij} = \lambda w_{ij} \left(1 - p_{ij}\right)$ to the rationing constraint, normalized by the household budget multiplier λ:

$$U_0 = \max_{\{n_{ij}, c_{ij}\}} u\left(C - \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)$$

$$\begin{aligned}
C &= \int \sum_{i\in j} w_{ij} n_{ij}\, dj + \Pi \qquad [\lambda] \\
n_{ij} &\leq \overline{n}_{ij} \qquad [\lambda w_{ij}\left(1 - p_{ij}\right)] \\
C &= \int \sum_{i\in j} c_{ij} dj \quad , \quad N = \left[\int n_j^{\frac{\theta+1}{\theta}}\, dj\right]^{\frac{\theta}{\theta+1}} \quad , \quad n_j = \left[\sum_{i\in j} n_j^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}
\end{aligned}$$

The first order condition for n_{ij} yields

$$\lambda w_{ij} - \lambda w_{ij}\left(1 - p_{ij}\right) = u'\left(\cdot\right)\left(\frac{\partial n_j}{\partial n_{ij}}\right)\left(\frac{\partial N}{\partial n_j}\right) N^{\frac{1}{\varphi}}$$

$$\lambda w_{ij} p_{ij} = u'\left(\cdot\right)\left(\frac{\partial n_j}{\partial n_{ij}}\right)\left(\frac{\partial N}{\partial n_j}\right) N^{\frac{1}{\varphi}}$$

The first order condition for consumption yields $u'\left(\cdot\right) = \lambda$. Define the *shadow wage* $\widetilde{w}_{ij} = p_{ij} w_{ij}$. Use the first order condition for consumption $u'\left(\cdot\right) = \lambda$, and use the derivatives of N and n_j:

$$\widetilde{w}_{ij} = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \qquad (*)$$

Now define the *shadow wage indexes*

$$\widetilde{w}_j = \left[\sum_{i\in j} \widetilde{w}_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \widetilde{W} = \left[\int \widetilde{w}_j^{1+\theta} dj\right]^{\frac{1}{1+\theta}} .$$

Using these definitions in $(*)$ along with the definition of n_j:

$$\sum_{i\in j} \widetilde{w}_{ij}^{1+\eta} = \left[\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right]^{1+\eta} \sum_{i\in j} \left(\frac{n_{ij}}{n_j}\right)^{\frac{1+\eta}{\eta}}$$

$$\widetilde{w}_j = \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$$

Using this along with the definition of N:

$$\int \widetilde{w}_j^{1+\theta} dj = \left[N^{\frac{1}{\varphi}}\right]^{1+\theta} \int \left(\frac{n_j}{N}\right)^{\frac{1+\theta}{\theta}} dj$$

$$\widetilde{W} = N^{\frac{1}{\varphi}}$$

Note that $\widetilde{W}N \neq \int \sum_{i\in j} w_{ij}n_{ij}dj$, however the aggregate labor supply $N = \widetilde{W}^{\varphi}$ is *as if*, the household had maximized

$$U_0 = \max_{C,N} u\left(C - \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right) \quad \text{subject to} \quad C = \widetilde{W}N + \Pi.$$

This makes clear the extent to which the shadow wage index $\widetilde{W}$ captures the full distribution of binding minimum wages. Note that shadow wages aggregate:

$$\begin{aligned}
\widetilde{w}_{ij}n_{ij} &= n_{ij}^{\frac{1+\eta}{\eta}} \left(\frac{1}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \\
\sum_{i\in j} \widetilde{w}_{ij}n_{ij} &= \left[\sum_{i\in j} n_{ij}^{\frac{1+\eta}{\eta}}\right] \left(\frac{1}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \\
\sum_{i\in j} \widetilde{w}_{ij}n_{ij} &= n_j\widetilde{w}_j
\end{aligned}$$

Shadow shares - We can define the shadow share $\widetilde{s}_{ij}$ as

$$\widetilde{s}_{ij} := \frac{\widetilde{w}_{ij}n_{ij}}{\sum_{i\in j} \widetilde{w}_{ij}n_{ij}}.$$

Substituting in the labor supply system $(*)$ for $\widetilde{w}_{ij}$

$$\widetilde{s}_{ij} := \frac{n_{ij}^{\frac{1+\eta}{\eta}}}{\sum_{i\in j} n_{ij}^{\frac{1+\eta}{\eta}}} = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1+\eta}{\eta}} = \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{1+\eta}$$

The firm's problem is

$$\pi_{ij} = \max_{n_{ij}} z_{ij}n_{ij}^{\alpha} - w_{ij}n_{ij}$$

subject to

$$\begin{aligned}
n_{ij} &= \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} N \\
w_{ij} &\geq \underline{w}
\end{aligned}$$

Let $r_{ij} \in \{1,2,3\}$ denote the region that the firm is in.

Region I - If the firm is in Region I, then its wage is the optimal markdown on the marginal revenue product of labor

$$w_{ij} = \mu_{ij}\alpha z_{ij}n_{ij}^{\alpha-1} \quad , \quad p_{ij} = 1 \quad , \quad \widetilde{w}_{ij} = w_{ij} \quad , \quad n_{ij} = \left(\frac{w_{ij}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} \widetilde{W}^{\varphi}$$

where the markdown depends on its shadow share of the labor market. That is, $\mu_{ij} =$

$\mu\left(\widetilde{s}_{ij}\right)$, where $\mu\left(\widetilde{s}_{ij}\right)=\frac{\varepsilon\left(\widetilde{s}_{ij}\right)}{\varepsilon\left(\widetilde{s}_{ij}\right)+1}$. We have shown that

$$\widetilde{s}_{ij}=\left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{1+\eta}\qquad\implies\widetilde{w}_j=\widetilde{w}_{ij}\widetilde{s}_{ij}^{-\frac{1}{1+\eta}}$$

Using these, we can write:

$$w_{ij}=\left[\mu\left(\widetilde{s}_{ij}\right)\alpha z_{ij}\widetilde{s}_{ij}^{-\frac{(1-\alpha)(\eta-\theta)}{1+\eta}}\widetilde{W}^{(1-\alpha)(\theta-\varphi)}\right]^{\frac{1}{1+\theta(1-\alpha)}}$$

Region II - In Region II, then

$$w_{ij}=\underline{w}\quad,\quad p_{ij}=1\quad,\quad\widetilde{w}_{ij}=\underline{w}\quad,\quad n_{ij}=\left(\frac{\underline{w}}{\widetilde{w}_j}\right)^{\eta}\left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta}N$$

Region III - In Region III, then

$$w_{ij}=\alpha z_{ij}n_{ij}^{\alpha-1}\quad,\quad p_{ij}<1\quad,\quad\widetilde{w}_{ij}=p_{ij}\underline{w}\quad,\quad n_{ij}=\left(\frac{p_{ij}\underline{w}}{\widetilde{w}_j}\right)^{\eta}\left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta}N$$

F.2 Minimum wage solution algorithm

We implement the following solution algorithm. We denote the *Region* that a firm is in by $r_{ijt}\in\{I,II,III\}$. Initialize the algorithm by (i) guessing a value for $\widetilde{W}^{(0)}$, (ii) assuming all firms are in *Region I*, $r_{ij}^{(0)}=I$, which implies guessing $p_{ij}^{(0)}=1$. These will all be updated in the algorithm.

1. Solve all market equilibria in shadow shares.

1. Guess shadow shares $\widetilde{s}_{ij}^{(0)}$.
2. *Region I* - Using the above optimality condition

$$w_{ij}=\left[\mu\left(\widetilde{s}_{ij}\right)\alpha z_{ij}\widetilde{s}_{ij}^{(0)-\frac{(1-\alpha)(\eta-\theta)}{1+\eta}}\widetilde{W}^{(0)(1-\alpha)(\theta-\varphi)}\right]^{\frac{1}{1+\theta(1-\alpha)}}$$

3. *Regions II, III* - Here the minimum wage is binding so set $w_{ij}=\underline{w}$.
4. Given the guess $p_{ij}^{(k)}$ and w_{ij}, compute the shadow wage: $\widetilde{w}_{ij}=p_{ij}w_{ij}$.
5. With all shadow wages in hand, update shadow shares using $\widetilde{w}_{ijt}$:

$$\widetilde{s}_{ij}^{(l+1)}=\frac{\widetilde{w}_{ij}^{1+\eta}}{\sum_{i\in j}\widetilde{w}_{ij}^{1+\eta}}.$$

6. Iterate over (b)-(e) until shadow shares converge: $\widetilde{s}_{ij}^{(l+1)}=\widetilde{s}_{ij}^{(l)}$.

2. Recover employment. Here we use the wages from the previous step plus the current guess of each firms' region. First aggregate $\widetilde{w}_{ij}$ to compute $\widetilde{w}_j$ and $\widetilde{W}$. Then by region $r_{ijt}^{(k)}$:

1. *Region I* - Firm is unconstrained:
$$n_{ij} = \left(\frac{w_{ij}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} \widetilde{W}^{\varphi}$$
2. *Region II* - Firm is constrained and n_{ij} determined by household labor supply curve at $\underline{w}$:
$$n_{ij} = \left(\frac{\underline{w}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} \widetilde{W}^{\varphi}$$
3. *Region III* - Firm is constrained and n_{ijt} determined by firm labor demand curve at $\underline{w}$:
$$\underline{w} = \alpha z_{ij} n_{ij}^{\alpha-1} \quad \Longrightarrow \quad n_{ij} = \left(\frac{\alpha z_{ij}}{\underline{w}}\right)^{\frac{1}{1-\alpha}}.$$

3. Update the multipliers: $p_{ij}^{(k)}$.

1. Aggregate n_{ij} to compute n_j and N.
2. Update p_{ij} from the *household's* first order conditions: $\widetilde{w}_{ij} = p_{ij} w_{ij}$
$$p_{ij}^{(k+1)} = \frac{\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}}{w_{ij}}$$

4. Update $\widetilde{W}^{(k)}$.

1. Compute $\widetilde{w}_{ij} = p_{ij}^{(k+1)} w_{ij}$
2. Use $\widetilde{w}_{ij}$ to update the aggregate shadow wage index to $\widetilde{W}^{(k+1)}$.

5. Update firm regions. For each region:.

1. Compute the marginal product of labor of all firms $mrpl_{ij} = \alpha z_{ij} n_{ij}^{\alpha-1}$.
2. If in market j there exists a firm in *Region I* with $w_{ij} < \underline{w}$, then move the firm with the lowest wage into *Region II*
3. If in market j there exists a firm that was initially in *Region II* and has a marginal product of labor that is less than marginal cost ($\underline{w}$), move that firm into *Region III*

Iterate over (1) to (5) until $p_{ij}^{(k+1)} = p_{ij}^{(k)}$ and $\widetilde{W}^{(k+1)} = \widetilde{W}^{(k)}$ and $r_{ij}^{(k+1)} = r_{ij}^{(k)}$.

G Mathematical details - Full quantitative model

- We first derive results for the competitive equilibrium, then the government's allocation problem. We then use results from the competitive equilibrium to prove that the solution to the government's allocation problem can be decentralized in a competitive equilibrium with revenue neutral lump sum taxes

G.1 Competitive equilibrium

G.1.1 Household problem - Labor supply system, shadow wages

- In the competitive equilibrium, household h solves the following problem:

$$\max_{c_{ht},n_{ht}} \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_{ht}/\pi_h)^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_h^{1/\varphi}} \frac{n_{ht}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]$$

where $\widetilde{\varphi}_h = \overline{\varphi}_h \pi_h^{1+\varphi}$ is adjusted for the measure of workers of the household,

$$n_{ht} = \left[\int n_{jht}^{\frac{\theta+1}{\theta}} dj \right]^{\frac{\theta}{\theta+1}} \quad , \quad n_{jht} = \left[\sum_{i \in j} n_{ijht}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}$$

subject to the budget constraint

$$c_{ht} + k_{ht+1} = \int \sum_{i \in j} w_{ijht} n_{ijht} dj + R_t k_{ht} + (1-\delta) k_{ht} + \kappa_h \Pi_t.$$

with the initial condition $k_{h0} = \kappa_h K_0$.

- Since we focus on steady-state we normalize the price of consumption to one.

- In the text we refer to these preferences as $u^h\left(\frac{c_{ht}}{\pi_h}, n_{ht}\right)$:

$$u^h\left(\frac{c_{ht}}{\pi_h}, n_{ht}\right) = \frac{(c_{ht}/\pi_h)^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_h^{1/\varphi}} \frac{n_{ht}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

- The household is also subject to the firm by firm rationing constraints: $n_{ijht} \leq \overline{n}_{ijht}$.

- Let $\beta^t \nu_{ht}$ be the multiplier on the household's budget constraint and write the multiplier on the rationing constraint as $\zeta_{ijht} = \beta^t \nu_{ht} w_{ijht} \left(1 - p_{ijht}\right)$.

- The the household's Lagrangean features the following terms in n_{ijht}

$$\mathcal{L} = \cdots + \beta^t u^h \left(\frac{c_{ht}}{\pi_h}, n_{ht}\right) + \cdots + \beta^t \nu_{ht} w_{ijht} n_{ijht} + \beta^t \nu_{ht} w_{ijht} \left(1 - p_{ijht}\right) \left[\overline{n}_{ijht} - n_{ijht}\right] + \ldots$$

$$\mathcal{L} = \cdots + u^h \left(\frac{c_{ht}}{\pi_h}, n_{ht}\right) + \cdots + \beta^t \nu_{ht} \left\{w_{ijht} p_{ijht}\right\} n_{ijht} + \beta^t \nu_{ht} w_{ijht} \left(1 - p_{ijht}\right) \overline{n}_{ijht} + \ldots$$

- The first order condition for consumption is

$$u_c^h \left(\frac{c_{ht}}{\pi_h}, n_{ht}\right) = \nu_{ht}$$

- The first order condition for labor supply is

$$\begin{aligned} \nu_{ht} w_{ijht} p_{ijht} &= -u_n^h \left(\frac{c_{ht}}{\pi_h}, n_{ht}\right) \frac{\partial n_{ht}}{\partial n_{jht}} \frac{\partial n_{jht}}{\partial n_{ijht}} \\ w_{ijht} p_{ijht} &= -\frac{u_n^h \left(\frac{c_{ht}}{\pi_h}, n_{ht}\right)}{u_c^h \left(\frac{c_{ht}}{\pi_h}, n_{ht}\right)} \left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1}{\theta}} \left(\frac{n_{ijht}}{n_{jht}}\right)^{\frac{1}{\eta}} \end{aligned}$$

- Define the *shadow wage* by $\widetilde{w}_{ijht} := w_{ijht} p_{ijht}$.

- Then

$$\widetilde{w}_{ijht} = -\frac{u_n^h \left(\frac{c_{ht}}{\pi_h}, n_{ht}\right)}{u_c^h \left(\frac{c_{ht}}{\pi_h}, n_{ht}\right)} \left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1}{\theta}} \left(\frac{n_{ijht}}{n_{jht}}\right)^{\frac{1}{\eta}}.$$

- Now define the following *shadow wage indexes*:

$$\widetilde{w}_{jht} = \left[\sum_{i \in j} \widetilde{w}_{ijht}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \widetilde{w}_{ht} = \left[\int \widetilde{w}_{jht}^{1+\theta} dj\right]^{\frac{1}{1+\eta}}$$

- Using this

$$
\begin{aligned}
\sum_{i\in j}\widetilde{w}_{ijht}^{1+\eta} &= \left[-\frac{u_n^h\left(\frac{c_{ht}}{\pi_h},n_{ht}\right)}{u_c^h\left(\frac{c_{ht}}{\pi_h},n_{ht}\right)}\left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1}{\theta}}\right]^{1+\eta}\sum_{i\in j}\left(\frac{n_{ijht}}{n_{jht}}\right)^{\frac{1+\eta}{\eta}} \\
\widetilde{w}_{jht} &= -\frac{u_n^h\left(\frac{c_{ht}}{\pi_h},n_{ht}\right)}{u_c^h\left(\frac{c_{ht}}{\pi_h},n_{ht}\right)}\left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1}{\theta}} \\
\widetilde{w}_{jht}^{1+\theta} &= \left[-\frac{u_n^h\left(\frac{c_{ht}}{\pi_h},n_{ht}\right)}{u_c^h\left(\frac{c_{ht}}{\pi_h},n_{ht}\right)}\right]\left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1+\theta}{\theta}} \\
\int\widetilde{w}_{jht}^{1+\theta}dj &= \left[-\frac{u_n^h\left(\frac{c_{ht}}{\pi_h},n_{ht}\right)}{u_c^h\left(\frac{c_{ht}}{\pi_h},n_{ht}\right)}\right]\int\left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1+\theta}{\theta}}dj \\
\widetilde{w}_{ht} &= -\frac{u_n^h\left(\frac{c_{ht}}{\pi_h},n_{ht}\right)}{u_c^h\left(\frac{c_{ht}}{\pi_h},n_{ht}\right)}
\end{aligned}
$$

- Using our form of preferences, this gives the household h labor supply curve:

$$
n_{ht} = \overline{\varphi}_h\pi_h\widetilde{w}_{ht}^{\varphi}\left(\frac{c_{ht}}{\pi_h}\right)^{-\varphi\sigma}
$$

- Using this we can show that shadow wages aggregate, as claimed in the text,
- First across markets:

$$
\begin{aligned}
\widetilde{w}_{ijht} &= \widetilde{w}_{ht}\left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1}{\theta}}\left(\frac{n_{ijht}}{n_{jht}}\right)^{\frac{1}{\eta}}. \\
\widetilde{w}_{ijht}^{1+\eta} &= \left[\widetilde{w}_{ht}\left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1}{\theta}}\right]^{1+\eta}\left(\frac{n_{ijht}}{n_{jht}}\right)^{\frac{1+\eta}{\eta}} \\
\left[\sum_{i\in j}\widetilde{w}_{ijht}^{1+\eta}\right]^{\frac{1}{1+\eta}} &= \widetilde{w}_{ht}\left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1}{\theta}} \\
\widetilde{w}_{jht} &= \widetilde{w}_{ht}\left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1}{\theta}} \\
\widetilde{w}_{jht}n_{jht} &= \widetilde{w}_{ht}n_{jht}\times\left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1+\theta}{\theta}} \\
\int\widetilde{w}_{jht}n_{jht}dj &= \widetilde{w}_{ht}n_{jht}
\end{aligned}
$$

- Then using these results, across firms within a market:

$$\begin{aligned}
\widetilde{w}_{ijht} &= \widetilde{w}_{ht}\left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1}{\theta}}\left(\frac{n_{ijht}}{n_{jht}}\right)^{\frac{1}{\eta}} \\
\widetilde{w}_{ijht} &= \widetilde{w}_{jht}\left(\frac{n_{ijht}}{n_{jht}}\right)^{\frac{1}{\eta}} \\
\widetilde{w}_{ijht}n_{ijht} &= \widetilde{w}_{jht}n_{jht} \times \left(\frac{n_{ijht}}{n_{jht}}\right)^{\frac{1+\eta}{\eta}} \\
\sum_{i\in j}\widetilde{w}_{ijht}n_{ijht} &= \widetilde{w}_{jht}n_{jht}
\end{aligned}$$

- Summarizing results so far, we have:

$$\begin{aligned}
\widetilde{w}_{ijht} &= \left(\frac{n_{ijht}}{n_{jht}}\right)^{\frac{1}{\eta}}\widetilde{w}_{jht} \\
\widetilde{w}_{jht} &= \left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1}{\theta}}\widetilde{w}_{ht} \\
\widetilde{w}_{ht}n_{jht} &= \int \widetilde{w}_{jht}n_{jht}dj \\
\widetilde{w}_{jht}n_{jht} &= \sum_{i\in j}\widetilde{w}_{ijht}n_{ijht}
\end{aligned}$$

- Note that these can be combined to give the entire labor supply system of household h in shadow wages:

$$\begin{aligned}
n_{ijht} &= \left(\frac{\widetilde{w}_{ijht}}{\widetilde{w}_{jht}}\right)^{\eta}\left(\frac{\widetilde{w}_{jht}}{\widetilde{w}_{ht}}\right)^{\theta} n_{ht} \\
n_{ht} &= \overline{\varphi}_h \pi_h^{1+\varphi\sigma}\widetilde{w}_{ht}^{\varphi}c_{ht}^{-\varphi\sigma}
\end{aligned}$$

- A hey result, used below, is that if the household received lump sum transfers T_h, then the same labor supply system would be obtained.

- Now consider our results regarding shadow shares. We define the *shadow share* as

$$\widetilde{s}_{ijht} := \frac{\widetilde{w}_{ijht}n_{ijht}}{\sum_{i\in j}\widetilde{w}_{ijht}n_{ijht}}.$$

- Using the above aggregation results, labor supply system, and definition of the aggregator n_{jht}:

$$\widetilde{s}_{ijht} = \frac{\widetilde{w}_{ijht} n_{ijht}}{\widetilde{w}_{jht} n_{jht}} = \left(\frac{\widetilde{w}_{ijht}}{\widetilde{w}_{jht}} \right)^{1+\eta} = \left(\frac{n_{ijht}}{n_{jht}} \right)^{\frac{1+\eta}{\eta}} = \frac{\partial \log n_{ijht}}{\partial \log n_{jht}}$$

which we use below in the firm optimality conditions.

G.1.2 Firm optimality

- **Simplifying the firm problem -** First we simplify the firm problem by separating it out across types and optimizing out capital for each type of worker:
- Consider the maximization problem of the firm in the text:

$$\pi_{ij} = \max_{\left\{ n_{ijh}, h_{ijh} \right\}_{h=1}^{h}} \overline{Z} z_{ij} \sum_{h=1}^{H} \left(\left[\xi_k n_{ijk} \right]^{\gamma} k_{ijk}^{1-\gamma} \right)^{\alpha} - R \sum_{h=1}^{H} k_{ijk} - \sum_{h=1}^{H} w_{ijh} n_{ijh}$$

  subject to the labor supply system and minimum wage constraints.

- First observe that this can separated out by type of worker h.
- The problem for type h labor at the firm is

$$\pi_{ijh} = \max_{n_{ijh}, k_{ijh}} \overline{Z} z_{ij} \left(\left[\xi_h n_{ijh} \right]^{\gamma} k_{ijh}^{1-\gamma} \right)^{\alpha} - R k_{ijh} - w_{ijh} n_{ijh}$$

- We first optimize out capital. This yields the objective function

$$\pi_{ijh} = \max_{n_{ijh}} \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_h n_{ijh}^{\widetilde{\alpha}} - w_{ijh} n_{ijh}$$

  where

$$\begin{aligned}
\widetilde{Z} &= \overline{Z}^{\frac{1}{1-(1-\gamma)\alpha}} \\
\widetilde{z}_{ij} &= \left[1 - (1-\gamma)\alpha \right] \left(\frac{(1-\gamma)\alpha}{R} \right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} z_{ij}^{\frac{1}{1-(1-\gamma)\alpha}} \\
\widetilde{\xi}_h &= \xi_h^{\widetilde{\alpha}} \\
\widetilde{\alpha} &= \frac{\gamma\alpha}{1-(1-\gamma)\alpha}
\end{aligned}$$

- We denote output net of capital expenses as $\widetilde{y}_{ijh} := \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_h n_{ijh}^{\widetilde{\alpha}}$.

- We can also define a market-level aggregate $\widetilde{y}_{jh} = \sum_{i\in j} \widetilde{y}_{ijh}$, and a type-level aggregate $\widetilde{y}_h = \int \widetilde{y}_{jh} dj$.

- Note that

$$y_{ijh} = \frac{\widetilde{y}_{ijh}}{1-(1-\gamma)\alpha} \quad , \quad y_{jh} = \frac{\widetilde{y}_{jh}}{1-(1-\gamma)\alpha} \quad , \quad y_h = \frac{\widetilde{y}_h}{1-(1-\gamma)\alpha}.$$

- Using the simplified problem we now consider optimality of the firm in each of the three regions described in the text.

- **Region I - Unconstrained**

  - Consider an unconstrained firm. Its problem is

$$\pi_{ijh} = \max_{n_{ijh}} \widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_h n_{ijh}^{\widetilde{\alpha}} - w_{ijh}n_{ijh}$$

    subject to its wage being given by the above labor supply system:

$$w\left(n_{ijht}\right) = \left(\frac{n_{ijht}}{n_{jht}}\right)^{\frac{1}{\eta}} \left(\frac{n_{jht}}{n_{ht}}\right)^{\frac{1}{\theta}} \widetilde{w}_{ht}.$$

  - The first order condition is

$$\begin{aligned}
w_{ijh} + w'\left(n_{ijh}\right)n_{ijh} &= \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_h n_{ijh}^{\widetilde{\alpha}-1} \\
w_{ijh}\left(1+\frac{w'\left(n_{ijh}\right)n_{ijh}}{w_{ijh}}\right) &= \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_h n_{ijh}^{\widetilde{\alpha}-1} \\
w_{ijh}\left(1+\frac{1}{\varepsilon_{ijh}}\right) &= \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_h n_{ijh}^{\widetilde{\alpha}-1} \\
w_{ijh} &= \frac{\varepsilon_{ijh}}{1+\varepsilon_{ijh}}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_h n_{ijh}^{\widetilde{\alpha}-1}
\end{aligned}$$

    where using the inverse labor supply curve gives

$$\begin{aligned}
\frac{1}{\varepsilon_{ijh}} &:= \frac{w'\left(n_{ijh}\right)n_{ijh}}{w_{ijh}} = \frac{\partial \log w_{ijh}}{\partial \log n_{ijh}} = \frac{1}{\eta} + \left(\frac{1}{\theta}-\frac{1}{\eta}\right)\frac{\partial \log n_{jh}}{\partial \log n_{ijh}} = \frac{1}{\eta} + \left(\frac{1}{\theta}-\frac{1}{\eta}\right)\widetilde{s}_{ijh} \\
\varepsilon_{ijh} &= \left[\frac{1}{\eta} + \left(\frac{1}{\theta}-\frac{1}{\eta}\right)\widetilde{s}_{ijh}\right]^{-1}.
\end{aligned}$$

– Therefore

$$w_{ijh} = \mu_{ijh}\widetilde{\alpha}\widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\widetilde{\xi}}_h n_{ijh}^{\widetilde{\alpha}-1}$$

where the markdown depends on the firms' elasticity of labor supply.

– Note that since $p_{ijh} = 1$ since the firm is unconstrained, then $\widetilde{w}_{ijh} = p_{ijh}w_{ijh} = w_{ijh}$, so

$$\widetilde{w}_{ijh} = \mu_{ijh} \times \widetilde{\alpha}\widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\widetilde{\xi}}_h n_{ijh}^{\widetilde{\alpha}-1}$$

- **Region III - Constrained, on labor demand curve**

  – Now consider a constrained firm in Region III, this firm's problem is

$$\pi_{ijh} = \max_{n_{ijh}} \widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\widetilde{\xi}}_h n_{ijh}^{\widetilde{\alpha}} - \underline{w}n_{ijh}$$

  – The solution to this problem is to choose employment to equate the marginal revenue product of labor to the minimum wage:

$$\underline{w} = \widetilde{\alpha}\widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\widetilde{\xi}}_h n_{ijh}^{\widetilde{\alpha}-1}$$

  – For convenience when aggregating, we can express this in terms of shadow wages by multiplying through by the equilibrium multiplier on the rationing constraint

$$\begin{aligned}\underline{w}p_{ijh} &= p_{ijh}\widetilde{\alpha}\widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\widetilde{\xi}}_h n_{ijh}^{\widetilde{\alpha}-1} \\ \widetilde{w}_{ijh} &= p_{ijh} \times \widetilde{\alpha}\widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\widetilde{\xi}}_h n_{ijh}^{\widetilde{\alpha}-1}\end{aligned}$$

- **Region II - Constrained, on labor supply curve**

  – Now consider a constrained firm in Region II, this firm simply has labor determined by the labor supply curve, but since the rationing constraint is slack, $\widetilde{w}_{ijh} = p_{ijh}w_{ijh} = \underline{w}$.

$$n_{ijh} = \left(\frac{\underline{w}}{\widetilde{w}_{jh}}\right)^{\eta}\left(\frac{\widetilde{w}_{jh}}{\widetilde{W}_h}\right)^{\theta} n_h.$$

– Nonetheless, we can express the shadow wage of the firm as

$$\widetilde{w}_{ijh} = \widetilde{\mu}_{ijh}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_h n_{ijh}^{\widetilde{\alpha}-1}$$
$$\widetilde{\mu}_{ijh} = \frac{w}{\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_h n_{ijh}^{\widetilde{\alpha}-1}} \quad , \quad n_{ijh} = \left(\frac{w}{\widetilde{w}_{jh}}\right)^{\eta}\left(\frac{\widetilde{w}_{jh}}{\widetilde{W}_h}\right)^{\theta} n_h.$$

– Therefore, in all three regions, we can express the *shadow wage* as a *shadow markdown* on the marginal revenue product of labor:

$$\widetilde{w}_{ijh} = \widetilde{\mu}_{ijh}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_h n_{ijh}^{\widetilde{\alpha}-1}.$$

G.1.3 Aggregation of output and labor demand conditions

- Using the above results for firm optimality and the household's labor supply system we can aggregate the optimality conditions of agents. This is a key step in solving the government problem and optimal transfers, which we describe below.

- **Aggregation - Firm-Type to Market-Type**

  – From the above we have the following set of five conditions at the firm and market level:

  – **Firm level**:

$$\widetilde{y}_{ijh} = \widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_h n_{ijh}^{\widetilde{\alpha}}$$
$$\widetilde{w}_{ijh} = \widetilde{\mu}_{ijh}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_h n_{ijh}^{\widetilde{\alpha}-1}.$$
$$n_{ijh} = \left(\frac{\widetilde{w}_{ijh}}{\widetilde{w}_{jh}}\right)^{\eta} n_{jh}$$

  – **Aggregates**:

$$\widetilde{y}_{jh} = \sum_{i\in j}\widetilde{y}_{ijh}$$
$$\widetilde{w}_{jh} = \left[\sum_{i\in j}\widetilde{w}_{ijh}^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

  – Following steps from Berger, Herkenhoff, Mongey (2022), these can be combined to yield:

$$\widetilde{y}_{jh} = \omega_{jh}\widetilde{Z}\widetilde{\xi}_h\widetilde{z}_j n_{jh}^{\widetilde{\alpha}}$$
$$\widetilde{w}_{jh} = \widetilde{\mu}_{jh}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_j\widetilde{\xi}_h n_{jh}^{\widetilde{\alpha}-1}$$
$$n_{jh} = \left(\frac{\widetilde{w}_{jh}}{\widetilde{w}_h}\right)^{\theta} n_h$$

where the three wedges $\left\{\widetilde{z}_j, \widetilde{\mu}_{jh}, \omega_{jh}\right\}$ are given by

$$\widetilde{z}_j = \left[\sum_{i\in j}\widetilde{z}_{ij}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}}$$
$$\widetilde{\mu}_{jh} = \left[\sum_{i\in j}\left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\widetilde{\mu}_{ijh}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}}$$
$$\omega_{jh} = \left[\sum_{i\in j}\left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\left(\frac{\widetilde{\mu}_{ijh}}{\widetilde{\mu}_{jh}}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}}$$

  - Note that this implies that if $\left\{\widetilde{z}_j, \widetilde{\mu}_{jh}, \widetilde{\omega}_{jh}\right\}$ are known, then $\left\{n_{jh}, \widetilde{w}_{jh}, \widetilde{y}_{jh}\right\}$ can be determined.

- **Aggregation - Market-Type to Type**

  - The same approach can be followed to aggregate to the household level, which delivers:

$$\widetilde{y}_h = \omega_h\widetilde{Z}\widetilde{\xi}_h\widetilde{z}_h n_h^{\widetilde{\alpha}}$$
$$\widetilde{w}_h = \widetilde{\mu}_h\widetilde{\alpha}\widetilde{Z}\widetilde{z}_h\widetilde{\xi}_h n_h^{\widetilde{\alpha}-1}$$

where

$$\widetilde{z}_h = \left[\int \widetilde{z}_j^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$

$$\widetilde{\mu}_h = \left[\int \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \widetilde{\mu}_{jh}^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$

$$\omega_h = \left[\int \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \left(\frac{\widetilde{\mu}_{jh}}{\widetilde{\mu}_h}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \omega_{jh}\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$

- The conditions derived thus far all hold in a competitive equilibrium with lump sum transfers.
- In a competitive equilibrium, the above conditions are satisfied and budget constraints clear for each household.

I Mathematical details - Tax and Transfer System

In this appendix, we derive the household labor supply curve presented in Section 6.

Household. We adopt the Benabou (2003) and Heathcote et al (2017) tax and transfer system. We assume that after-tax income is a log-linear function of pre-tax income, where τ denotes the marginal tax rate and λ determines the level of transfers. We assume labor income per worker is w_{ij}, there are n_{ij} workers in the stand-in household, and thus after-tax income is $\left(\lambda w_{ij}^{1-\tau}\right) n_{ij}$. We present the economy with homogeneous workers, linear preferences over consumption, and omitting capital to minimize clutter. The household solves the following problem:

$$\max_{\{n_i\}_{i\in[0,1]}} U\left(C,N\right) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{1}{\overline{\varphi}^{1/\varphi}}\frac{N^{1+1/\varphi}}{1+1/\varphi}$$

$$N = \left[\int_j n_j^{\frac{1+\theta}{\theta}} dj\right]^{\frac{\theta}{1+\theta}} \qquad n_j = \left[\sum_i n_{ij}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$$

subject to

$$C \leq \int \sum_i \left(\lambda w_{ij}^{1-\tau}\right) n_{ij} dj + \Pi \qquad , \qquad n_{ij} \leq \overline{n}_{ij}.$$

The corresponding Lagrangian is given by,

$$\mathcal{L} = \left[\frac{C^{1-\sigma}}{1-\sigma} - \frac{1}{\overline{\varphi}^{1/\varphi}}\frac{N^{1+1/\varphi}}{1+1/\varphi}\right] + \Lambda\left[\int_j \sum_i \left(\lambda w_{ij}^{1-\tau}\right) n_{ij} dj + \Pi - C\right] + \int_j \sum_i \phi_{ij}\left[\overline{n}_{ij} - n_{ij}\right] dj.$$

We then rewrite the *normalized* multiplier as follows:

$$\phi_{ij} = \left(1 - p_{ij}^{1-\tau}\right) \Lambda \lambda w_{ij}^{1-\tau}.$$

After substituting for ϕ_{ij}, collecting terms, and letting let $\widetilde{w}_{ij} := w_{ij} p_{ij}$, we have the following Lagrangian:

$$\mathcal{L} = \left[\frac{C^{1-\sigma}}{1-\sigma} - \frac{1}{\overline{\varphi}^{1/\varphi}}\frac{N^{1+1/\varphi}}{1+1/\varphi}\right] + \Lambda\left[\int_j \sum_i \lambda \widetilde{w}_{ij}^{1-\tau} n_{ij} dj + \Pi - C\right] + \int_j \sum_i \left(1 - p_{ij}^{1-\tau}\right) \Lambda \lambda w_{ij}^{1-\tau} \overline{n}_{ij} dj.$$

The FOC for consumption yields

$$U_C = \Lambda.$$

The FOC for n_{ij} yields

$$-U_N \frac{\partial N}{\partial n_j}\frac{\partial n_j}{\partial n_{ij}} = U_C \lambda \widetilde{w}_{ij}^{1-\tau}$$

We define $\widetilde{w}_j$ and $\widetilde{W}$ such that

$$\lambda \widetilde{w}_j^{1-\tau} n_j = \sum_{i\in j} \lambda \widetilde{w}_{ij}^{1-\tau} n_{ij} \qquad , \qquad \lambda \widetilde{W}^{1-\tau} N = \sum_j \lambda \widetilde{w}_j^{1-\tau} n_j$$

Aggregating to the household-level the first order condition for n_{ij},we can then solve for the labor supply curve of representative household

$$-\frac{U_N N}{U_C} = \lambda \widetilde{W}^{1-\tau} N.$$

Then substituting this back into our first order condition for n_{ij} and aggregating to the market-level yields:

$$n_j = \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta(1-\tau)} N$$

Substituting back into n_{ij}yields the inverse labor supply curve:

$$\widetilde{w}_{ij} = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{(1-\tau)\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{(1-\tau)\theta}} \widetilde{W}$$

Inversion yields the labor supply curve shown in the text:

$$n_{ij} = \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{(1-\tau)\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta(1-\tau)} N$$

Firm. Abstracting from capital, the firm problem is to choose n_{ij},and the rationing constraint $\overline{n}_{ij}$ to maximize profits

$$\pi_{ij} = \max_{n_{ij},\overline{n}_{ij}} z_{ij} n_{ij}^{\alpha} - w\left(n_{ij}\right) n_{ij},$$

where $w\left(n_{ij}\right)$ is the household labor supply curve under HSV taxes, $w\left(n_{ij}\right) \geq \underline{w}$, and $n_{ij} \leq \overline{n}_{ij}$.

Characterization. Dividing the firm problem into three regions, we arrive at a similar characterization of wage setting as our baseline economy.

Region I wages and allocations:

$$w_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{ij}+1}\alpha z_{ij} n_{ij}^{\alpha-1}, \qquad \varepsilon_{ij}^{-1} = \frac{1}{1-\tau}\left[\frac{1}{\eta} + \left(\frac{1}{\theta}-\frac{1}{\eta}\right)\left(\frac{w_{ij}}{\widetilde{w}_j}\right)^{(1+\eta)(1-\tau)}\right],$$

$$n_{ij} = \left(\frac{w_{ij}}{\widetilde{w}_j}\right)^{(1-\tau)\eta}\left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta(1-\tau)} N, \qquad p_{ij} = 1$$

Region II wages and allocations:

$$w_{ij} = \underline{w}, \qquad n_{ij} = \left(\frac{\underline{w}}{\widetilde{w}_j}\right)^{(1-\tau)\eta}\left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta(1-\tau)} N, \qquad p_{ij} = 1$$

Region III wages and allocations:

$$w_{ij} = \underline{w}, \qquad n_{ij} = \left(\frac{\alpha z_{ij}}{\underline{w}}\right)^{1/(1-\alpha)}, \qquad p_{ij} = \frac{\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{(1-\tau)\eta}}\left(\frac{n_j}{N}\right)^{\frac{1}{(1-\tau)\theta}}\widetilde{W}}{\underline{w}}$$

where the wage indeces are given by,

$$\widetilde{w}_{ij} = p_{ij}w_{ij}, \qquad \widetilde{w}_j = \left[\sum_{i\in j}\widetilde{w}_{ij}^{(1+\eta)(1-\tau)}\right]^{\frac{1}{(1+\eta)(1-\tau)}}, \qquad \widetilde{W} = \left[\sum_{j}\widetilde{w}_{j}^{(1+\eta)(1-\tau)}\right]^{\frac{1}{(1+\eta)(1-\tau)}}.$$

The solution algorithm proceeds in an identical manner to our baseline economy.

I Optimal Tax and Transfer System

We thank an anonymous referee for suggesting this exercise. We compute the total potential welfare gains from changes in the tax and transfer policy in an economy with $\underline{w} = 0$. We limit our exercise to optimizing the Heathcote, Storesletten, and Violante (2017) (henceforth, HSV) tax function. Mechanically, we proceeds as follows:

1. The tax policy consists of three parameters: τ progressivity, λ the shifter on the tax function which determines the point at which it goes from a subsidy to a tax, and g the implied share of output that is accounted for by the net government spending position of the tax and transfer system.

2. We choose (τ, λ) to match the data and recalibrated the parameters of the economy

Figure I1: Optimal Progressivity

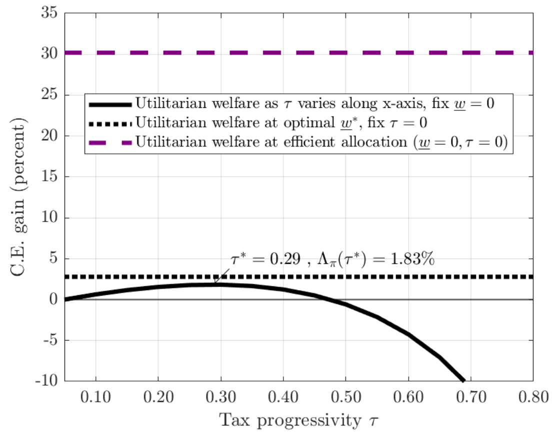
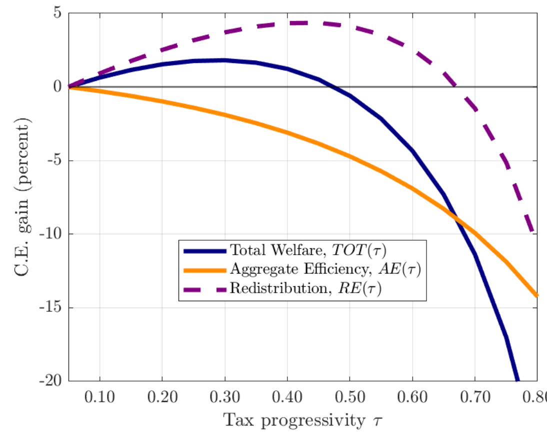


Figure I2: Efficiency and Redistribution



so that we match the same wage moments *pre-tax*, and backed out the implied g, which was small and positive.

3. We then *adjust the progressivity of the tax system*. We consider alternative values of $\tau' \in [0.05, 0.80]$, computing for each τ' the required change in λ' to deliver the same balance g. In this case, raising τ raises more taxes on high income workers, which allows for an expansion of the cut-off of the policy and expansion of the maximum transfer.

Results. Figure I1 summarizes our main result. The optimal degree of progressivity and subsidy/tax cut-off are $\tau^* = 0.29$ and $\lambda^* = 2.39$. This is more progressivity than the empirical baseline of $\tau = 0.18$, and a larger threshold for receipt of a net subsidy than the empirical baseline of $\lambda = 1.74$. The overall welfare gain relative to the empirical (τ, λ) is 1.83% to the Utilitarian planner. This remains small relative to the gains an unrestricted planner would achieve of 30.2%. Relative to the baseline (τ, λ) we find that the gains from (τ^*, λ^*) decompose into roughly a 4% *Redistribution* gain and 3% *Efficiency* loss. See Figure I2.

Discussion. Why are the welfare gains from optimizing the tax policy parameters—subject to the fiscal position of the government—small relative to the welfare gains a planner could achieve? There are two main reasons. First, the empirical distribution of consumption, labor, labor income and non-labor income in the economy—which the model is calibrated to— is far from what a Utilitarian planner would choose. For example, 33 per-

Figure I3: C_h for $\tau = 0.18$ vs $\tau^* = 0.29$

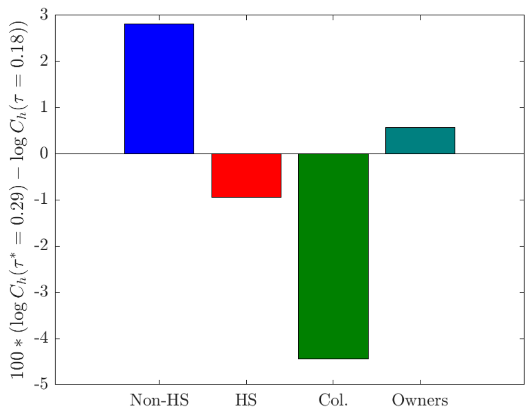
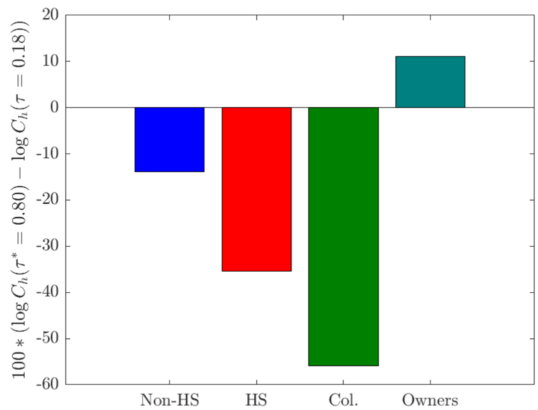


Figure I4: C_h for $\tau = 0.18$ vs $\tau' = 0.80$



cent of households have a college degree, yet they account for 72 percent of consumption. A tax and transfer system that maximizes a Utilitarian objective would involve massive transfers away from these households. Unrestricted, subsidies are going to look nothing like tax policy that we see in the data.

Second, a higher τ widens markdowns as firms internalize it being more expensive to hire labor on the margin. This can then generate efficiency losses. For $\tau = 0.29$, there is redistribution toward lower income households which would be the 'intended' effect of the policy (see Figure I3). As τ increases further, the 'unintended consequences' kick in limiting the redistributive gains. In Figure I4, we consider $\tau = 0.80$. At this level markdowns are wider and profits are flowing to the highest income business owners. Hence on top of the lower efficiency, redistributive gains are now undone.

Caveats. First, we don't know what the correct social welfare weights are and results depend critically on welfare weights.[52] Second, and related, the U.S. equilibrium—which we calibrate our model to—is inconsistent with an allocation of resources that a social planner with Utilitarian weights would choose. To maximize the *redistribution component* of welfare, requires massively increasing the consumption of the very large mass of high-school and non-high-school educated households in the economy that consume very little. An EITC/HSV tax function is not going to achieve this, only a massive overhaul of the tax and transfer system. Third, the tax and transfer system is not just about redistribution, but

[52]That the competitive equilibrium is so far from the Utilitarian optimal should give us some pause that Utilitarian weights are not the right ones for policy analysis, but we don't know what the correct ones are either.

Figure J1: Lower elasticities $(\eta', \theta') = (0.7 \times \eta, 0.7 \times \theta)$

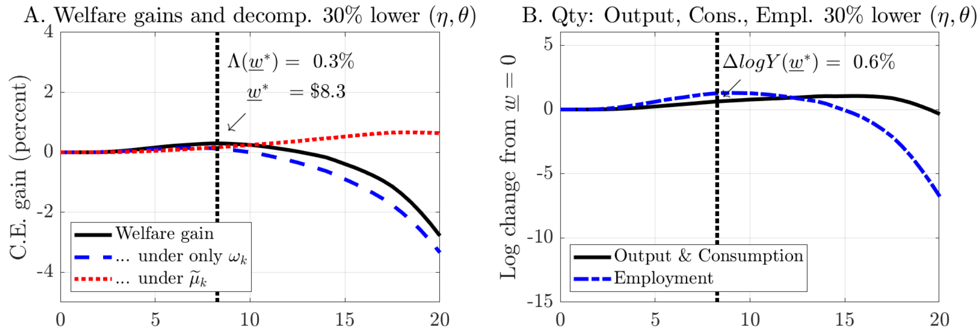
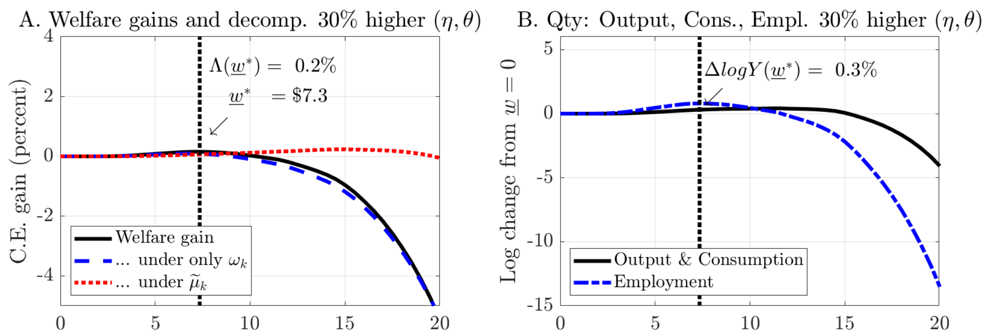


Figure J2: Higher elasticities $(\eta', \theta') = (1.3 \times \eta, 1.3 \times \theta)$



also insurance. Heathcote, Storesletten, and Violante (2017) show persuasively that in an economy with idiosyncratic risk, the optimal degree of progressivity—in a formulation of taxes that we use in this paper—increases welfare primarily through providing insurance. Our economy does not have such idiosyncratic risk, and hence abstracts from this key benefit.

J Robustness to η and θ

Higher (Figure J2) and lower (Figure J1) values of labor supply elasticities do very little to the magnitude of efficiency gains. Decreasing (Increasing) elasticities by 30% increases (decreases) the optimal minimum wage by 70c (30c), and leaves welfare gains almost unchanged. The intuition is that firm productivity dispersion limits the effectiveness of minimum wages, even when markdowns are greater at smaller firms (see Figure J1).