SUPPLEMENTAL MATERIALS

The indexing of the Supplemental Material Appendix follows on from the Online Appendix. This additional appendix is organized as follows: Section F details the algorithm for solving the minimum wage economy. Section G contains mathematical derivations for the full quantitative model from Section 1.2 and derivations associated with the solution of the government problem and its implementation via lump sum transfers. Section H analyzes a search model with frictional rationing that closely resembles our model.

F Algorithm for the minimum wage economy

The aim of this section is to clearly lay out the algorithm for solving the minimum wage equilibrium, and to present a full solultion of a simplified model, which may be pedagogically useful relative to the extensive derivations in Appendix D. The algorithm for the minimum wage equilibrium is nested in the broader solution to the equilibrium of the model described in Appendix G.

For ease of exposition, we lay out the minimum wage problem (i) ignoring capital, (ii) consider an economy with a single type of household, (iii) to simplify exposition we also consider GHH preferences, which are not used in the main text, (iv) as well as a static environment, (v) set the coefficient on labor in utility $\overline{\varphi} = 1$. We derive conditions for this simplified economy and then present the algorithm.

F.1 Model

- Consider the household problem with the rationing constraint $n_{ij} \leq \overline{n}_{ij}$. For ease of interpretation we attach multiplier $\zeta_{ij} = \lambda w_{ij}\left(1 - p_{ij}\right)$ to the rationing constraint, normalized by the household budget multiplier λ:

$$U_0 = \max_{\left\{n_{ij}, c_{ij}\right\}} u\left(C - \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)$$

$$\begin{aligned}
C &= \int \sum_{i\in j} w_{ij} n_{ij}\, dj + \Pi & & [\lambda] \\
n_{ij} &\leq \overline{n}_{ij} & & \left[\lambda w_{ij}\left(1 - p_{ij}\right)\right] \\
C &= \int \sum_{i\in j} c_{ij} dj \\
N &= \left[\int n_j^{\frac{\theta+1}{\theta}}\, dj\right]^{\frac{\theta}{\theta+1}} \\
n_j &= \left[\sum_{i\in j} n_j^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}
\end{aligned}$$

- The first order condition for n_{ij} yields

$$\lambda w_{ij} - \lambda w_{ij}\left(1-p_{ij}\right) = u'(\cdot)\left(\frac{\partial n_j}{\partial n_{ij}}\right)\left(\frac{\partial N}{\partial n_j}\right) N^{\frac{1}{\varphi}}$$
$$\lambda w_{ij} p_{ij} = u'(\cdot)\left(\frac{\partial n_j}{\partial n_{ij}}\right)\left(\frac{\partial N}{\partial n_j}\right) N^{\frac{1}{\varphi}}$$

- The first order condition for consumption yields $u'(\cdot) = \lambda$.
- Define the *shadow wage* $\widetilde{w}_{ij} = p_{ij} w_{ij}$, use the first order condition for consumption $u'(\cdot) = \lambda$, and use the derivatives of N and n_j:

$$\widetilde{w}_{ij} = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \qquad (*)$$

- Now define the *shadow wage indexes*

$$\widetilde{w}_j = \left[\sum_{i\in j} \widetilde{w}_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \widetilde{W} = \left[\int \widetilde{w}_j^{1+\theta} dj\right]^{\frac{1}{1+\theta}} .$$

- Using these definitions in $(*)$ along with the definition of n_j:

$$\sum_{i\in j} \widetilde{w}_{ij}^{1+\eta} = \left[\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right]^{1+\eta} \sum_{i\in j} \left(\frac{n_{ij}}{n_j}\right)^{\frac{1+\eta}{\eta}}$$
$$\widetilde{w}_j = \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$$

- Using this along with the definition of N:

$$\int \widetilde{w}_j^{1+\theta} dj = \left[N^{\frac{1}{\varphi}}\right]^{1+\theta} \int \left(\frac{n_j}{N}\right)^{\frac{1+\theta}{\theta}} dj$$
$$\widetilde{W} = N^{\frac{1}{\varphi}}$$

- Note that $\widetilde{W}N \neq \int \sum_{i\in j} w_{ij} n_{ij} dj$, however the aggregate labor supply $N = \widetilde{W}^{\varphi}$ is *as if*, the household had maximized

$$U_0 = \max_{C,N} u\left(C - \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right) \quad \text{subject to} \quad C = \widetilde{W}N + \Pi.$$

  This makes clear the extent to which the shadow wage index $\widetilde{W}$ captures the full distribution of binding minimum wages.

- Note that shadow wages aggregate:

$$\widetilde{w}_{ij} n_{ij} = n_{ij}^{\frac{1+\eta}{\eta}} \left(\frac{1}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$$

$$\sum_{i\in j} \widetilde{w}_{ij} n_{ij} = \left[\sum_{i\in j} n_{ij}^{\frac{1+\eta}{\eta}}\right] \left(\frac{1}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$$

$$\sum_{i\in j} \widetilde{w}_{ij} n_{ij} = n_j \widetilde{w}_j$$

- **Shadow shares** - We can define the shadow share $\widetilde{s}_{ij}$ as

$$\widetilde{s}_{ij} := \frac{\widetilde{w}_{ij} n_{ij}}{\sum_{i\in j} \widetilde{w}_{ij} n_{ij}}.$$

  Substituting in the labor supply system $(*)$ for $\widetilde{w}_{ij}$

$$\widetilde{s}_{ij} := \frac{n_{ij}^{\frac{1+\eta}{\eta}}}{\sum_{i\in j} n_{ij}^{\frac{1+\eta}{\eta}}} = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1+\eta}{\eta}} = \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{1+\eta}$$

- The firm's problem is

$$\pi_{ij} = \max_{n_{ij}} z_{ij} n_{ij}^{\alpha} - w_{ij} n_{ij}$$

  subject to

$$n_{ij} = \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} N$$

$$w_{ij} \geq \underline{w}$$

- Let $r_{ij} \in \{1, 2, 3\}$ denote the region that the firm is in.
- **Region I -** If the firm is in Region I, then its wage is the optimal markdown on the marginal revenue product of labor

$$w_{ij} = \mu_{ij} \alpha z_{ij} n_{ij}^{\alpha-1}$$

$$p_{ij} = 1$$

$$\widetilde{w}_{ij} = w_{ij}$$

$$n_{ij} = \left(\frac{w_{ij}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} \widetilde{W}^{\varphi}$$

  where the markdown depends on its shadow share of the labor market. That is, $\mu_{ij} = \mu\left(\widetilde{s}_{ij}\right)$, where

$\mu\left(\widetilde{s}_{ij}\right)=\frac{\varepsilon\left(\widetilde{s}_{ij}\right)}{\varepsilon\left(\widetilde{s}_{ij}\right)+1}$. We have shown that

$$\widetilde{s}_{ij}=\left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{1+\eta} \quad \Longrightarrow \widetilde{w}_j=\widetilde{w}_{ij}\widetilde{s}_{ij}^{-\frac{1}{1+\eta}}$$

Using these, we can write:

$$w_{ij}=\left[\mu\left(\widetilde{s}_{ij}\right)\alpha z_{ij}\widetilde{s}_{ij}^{-\frac{(1-\alpha)(\eta-\theta)}{1+\eta}}\widetilde{W}^{(1-\alpha)(\theta-\varphi)}\right]^{\frac{1}{1+\theta(1-\alpha)}}$$

- **Region II -** In Region II, then

$$\begin{aligned} w_{ij} &= \underline{w} \\ p_{ij} &= 1 \\ \widetilde{w}_{ij} &= \underline{w} \\ n_{ij} &= \left(\frac{\underline{w}}{\widetilde{w}_j}\right)^{\eta}\left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} N \end{aligned}$$

- **Region III -** In Region III, then

$$\begin{aligned} w_{ij} &= \alpha z_{ij} n_{ij}^{\alpha-1} \\ p_{ij} &< 1 \\ \widetilde{w}_{ij} &= p_{ij}\underline{w} \\ n_{ij} &= \left(\frac{p_{ij}\underline{w}}{\widetilde{w}_j}\right)^{\eta}\left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} N \end{aligned}$$

F.2 Minimum wage solution algorithm

We implement the following solution algorithm. We denote the *Region* that a firm is in by $r_{ijt} \in \{I, II, III\}$. Initialize the algorithm by (i) guessing a value for $\widetilde{W}^{(0)}$, (ii) assuming all firms are in *Region I*, $r_{ij}^{(0)}=I$, which implies guessing $p_{ij}^{(0)}=1$. These will all be updated in the algorithm.

1. **Solve all market equilibria in shadow shares**

   (a) Guess shadow shares $\widetilde{s}_{ij}^{(0)}$.

   (b) *Region I* - Using the above optimality condition

$$w_{ij}=\left[\mu\left(\widetilde{s}_{ij}\right)\alpha z_{ij}\widetilde{s}_{ij}^{(0)-\frac{(1-\alpha)(\eta-\theta)}{1+\eta}}\widetilde{W}^{(0)(1-\alpha)(\theta-\varphi)}\right]^{\frac{1}{1+\theta(1-\alpha)}}$$

   (c) *Regions II, III* - Here the minimum wage is binding so set $w_{ij}=\underline{w}$.

(d) Given the guess $p_{ij}^{(k)}$ and w_{ij}, compute the shadow wage: $\widetilde{w}_{ij} = p_{ij} w_{ij}$.

(e) With all shadow wages in hand, update shadow shares using $\widetilde{w}_{ijt}$:

$$\widetilde{s}_{ij}^{(l+1)} = \frac{\widetilde{w}_{ij}^{1+\eta}}{\sum_{i \in j} \widetilde{w}_{ij}^{1+\eta}}.$$

(f) Iterate over (b)-(e) until shadow shares converge: $\widetilde{s}_{ij}^{(l+1)} = \widetilde{s}_{ij}^{(l)}$.

2. **Recover employment** - Here we use the wages from the previous step plus the current guess of each firms' region. First aggregate $\widetilde{w}_{ij}$ to compute $\widetilde{w}_j$ and $\widetilde{W}$. Then by region $r_{ijt}^{(k)}$:

   (a) *Region I* - Firm is unconstrained:

   $$n_{ij} = \left(\frac{w_{ij}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} \widetilde{W}^{\varphi}$$

   (b) *Region II* - Firm is constrained and n_{ij} determined by household labor supply curve at $\underline{w}$:

   $$n_{ij} = \left(\frac{\underline{w}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} \widetilde{W}^{\varphi}$$

   (c) *Region III* - Firm is constrained and n_{ijt} determined by firm labor demand curve at $\underline{w}$:

   $$\underline{w} = \alpha z_{ij} n_{ij}^{\alpha-1} \quad \Longrightarrow \quad n_{ij} = \left(\frac{\alpha z_{ij}}{\underline{w}}\right)^{\frac{1}{1-\alpha}}.$$

3. Update the multipliers: $p_{ij}^{(k)}$

   (a) Aggregate n_{ij} to compute n_j and N.

   (b) Update p_{ij} from the *household's* first order conditions: $\widetilde{w}_{ij} = p_{ij} w_{ij}$

   $$p_{ij}^{(k+1)} = \frac{\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}}{w_{ij}}$$

4. Update $\widetilde{W}^{(k)}$:

   (a) Compute $\widetilde{w}_{ij} = p_{ij}^{(k+1)} w_{ij}$

   (b) Use $\widetilde{w}_{ij}$ to update the aggregate shadow wage index to $\widetilde{W}^{(k+1)}$.

5. Update firm regions. For each region:

   (a) Compute the marginal product of labor of all firms $mrpl_{ij} = \alpha z_{ij} n_{ij}^{\alpha-1}$.

(b) If in market j there exists a firm in *Region I* with $w_{ij} < \underline{w}$, then move the firm with the lowest wage into *Region II*.

(c) If in market j there exists a firm that was initially in *Region II* and has a marginal product of labor that is less than marginal cost ($\underline{w}$), move that firm into *Region III*.

6. Iterate over (1) to (5) until $p_{ij}^{(k+1)} = p_{ij}^{(k)}$ and $\widetilde{W}^{(k+1)} = \widetilde{W}^{(k)}$ and $r_{ij}^{(k+1)} = r_{ij}^{(k)}$.

G Mathematical details - Full quantitative model

- We first derive results for the competitive equilibrium, then the government's allocation problem. We then use results from the competitive equilibrium to prove that the solution to the government's allocation problem can be decentralized in a competitive equilibrium with revenue neutral lump sum taxes

G.1 Competitive equilibrium

G.1.1 Household problem - Labor supply system, shadow wages

- In the competitive equilibrium, household k solves the following problem:

$$\max_{c_{kt}, h_{kt}} \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_{kt}/\pi_k)^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{n_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]$$

where $\widetilde{\varphi}_k = \overline{\varphi}_k \pi_k^{1+\varphi}$ is adjusted for the measure of workers of the household,

$$n_{kt} = \left[\int n_{jkt}^{\frac{\theta+1}{\theta}} dj \right]^{\frac{\theta}{\theta+1}} \quad , \quad n_{jkt} = \left[\sum_{i \in j} n_{ijkt}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}$$

subject to the budget constraint

$$c_{kt} + k_{kt+1} = \int \sum_{i \in j} w_{ijkt} n_{ijkt} dj + R_t k_{kt} + (1-\delta) k_{kt} + \kappa_k \Pi_t.$$

with the initial condition $k_{k0} = \kappa_k K_0$.

- Since we focus on steady-state we normalize the price of consumption to one.
- In the text we refer to these preferences as $u^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)$:

$$u^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right) = \frac{(c_{kt}/\pi_k)^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{n_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

- The household is also subject to the firm by firm rationing constraints: $n_{ijkt} \leq \overline{n}_{ijkt}$.
- Let $\beta^t \nu_{kt}$ be the multiplier on the household's budget constraint and write the multiplier on the ratioining constraint as $\zeta_{ijkt} = \beta^t \nu_{kt} w_{ijkt} \left(1 - p_{ijkt}\right)$.

- The the household's Lagrangean features the following terms in n_{ijkt}

$$\mathcal{L} = \cdots + \beta^t u^k \left(\frac{c_{kt}}{\pi_k}, n_{kt}\right) + \cdots + \beta^t \nu_{kt} w_{ijkt} n_{ijkt} + \beta^t \nu_{kt} w_{ijkt} \left(1 - p_{ijkt}\right) \left[\overline{n}_{ijkt} - n_{ijkt}\right] + \ldots$$
$$\mathcal{L} = \cdots + u^k \left(\frac{c_{kt}}{\pi_k}, n_{kt}\right) + \cdots + \beta^t \nu_{kt} \left\{w_{ijkt} p_{ijkt}\right\} n_{ijkt} + \beta^t \nu_{kt} w_{ijkt} \left(1 - p_{ijkt}\right) \overline{n}_{ijkt} + \ldots$$

- The first order condition for consumption is

$$u_c^k \left(\frac{c_{kt}}{\pi_k}, n_{kt}\right) = \nu_{kt}$$

- The first order condition for labor supply is

$$\nu_{kt} w_{ijkt} p_{ijkt} = -u_n^k \left(\frac{c_{kt}}{\pi_k}, n_{kt}\right) \frac{\partial n_{kt}}{\partial n_{jkt}} \frac{\partial n_{jkt}}{\partial n_{ijkt}}$$
$$w_{ijkt} p_{ijkt} = -\frac{u_n^k \left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)}{u_c^k \left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}}$$

- Define the *shadow wage* by $\widetilde{w}_{ijkt} := w_{ijkt} p_{ijkt}$.
- Then

$$\widetilde{w}_{ijkt} = -\frac{u_n^k \left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)}{u_c^k \left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}}.$$

- Now define the following *shadow wage indexes*:

$$\widetilde{w}_{jkt} = \left[\sum_{i \in j} \widetilde{w}_{ijkt}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \widetilde{w}_{kt} = \left[\int \widetilde{w}_{jkt}^{1+\theta} dj\right]^{\frac{1}{1+\eta}}$$

- Using this

$$\sum_{i\in j}\widetilde{w}_{ijkt}^{1+\eta} = \left[-\frac{u_n^k\left(\frac{c_{kt}}{\pi_k},n_{kt}\right)}{u_c^k\left(\frac{c_{kt}}{\pi_k},n_{kt}\right)}\left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}\right]^{1+\eta}\sum_{i\in j}\left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1+\eta}{\eta}}$$

$$\widetilde{w}_{jkt} = -\frac{u_n^k\left(\frac{c_{kt}}{\pi_k},n_{kt}\right)}{u_c^k\left(\frac{c_{kt}}{\pi_k},n_{kt}\right)}\left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}$$

$$\widetilde{w}_{jkt}^{1+\theta} = \left[-\frac{u_n^k\left(\frac{c_{kt}}{\pi_k},n_{kt}\right)}{u_c^k\left(\frac{c_{kt}}{\pi_k},n_{kt}\right)}\right]\left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1+\theta}{\theta}}$$

$$\int\widetilde{w}_{jkt}^{1+\theta}dj = \left[-\frac{u_n^k\left(\frac{c_{kt}}{\pi_k},n_{kt}\right)}{u_c^k\left(\frac{c_{kt}}{\pi_k},n_{kt}\right)}\right]\int\left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1+\theta}{\theta}}dj$$

$$\widetilde{w}_{kt} = -\frac{u_n^k\left(\frac{c_{kt}}{\pi_k},n_{kt}\right)}{u_c^k\left(\frac{c_{kt}}{\pi_k},n_{kt}\right)}$$

- Using our form of preferences, this gives the household k labor supply curve:

$$n_{kt} = \overline{\varphi}_k\pi_k\widetilde{w}_{kt}^{\varphi}\left(\frac{c_{kt}}{\pi_k}\right)^{-\varphi\sigma}$$

- Using this we can show that shadow wages aggregate, as claimed in the text,
- First across markets:

$$\widetilde{w}_{ijkt} = \widetilde{w}_{kt}\left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}\left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}}.$$

$$\widetilde{w}_{ijkt}^{1+\eta} = \left[\widetilde{w}_{kt}\left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}\right]^{1+\eta}\left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1+\eta}{\eta}}$$

$$\left[\sum_{i\in j}\widetilde{w}_{ijkt}^{1+\eta}\right]^{\frac{1}{1+\eta}} = \widetilde{w}_{kt}\left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}$$

$$\widetilde{w}_{jkt} = \widetilde{w}_{kt}\left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}$$

$$\widetilde{w}_{jkt}n_{jkt} = \widetilde{w}_{kt}n_{jkt}\times\left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1+\theta}{\theta}}$$

$$\int\widetilde{w}_{jkt}n_{jkt}dj = \widetilde{w}_{kt}n_{jkt}$$

- Then using these results, across firms within a market:

$$\widetilde{w}_{ijkt} = \widetilde{w}_{kt} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}}$$

$$\widetilde{w}_{ijkt} = \widetilde{w}_{jkt} \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}}$$

$$\widetilde{w}_{ijkt} n_{ijkt} = \widetilde{w}_{jkt} n_{jkt} \times \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1+\eta}{\eta}}$$

$$\sum_{i \in j} \widetilde{w}_{ijkt} n_{ijkt} = \widetilde{w}_{jkt} n_{jkt}$$

- Summarizing results so far, we have:

$$\widetilde{w}_{ijkt} = \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}} \widetilde{w}_{jkt}$$

$$\widetilde{w}_{jkt} = \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \widetilde{w}_{kt}$$

$$\widetilde{w}_{kt} n_{jkt} = \int \widetilde{w}_{jkt} n_{jkt} dj$$

$$\widetilde{w}_{jkt} n_{jkt} = \sum_{i \in j} \widetilde{w}_{ijkt} n_{ijkt}$$

- Note that these can be combined to give the entire labor supply system of household k in shadow wages:

$$n_{ijkt} = \left(\frac{\widetilde{w}_{ijkt}}{\widetilde{w}_{jkt}}\right)^{\eta} \left(\frac{\widetilde{w}_{jkt}}{\widetilde{w}_{kt}}\right)^{\theta} n_{kt}$$

$$n_{kt} = \overline{\varphi}_k \pi_k^{1+\varphi\sigma} \widetilde{w}_{kt}^{\varphi} c_{kt}^{-\varphi\sigma}$$

- A key result, used below, is that if the household received lump sum transfers T_k, then the same labor supply system would be obtained.
- Now consider our results regarding shadow shares. We define the *shadow share* as

$$\widetilde{s}_{ijkt} := \frac{\widetilde{w}_{ijkt} n_{ijkt}}{\sum_{i \in j} \widetilde{w}_{ijkt} n_{ijkt}}.$$

- Using the above aggregation results, labor supply system, and definition of the aggregator n_{jkt}:

$$\widetilde{s}_{ijkt} = \frac{\widetilde{w}_{ijkt} n_{ijkt}}{\widetilde{w}_{jkt} n_{jkt}} = \left(\frac{\widetilde{w}_{ijkt}}{\widetilde{w}_{jkt}}\right)^{1+\eta} = \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1+\eta}{\eta}} = \frac{\partial \log n_{ijkt}}{\partial \log n_{jkt}}$$

which we use below in the firm optimality conditions.

G.1.2 Firm optimality

- **Simplifying the firm problem -** First we simplify the firm problem by separating it out across types and optimizing out capital for each type of worker:
- Consider the maximization problem of the firm in the text:

$$\pi_{ij} = \max_{\left\{n_{ijk}, k_{ijk}\right\}_{k=1}^{K}} \overline{Z} z_{ij} \sum_{k=1}^{K} \left(\left[\xi_k n_{ijk}\right]^{\gamma} k_{ijk}^{1-\gamma} \right)^{\alpha} - R \sum_{k=1}^{K} k_{ijk} - \sum_{k=1}^{K} w_{ijk} n_{ijk}$$

  subject to the labor supply system and minimum wage constraints.
- First observe that this can separated out by type of worker k.
- The problem for type k labor at the firm is

$$\pi_{ijk} = \max_{n_{ijk}, k_{ijk}} \overline{Z} z_{ij} \left(\left[\xi_k n_{ijk}\right]^{\gamma} k_{ijk}^{1-\gamma} \right)^{\alpha} - R k_{ijk} - w_{ijk} n_{ijk}$$

- We first optimize out capital. This yields the objective function

$$\pi_{ijk} = \max_{n_{ijk}} \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}} - w_{ijk} n_{ijk}$$

  where

$$\begin{aligned}
\widetilde{Z} &= \overline{Z}^{\frac{1}{1-(1-\gamma)\alpha}} \\
\widetilde{z}_{ij} &= \left[1 - (1-\gamma)\alpha\right] \left(\frac{(1-\gamma)\alpha}{R} \right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} z_{ij}^{\frac{1}{1-(1-\gamma)\alpha}} \\
\widetilde{\xi}_k &= \xi_k^{\widetilde{\alpha}} \\
\widetilde{\alpha} &= \frac{\gamma\alpha}{1-(1-\gamma)\alpha}
\end{aligned}$$

- We denote output net of capital expenses as $\widetilde{y}_{ijk} := \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}}$.
- We can also define a market-level aggregate $\widetilde{y}_{jk} = \sum_{i \in j} \widetilde{y}_{ijk}$, and a type-level aggregate $\widetilde{y}_k = \int \widetilde{y}_{jk} dj$.
- Note that

$$y_{ijk} = \frac{\widetilde{y}_{ijk}}{1-(1-\gamma)\alpha} \quad , \quad y_{jk} = \frac{\widetilde{y}_{jk}}{1-(1-\gamma)\alpha} \quad , \quad y_k = \frac{\widetilde{y}_k}{1-(1-\gamma)\alpha}.$$

- Using the simplified problem we now consider optimality of the firm in each of the three regions described in the text.
- **Region I - Unconstrained**
  - Consider an unconstrained firm. Its problem is

$$\pi_{ijk} = \max_{n_{ijk}} \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}} - w_{ijk} n_{ijk}$$

subject to its wage being given by the above labor supply system:

$$w\left(n_{ijkt}\right) = \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} X_{kt}.$$

where X_{kt} are aggregates the firm takes as given.

– The first order condition is

$$\begin{aligned}
w_{ijk} + w'\left(n_{ijk}\right) n_{ijk} &= \widetilde{\alpha}\widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1} \\
w_{ijk}\left(1 + \frac{w'\left(n_{ijk}\right) n_{ijk}}{w_{ijk}}\right) &= \widetilde{\alpha}\widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1} \\
w_{ijk}\left(1 + \frac{1}{\varepsilon_{ijk}}\right) &= \widetilde{\alpha}\widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1} \\
w_{ijk} &= \frac{\varepsilon_{ijk}}{1+\varepsilon_{ijk}}\widetilde{\alpha}\widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}
\end{aligned}$$

where using the inverse labor supply curve gives

$$\begin{aligned}
\frac{1}{\varepsilon_{ijk}} &:= \frac{w'\left(n_{ijk}\right) n_{ijk}}{w_{ijk}} = \frac{\partial \log w_{ijk}}{\partial \log n_{ijk}} = \frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\frac{\partial \log n_{jk}}{\partial \log n_{ijk}} = \frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\widetilde{s}_{ijk} \\
\varepsilon_{ijk} &= \left[\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\widetilde{s}_{ijk}\right]^{-1}.
\end{aligned}$$

– Therefore

$$w_{ijk} = \mu_{ijk}\widetilde{\alpha}\widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$

where the markdown depends on the firms' elasticity of labor supply.

– Note that since $p_{ijk} = 1$ since the firm is unconstrained, then $\widetilde{w}_{ijk} = p_{ijk}w_{ijk} = w_{ijk}$, so

$$\widetilde{w}_{ijk} = \mu_{ijk} \times \widetilde{\alpha}\widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$

- **Region III - Constrained, on labor demand curve**
  – Now consider a constrained firm in Region III, this firm's problem is

$$\pi_{ijk} = \max_{n_{ijk}} \widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}} - \underline{w}n_{ijk}$$

  – The solution to this problem is to choose employment to equate the marginal revenue product of labor to the minimum wage:

$$\underline{w} = \widetilde{\alpha}\widetilde{\widetilde{Z}}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$

  – For convenience when aggregating, we can express this in terms of shadow wages by multiplying through by the equilibrium multiplier on the rationing constraint

$$\underline{w}p_{ijk} = p_{ijk}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$
$$\widetilde{w}_{ijk} = p_{ijk} \times \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$

- **Region II - Constrained, on labor supply curve**
  - Now consider a constrained firm in Region II, this firm simply has labor determined by the labor supply curve, but since the rationing constraint is slack, $\widetilde{w}_{ijk} = p_{ijk}w_{ijk} = \underline{w}$. Using our characterization results we can write this in terms of shadow wages:

$$n_{ijk} = \left(\frac{\underline{w}}{\widetilde{w}_{jk}}\right)^{\eta}\left(\frac{\widetilde{w}_{jk}}{\widetilde{W}_k}\right)^{\theta} n_k.$$

  - Nonetheless, we can express the shadow wage of the firm as

$$\widetilde{w}_{ijk} = \widetilde{\mu}_{ijk}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$
$$\widetilde{\mu}_{ijk} = \frac{\underline{w}}{\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}} \quad , \quad n_{ijk} = \left(\frac{\underline{w}}{\widetilde{w}_{jk}}\right)^{\eta}\left(\frac{\widetilde{w}_{jk}}{\widetilde{W}_k}\right)^{\theta} n_k.$$

  - Therefore, in all three regions, we can express the *shadow wage* as a *shadow markdown* on the marginal revenue product of labor:

$$\widetilde{w}_{ijk} = \widetilde{\mu}_{ijk}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}.$$

G.1.3 Aggregation of output and labor demand conditions

- Using the above results for firm optimality and the household's labor supply system, where the first order conditions have been characterized in terms of shadow wages, we can aggregate the optimality conditions of agents. This is a key step in solving the government problem and optimal transfers, which we describe below.
- **Aggregation - Firm-Type to Market-Type**
  - From the above we have the following set of five conditions at the firm and market level:
  - **Firm level**:

$$\widetilde{y}_{ijk} = \widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}}$$
$$\widetilde{w}_{ijk} = \widetilde{\mu}_{ijk}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}.$$
$$n_{ijk} = \left(\frac{\widetilde{w}_{ijk}}{\widetilde{w}_{jk}}\right)^{\eta} n_{jk}$$

– **Aggregates**:

$$\widetilde{y}_{jk} = \sum_{i \in j} \widetilde{y}_{ijk}$$

$$\widetilde{w}_{jk} = \left[\sum_{i \in j} \widetilde{w}_{ijk}^{1+\eta} \right]^{\frac{1}{1+\eta}}$$

– Following steps from Berger, Herkenhoff, Mongey (2022), these can be combined to yield:

$$\widetilde{y}_{jk} = \omega_{jk} \widetilde{Z} \widetilde{\xi}_k \widetilde{z}_j n_{jk}^{\widetilde{\alpha}}$$

$$\widetilde{w}_{jk} = \widetilde{\mu}_{jk} \widetilde{\alpha} \widetilde{Z} \widetilde{z}_j \widetilde{\xi}_k n_{jk}^{\widetilde{\alpha}-1}$$

$$n_{jk} = \left(\frac{\widetilde{w}_{jk}}{\widetilde{w}_k} \right)^{\theta} n_k$$

where the three wedges $\left\{ \widetilde{z}_j, \widetilde{\mu}_{jk}, \omega_{jk} \right\}$ are given by

$$\widetilde{z}_j = \left[\sum_{i \in j} \widetilde{z}_{ij}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}}$$

$$\widetilde{\mu}_{jk} = \left[\sum_{i \in j} \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j} \right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \widetilde{\mu}_{ijk}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}}$$

$$\omega_{jk} = \sum_{i \in j} \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j} \right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \left(\frac{\widetilde{\mu}_{ijk}}{\widetilde{\mu}_{jk}} \right)^{\frac{\eta\widetilde{\alpha}}{1+\eta(1-\widetilde{\alpha})}}$$

– Note that this implies that if $\left\{ \widetilde{z}_j, \widetilde{\mu}_{jk}, \widetilde{\omega}_{jk} \right\}$ are known, then $\left\{ n_{jk}, \widetilde{w}_{jk}, \widetilde{y}_{jk} \right\}$ can be determined.

- **Aggregation - Market-Type to Type**

  – The same approach can be followed to aggregate to the household level, which delivers:

$$\widetilde{y}_k = \omega_k \widetilde{Z} \widetilde{\xi}_k \widetilde{z}_k n_k^{\widetilde{\alpha}}$$

$$\widetilde{w}_k = \widetilde{\mu}_k \widetilde{\alpha} \widetilde{Z} \widetilde{z}_k \widetilde{\xi}_k n_k^{\widetilde{\alpha}-1}$$

where

$$\widetilde{z}_k = \left[\int \widetilde{z}_j^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$

$$\widetilde{\mu}_k = \left[\int \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \widetilde{\mu}_{jk}^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$

$$\omega_k = \int \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \left(\frac{\widetilde{\mu}_{jk}}{\widetilde{\mu}_k}\right)^{\frac{\theta\widetilde{\alpha}}{1+\theta(1-\widetilde{\alpha})}} \omega_{jk}$$

- The conditions derived thus far all hold in a competitive equilibrium with lump sum transfers.

G.2 General equilibrium

Combining the above, we can state the general equilibrium conditions of the economy, where the wedges $\{\widetilde{\mu}_k, \omega_k\}_{k=1}^K$ are determined by the market-level Nash equilibria described above, aggregated up to the household k level.

1. **Macro to micro** - Suppose the following are determined by market equilibria for all types of workers k and in all markets j, where firms take aggregate quantities $\{C_k, N_k, Y_k, K_k\}_{k=1}^K$ and prices $\{\widetilde{W}_k\}_{k=1}^K$, R as given

$$\underbrace{\widetilde{z}_k := \left[\int \widetilde{z}_{jk}^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}}_{\text{1. Type productivity}}, \underbrace{\widetilde{\mu}_k := \left[\int \left(\frac{\widetilde{z}_{jk}}{\widetilde{z}_k}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \widetilde{\mu}_{jk}^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}}_{\text{2. Type shadow markdown}}, \underbrace{\omega_k := \int \left(\frac{\widetilde{z}_{jk}}{\widetilde{z}_k}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \left(\frac{\widetilde{\mu}_{jk}}{\widetilde{\mu}_k}\right)^{\frac{\theta\alpha}{1+\theta(1-\widetilde{\alpha})}} \omega_{jk}}_{\text{3. Type misallocation}}.$$

2. **Micro to macro** - For each k, under $\{\widetilde{z}_k, \widetilde{\mu}_k, \omega_k\}_{k=1}^K$, aggregate quantities $\{C_k, N_k, Y_k, K_k\}_{k=1}^K$ and prices $\{\widetilde{W}_k\}_{k=1}^K$, R satisfy:

$$\textit{Output}: \quad \widetilde{Y}_k = \omega_k \widetilde{Z}\widetilde{\xi}_k \widetilde{z}_k N_k^{\widetilde{\alpha}} \quad , \quad Y_k = \frac{1}{1-(1-\gamma)\alpha}\widetilde{Y}_k$$

$$\textit{Capital supply and demand}: \quad 1 = \beta(R + (1-\delta)) \quad , \quad R = \alpha(1-\gamma)\frac{Y_k}{K_k} \quad , \quad K_k = \kappa_k K$$

$$\textit{Labor supply and demand}: \quad N_k = \pi_k \widetilde{\varphi}_k \left(\frac{\widetilde{W}_k}{P}\right)^{\varphi} C_k^{-\sigma\varphi} \quad , \quad \widetilde{W}_k = \widetilde{\mu}_k \widetilde{\alpha}\widetilde{Z}\widetilde{\xi}_k \widetilde{z}_k N_k^{\widetilde{\alpha}-1}$$

$$\textit{Budget constraint}: \quad C_k + \delta K_k = \int \sum_{i\in k} w_{ijk} n_{ijk}\, dj + RK_k + \kappa_k \Pi$$

where aggregate profits are consistent:

$$\Pi = \sum_k Y_k - \int \sum_{i\in k} w_{ijk} n_{ijk}\, dj - R\sum_k K_k$$

These conditions yield three results. First, they show that the market-level lesson of focusing on the shadow markdown and misallocation carries over to the aggregate economy, when these wedges are appropriately aggregated. Second, they provide an algorithm to solve the competitive equilibrium, given $\{z_k, \widetilde{\mu}_k, \omega_k\}_{k=1}^K$, which later will allow us understand the role of different wedges in aggregate welfare. Third, they show how the shadow wages that we have constructed are allocative for quantities. Household labor supply N_k is pinned down by the shadow wage $\widetilde{W}_k$ that the household faces.

G.3 Government problem - Summary and brief results

This section should be read as a summary, with the full derivations in the following subsection.

To separate out the redistribution and efficiency effects of a minimum wage, we consider the problem of a government with social welfare weights $\{\psi_k\}_{k=1}^k$. The government faces prices determined by the imperfectly competitive labor market where firms are subject to the minimum wage. The government is given access to lump-sum taxes $\{T_k\}_{k=1}^K$, with the restriction that total lump sum taxes add to zero. We take the standard approach of solving for the optimal allocation, then the transfers that implement it.

Problem. The government chooses allocations of consumption and labor to maximize

$$\mathcal{U} = \sum_k \psi_k \sum_{t=0}^{\infty} u^k\left(\frac{C_{kt}}{\pi_k}, N_{kt}\right) = \sum_k \psi_k \sum_{t=0}^{\infty}\left[\frac{C_{kt}^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}}\frac{N_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right].$$

We can define the following aggregate consumption and labor indices, and use these to write social welfare as follows:

$$\mathcal{U} = \sum_{t=0}^{\infty}\beta^t\left[\frac{\mathcal{C}_t^{1-\sigma}}{1-\sigma} - \frac{1}{\overline{\varphi}^{1/\varphi}}\frac{\mathcal{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right] \quad , \quad \mathcal{C}_t := \left[\sum_k \psi_k\left(\frac{C_{kt}}{\pi_k}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \quad , \quad \mathcal{N}_t := \left[\sum_k \frac{\psi_k}{\widetilde{\varphi}_k^{1/\varphi}} N_{kt}^{\frac{1+\varphi}{\varphi}}\right]^{\frac{\varphi}{1+\varphi}}$$

This problem can be solved subject to an aggregate budget constraint, and then implemented using lump sum taxes. The government also takes labor rationing constraints into account. Under this approach, the government is endowed with K_0 units of capital, and maximizes social welfare subject to:

$$\sum_k C_{kt} + K_{t+1} = \sum_k \int \sum_{i\in j} w_{ijkt} n_{ijkt}\, dj + R_t K_t + (1-\delta)K_t + \Pi_t \quad , \quad n_{ijkt} \leq \overline{n}_{ijkt} \tag{G1}$$

Optimization delivers an identical set of labor supply conditions to firms as the competitive equilibrium given in the main text. These involve $\{\widetilde{W}_k, N_k, C_k\}$, which in the decentralized economy were determined by each household's labor supply curve and budget constraint. In the government's allocation problem these are instead determined by the government's optimality conditions:

$$C_{kt} = \pi_k\left(\frac{\psi_k}{\pi_k}\right)^{\frac{1}{\sigma}}\left(\frac{1}{\mathcal{P}_t}\right)^{-\frac{1}{\sigma}}\mathcal{C}_t \quad , \quad N_{kt} = \pi_k\widetilde{\varphi}_k\left(\frac{\psi_k}{\pi_k}\right)^{-\varphi}\left(\frac{\widetilde{W}_{kt}}{\widetilde{\mathcal{W}}_t}\right)^{\varphi}\mathcal{N}_t \quad , \quad \mathcal{N}_t = \overline{\varphi}\left(\frac{\widetilde{\mathcal{W}}_t}{\mathcal{P}_t}\right)^{\varphi}\mathcal{C}_t^{-\sigma\varphi}. \tag{G2}$$

Higher social welfare weights relative to population shares entail a higher share of consumption and less labor supply, where the latter is offset if relative wages of the type are higher, or disutility of work is lower (higher $\widetilde{\varphi}_k$). The aggregate shadow price $\mathcal{P}_t$—which is such that $\mathcal{P}_t\mathcal{C}_t = \sum_k C_{kt}$—and aggregate shadow wage $\widetilde{\mathcal{W}}_t$

indexes are given by

$$\mathcal{P}_t = \left[\sum_k \psi_k^{\frac{1}{\sigma}} \pi_k^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \quad , \quad \widetilde{\mathcal{W}}_t = \left[\sum_k \widetilde{\varphi}_k \pi_k \left(\frac{\psi_k}{\pi_k}\right)^{-\varphi} \widetilde{W}_{kt}^{1+\varphi}\right]^{\frac{1}{1+\varphi}}.$$

The derivation of all of these results are given in the following subsection.

Implementation. The planner can implement this allocation in a competitive economy in which households are endowed with shares of capital and profits, by choosing lump sum transfers T_k. These can be read off of each household's budget constraint under equilibrium prices and the government's desired allocation:

$$T_k = \int \sum_{i \in j} w_{ijk} n_{ijk} \, dj + (R + \delta)\kappa_k K + \kappa_k \Pi - C_k. \tag{G3}$$

To see that this implements the government's solution, observe that combining conditions in (G2) yields the decentralized household labor supply curves, and that the government's steady-state Euler equation coincides with each household's in the competitive equilibrium. Since taxes are lump-sum, their presence does not distort these conditions.[50] Finally, summing budget constraints (G3) obtains the planner's budget constraint (G1), and hence transfers sum to zero. The following subsection steps through these.

Aggregates. To solve the government's problem still requires the determination of aggregates $\mathcal{C}$, $\widetilde{\mathcal{W}}$, and $\mathcal{N}$. Under a given set of social welfare weights, market equilibria can be aggregated to obtain shadow markdowns for all types: $\{\widetilde{\mu}_k, \omega_k\}_{k=1}^{K}$. These can be further aggregated, where $\phi_k = \pi_k^{1+\varphi} \widetilde{\varphi}_k \psi_k^{-\varphi}$:

$$\widetilde{z} = \left[\sum_k (\xi_k \widetilde{z}_k)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}}\right]^{\frac{1+\varphi(1-\alpha)}{1+\varphi}}, \widetilde{\mu} = \left[\sum_k \left(\frac{\xi_k \widetilde{z}_k}{\widetilde{z}}\right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \widetilde{\mu}_k^{\frac{1+\varphi}{1+\varphi(1-\alpha)}}\right]^{\frac{1+\varphi(1-\alpha)}{1+\varphi}}, \omega = \sum_k \left(\frac{\xi_k \widetilde{z}_k}{\widetilde{z}}\right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \left(\frac{\widetilde{\mu}_k}{\widetilde{\mu}}\right)^{\frac{\varphi\alpha}{1+\varphi(1-\alpha)}} \omega_k$$

Weights now account for productivity shifters ξ_k, household measures π_k, labor disutility $\widetilde{\varphi}_k$ and social welfare weights ψ_k. Given $\{\mathcal{P}, \widetilde{z}, \widetilde{\mu}, \omega\}$, the following can be solved for $\mathcal{C}$, $\widetilde{\mathcal{W}}$, and $\mathcal{N}$ in closed form:

$$\underbrace{Y = \frac{\omega \widetilde{Z} \widetilde{z} \mathcal{N}^{\widetilde{\alpha}}}{1-(1-\gamma)\alpha}}_{\text{Output}} \quad , \quad \underbrace{\mathcal{P}\mathcal{C} = Y - \delta K}_{\text{Resource constraint}} \quad , \quad \underbrace{\widetilde{\mathcal{W}} = \widetilde{\mu}\widetilde{\alpha}\widetilde{Z}\widetilde{z}\mathcal{N}^{\widetilde{\alpha}-1}}_{\text{Labor demand}} \quad , \quad \underbrace{\mathcal{N} = \overline{\varphi}\left(\frac{\widetilde{\mathcal{W}}}{\mathcal{P}}\right)^{\varphi} \mathcal{C}^{-\varphi\sigma}}_{\text{Labor supply}} \tag{G4}$$

To allocate these aggregates across households, we use the government's first order conditions (G2).

Negishi weights. Our baseline calibration of the model is a competitive equilibrium with zero lump sum taxes. This yields an allocation of labor, consumption and capital. Note that there exists a vector of social welfare weights $\{\psi_k^*\}_{k=1}^{K}$ such that a government with these weights would choose the same allocation, also with zero lump sum taxes. As is standard, we refer to this vector of social welfare weights as the *Negishi weights*. Computing the Negishi weights associated with the benchmark competitive equilibrium is a key step in our welfare exercise. Optimal policy under this benchmark can be compared to optimal policy under

[50] As is standard, comparing households' and the planner's first order conditions for consumption reveal that the social welfare weights map into multipliers on households budget constraints, which are constant in steady-state. Denote these multipliers ν_k. Normalize $\psi_1 = 1$, then $\psi_k = \nu_1/\nu_k$. Hence, starting with some social welfare weights, the implied allocation can be decentralized by budget-neutral lump-sum taxes. Lump sum transfers tighten and loosen budget constraints, so can be chosen to align multipliers with the planner's social welfare weights.

alternative weights, such as Utilitarian weights. Incidentally, we also exploit the associated *Negishi algorithm* to make feasible the computation of the competitive equilibrium with K types.[51]

G.4 Government problem - Details

- We consider the government primal problem where it chooses an allocation of (i) labor from each household to all firms (ii) consumption of all households, (iii) investment. We then show that the government can decentralize this allocation in a competitive equilibrium by choosing appropriate lump sum transfers.
- This has the flavor of a 'partial' planning problem. 'Partial' in the sense that the government takes as given the prices of firms in the economy, and firms' rationing constraints, where these are due to the market power of firms. The government therefore faces a *budget constraint* rather than a resource constraint.

G.4.1 Allocation problem

- The government is endowed with K_0, takes prices $\left\{w_{ijkt}\right\}$, profits $\{\Pi_t\}$, rationing constraints $\left\{\overline{n}_{ijkt}\right\}$ as given and chooses directly $\left\{c_{kt}, n_{ijkt}, K_{t+1}\right\}$ to maximize

$$U_0 = \sum_{t=0}^{\infty} \beta^t \sum_k \psi_k \left[\frac{(c_{kt}/\pi_k)^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{n_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]$$

where

$$n_{kt} = \left[\int n_{jkt}^{\frac{\theta+1}{\theta}} dj \right]^{\frac{\theta}{\theta+1}}$$

$$n_{jkt} = \left[\sum_{i \in j} n_{ijkt}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}$$

subject to its budget constraint

$$\sum_k c_{kt} + K_{t+1} = \sum_k \int \sum_{i \in j} w_{ijkt} n_{ijkt} dj + R_t K_t + (1-\delta) K_t + \Pi_t$$

rationing constraints

$$n_{ijk} \leq \overline{n}_{ijk}$$

- Here $\widetilde{\varphi}_k = \overline{\varphi}_k \pi_k^{\varphi+1}$ is adjusted for the measure of workers of the household.

[51] In particular, we can guess a set of Negishi weights, normalizing $\psi_1^* = 1$. First, we solve market equilibria, to obtain $\{\widetilde{\mu}_k, \omega_k\}_{k=1}^K$. Using the guessed Negishi weights we can compute $\widetilde{z}, \widetilde{\mu}, \omega, \mathcal{P}$ from the above expressions, and then use these to solve for $Y, \mathcal{W}, \mathcal{C}, \mathcal{N}$ using equations (G4). Using the planner's first order conditions (G2), we can allocate $\mathcal{C}$ among households, and hence compute implied household consumption C_k. We can also compute firm wages and employment. Following the tradition of the Negishi algorithm, we then compute the implied residual in the household's budget constraint—under $T_k = 0$—and update our guess of $\{\psi_k^*\}_{k \neq 1}^K$, until this residual is zero. We lower ψ_k^* for households with a deficit, and increase ψ_k^* for households with a surplus.

- We can rewrite the objective function as

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left[\frac{\mathcal{C}_t^{1-\sigma}}{1-\sigma} - \frac{1}{\widehat{\varphi}^{1/\varphi}} \frac{\mathcal{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]$$

where

$$\mathcal{C}_t = \left[\sum_k \psi_k \left(\frac{c_{kt}}{\pi_k} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$
$$\mathcal{N}_t = \left[\sum_k \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}} \right) n_{kt}^{\frac{\varphi+1}{\varphi}} \right]^{\frac{\varphi}{\varphi+1}}$$

and $\widehat{\varphi}_k = \widetilde{\varphi}_k / \widehat{\varphi}$.

- Let $\beta^t \Lambda_t$ be the multiplier on the government's budget constraint.

G.4.2 Allocation problem - Consumption

- The first order condition for c_{kt} gives the following:

$$\psi_k \left(\frac{c_{kt}}{\pi_k} \right)^{1-\sigma} = \Lambda_t p_{kt} c_{kt}$$
$$c_{kt} = \pi_k \left(\frac{\Lambda_t p_{kt} \pi_k}{\psi_k} \right)^{-\frac{1}{\sigma}}$$

- Suppose there exists some $\mathcal{P}_t$ such that aggregate consumption $C_t = \sum_k c_{kt} = \mathcal{P}_t \mathcal{C}_t$.
- Using the first order condition we can obtain:

$$\Lambda_t = \frac{\mathcal{C}_t^{-\sigma}}{\mathcal{P}_t}$$
$$c_{kt} = \pi_k \left(\frac{\psi_k}{\pi_k} \right)^{\frac{1}{\sigma}} \left(\frac{1}{\mathcal{P}_t} \right)^{-\frac{1}{\sigma}} \mathcal{C}_t$$
$$\mathcal{P}_t = \left[\sum_k \psi_k^{\frac{1}{\sigma}} \pi_k^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}}$$

- We can substitute $\sum_k c_{kt} = \mathcal{P}_t \mathcal{C}_t$ into the planner's problem to obtain the following problem, where the distribution of $\mathcal{C}_t$ among households is determined by

$$c_{kt} = \pi_k \left(\frac{\psi_k}{\pi_k} \right)^{\frac{1}{\sigma}} \left(\frac{1}{\mathcal{P}_t} \right)^{-\frac{1}{\sigma}} \mathcal{C}_t$$

- The government's reduced problem is therefore to choose $\left\{\mathcal{C}_t, n_{ijkt}\right\}$ to maximize

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left[\frac{\mathcal{C}_t^{1-\sigma}}{1-\sigma} - \frac{1}{\widehat{\varphi}^{1/\varphi}} \frac{\mathcal{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right]$$

$$\mathcal{N}_t = \left[\sum_k \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) n_{kt}^{\frac{\varphi+1}{\varphi}}\right]^{\frac{\varphi}{\varphi+1}}$$

$$n_{kt} = \left[\int n_{jkt}^{\frac{\theta+1}{\theta}} dj\right]^{\frac{\theta}{\theta+1}}$$

$$n_{jkt} = \left[\sum_{i \in j} n_{ijkt}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$$

subject to

$$\mathcal{P}_t \mathcal{C}_t + K_{t+1} = \sum_k \int \sum_{i \in j} w_{ijkt} n_{ijkt} dj + R_t K_t + (1-\delta) K_t + \Pi_t$$

and rationing constraints

$$n_{ijk} \leq \overline{n}_{ijk}$$

- The planner's first order condition for $\mathcal{C}_t$ is then

$$U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right) = \Lambda_t \mathcal{P}_t$$

where

$$U\left(\mathcal{C}_t, \mathcal{N}_t\right) = \frac{\mathcal{C}_t^{1-\sigma}}{1-\sigma} - \frac{1}{\widehat{\varphi}^{1/\varphi}} \frac{\mathcal{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}.$$

G.4.3 Allocation problem - Labor

- Consider the terms in the government's Lagrangean that feature n_{ijkt}
- Write the multiplier on the rationing constraint as $\zeta_{ijkt} = \beta^t \Lambda_t w_{ijkt}\left(1 - p_{ijkt}\right)$
- These terms are

$$\mathcal{L} = \cdots + \beta^t U\left(\mathcal{C}_t, \mathcal{N}_t\right) + \ldots + \beta^t \Lambda_t w_{ijkt} n_{ijkt} + \beta^t \Lambda_t w_{ijkt}\left(1 - p_{ijkt}\right)\left[\overline{n}_{ijkt} - n_{ijkt}\right] + \ldots$$
$$\mathcal{L} = \cdots + \beta^t U\left(\mathcal{C}_t, \mathcal{N}_t\right) + \ldots + \beta^t \Lambda_t w_{ijkt} p_{ijkt} n_{ijkt} + \beta^t \Lambda_t w_{ijkt}\left(1 - p_{ijkt}\right) \overline{n}_{ijkt} + \ldots$$

- The first order condition for consumption is as above:

$$\Lambda_t = U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right) / \mathcal{P}_t$$

- The first order condition for n_{ijkt} is

$$w_{ijkt}p_{ijkt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t,\mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t,\mathcal{N}_t\right)/\mathcal{P}_t}\left(\frac{\partial \mathcal{N}_t}{\partial n_{kt}}\right)\left(\frac{\partial n_{kt}}{\partial n_{jkt}}\right)\left(\frac{\partial n_{jkt}}{\partial n_{ijkt}}\right)$$

- Using the definitions of aggregators $\mathcal{N}_t$, n_{kt}, n_{jkt}:

$$w_{ijkt}p_{ijkt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t,\mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t,\mathcal{N}_t\right)/\mathcal{P}_t}\left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right)\left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}}\left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}\left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}}$$

- Define the *shadow wage* $\widetilde{w}_{ijkt} := w_{ijkt}p_{ijkt}$.
- Using this definition:

$$\widetilde{w}_{ijkt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t,\mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t,\mathcal{N}_t\right)/\mathcal{P}_t}\left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right)\left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}}\left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}\left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}} \qquad (*)$$

- We now define the following *shadow wage indexes* at the market, type and aggregate level:

$$\widetilde{w}_{jkt} := \left[\sum_{i\in j}\widetilde{w}_{ijkt}^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

$$\widetilde{w}_{kt} := \left[\int \widetilde{w}_{jkt}^{1+\theta}dj\right]^{\frac{1}{1+\theta}}$$

$$\widetilde{\mathcal{W}}_t := \left[\sum_k\left(\frac{\widehat{\varphi}_k}{\psi_k^{\varphi}}\right)\widetilde{w}_{kt}^{1+\varphi}\right]^{\frac{1}{1+\varphi}}$$

- Using the definition of $\widetilde{w}_{jkt}$ and $n_{jkt} = \left[\sum_{i\in j} n_{ijkt}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$ in $(*)$ we have

$$\widetilde{w}_{jkt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t,\mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t,\mathcal{N}_t\right)/\mathcal{P}_t}\left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right)\left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}}\left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}$$

- Then using the definition of $\widetilde{w}_{kt}$ and $n_{kt} = \left[\int n_{jkt}^{\frac{\theta+1}{\theta}}dj\right]^{\frac{\theta}{\theta+1}}$:

$$\widetilde{w}_{kt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t,\mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t,\mathcal{N}_t\right)/\mathcal{P}_t}\left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right)\left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}}$$

- Then using the definition of $\widetilde{\mathcal{W}}_t$ and $\mathcal{N}_t = \left[\sum_k \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) n_{kt}^{\frac{\varphi+1}{\varphi}}\right]^{\frac{\varphi}{\varphi+1}}$:

$$\widetilde{\mathcal{W}}_t = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t, \mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right)/\mathcal{P}_t}$$

- Using $U\left(\mathcal{C}_t, \mathcal{N}_t\right)$ we can obtain what we will refer to as the *aggregate labor supply curve*

$$\widetilde{\mathcal{W}}_t = \widehat{\varphi}^{-\frac{1}{\varphi}} \mathcal{P}_t \mathcal{C}_t^{\sigma} \mathcal{N}_t^{\frac{1}{\varphi}}$$
$$\mathcal{N}_t = \widehat{\varphi}\left(\frac{\widetilde{\mathcal{W}}_t}{\mathcal{P}_t}\right)^{\varphi} \mathcal{C}_t^{-\varphi\sigma}$$

- A key result is that this is the labor supply curve that would obtain from a government that maximizes $U\left(\mathcal{C}_t, \mathcal{N}_t\right)$ subject to a budget constraint

$$\mathcal{P}_t \mathcal{C}_t + K_{t+1} = \widetilde{\mathcal{W}}_t \mathcal{N}_t + R_t K_t + (1-\delta) K_t + \Pi_t$$

and faced no rationing constraints. However such a budget constraint is incorrect, in that $\widetilde{\mathcal{W}}_t \mathcal{N}_t \neq \sum_k \int \sum_{i \in j} w_{ijkt} n_{ijkt} dj$. Nonetheless, the interpretation of the aggregate labor supply curves holds, and shows exactly the extent to which the economy supplies labor *as if* it faced a wage $\widetilde{\mathcal{W}}_t$.

G.4.4 Implied labor supply system to firms

- Using the above results we can refine the labor supply system.
- Using the aggregate labor supply curve in the type-level expression above, we have

$$\widetilde{w}_{kt} = \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right)\left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}} \widetilde{\mathcal{W}}_t$$
$$n_{kt} = \left(\frac{\widehat{\varphi}_k}{\psi_k^{\varphi}}\right)\left(\frac{\widetilde{w}_{kt}}{\widetilde{\mathcal{W}}_t}\right)^{\varphi} \mathcal{N}_t$$

- Using this in the market-type-level expression above:

$$\widetilde{w}_{jkt} = \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \widetilde{w}_{kt}$$
$$\Longrightarrow n_{jkt} = \left(\frac{\widetilde{w}_{jkt}}{\widetilde{w}_{kt}}\right)^{\theta} n_{kt}$$

- Using this in the firm-market-type-level expression $(*)$ above, we then recover the same labor supply system as the competitive equilibrium:

$$\widetilde{w}_{ijkt} = \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}\left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}} \widetilde{w}_{kt}$$

which can then be written:

$$n_{ijkt} = \left(\frac{\widetilde{w}_{ijkt}}{\widetilde{w}_{jkt}}\right)^{\eta} \left(\frac{\widetilde{w}_{jkt}}{\widetilde{w}_{kt}}\right)^{\theta} n_{kt}$$

$$\widetilde{w}_{jkt} = \left[\sum_{i\in j} \widetilde{w}_{ijkt}^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

$$\widetilde{w}_{kt} = \left[\int \widetilde{w}_{jkt}^{1+\theta} dj\right]^{\frac{1}{1+\theta}}$$

- **Result A** - *This corresponds to the labor supply system from type-k household optimality in a competitive equilibrium with lump sum transfers T_k.*
  - This follows immediately from our derivation in the competitive equilibrium, and the fact that the presence of lump sum transfers in household k 's budget constraint do not affect any such derivations.

G.4.5 Implied household labor supply curves

- Combining the planner's allocation of consumption

$$c_{kt} = \pi_k \left(\frac{\psi_k}{\pi_k}\right)^{\frac{1}{\sigma}} \left(\frac{1}{\mathcal{P}_t}\right)^{-\frac{1}{\sigma}} \mathcal{C}_t$$

the aggregate labor supply curve

$$\mathcal{N}_t = \widehat{\varphi} \left(\frac{\widetilde{\mathcal{W}}_t}{\mathcal{P}_t}\right)^{\varphi} \mathcal{C}_t^{-\varphi\sigma}$$

and the planner's allocation of labor

$$n_{kt} = \left(\frac{\widehat{\varphi}_k}{\psi_k^{\varphi}}\right) \left(\frac{\widetilde{w}_{kt}}{\widetilde{\mathcal{W}}_t}\right)^{\varphi} \mathcal{N}_t$$

by substituting out the planner's social welfare weight ψ_k, obtains

$$n_{kt} = \pi_k^{\varphi(\sigma-1)} \widehat{\varphi}\widehat{\varphi}_k \widetilde{w}_{kt}^{\varphi} c_{kt}^{-\varphi\sigma}$$

- Using definition of $\widehat{\varphi}_k = \widetilde{\varphi}_k / \widehat{\varphi}$

$$n_{kt} = \pi_k^{\varphi(\sigma-1)} \widetilde{\varphi}_k \widetilde{w}_{kt}^{\varphi} c_{kt}^{-\varphi\sigma}$$

- Using the definition of $\widetilde{\varphi}_k = \overline{\varphi}_k \pi_k^{\varphi+1}$:

$$n_{kt} = \pi_k \overline{\varphi}_k \widetilde{w}_{kt}^{\varphi} \left(\frac{c_{kt}}{\pi_k}\right)^{-\varphi\sigma}$$

- **Result B** - *This corresponds to the household labor supply curve from type-k household optimality in a competitive equilibrium with lump sum transfers T_k.*

– This follows immediately from our derivation in the competitive equilibrium, and the fact that the presence of lump sum transfers in household k 's budget constraint do not affect any such derivations.

G.4.6 Further conditions

- From the above we have obtained the aggregate supply curve. We also have the aggregate resource constraint, which in steady-state is:

$$Y_t = \sum_k c_{kt} + \delta K_t.$$

- Using the consumption results from above, this can be written

$$Y_t = \mathcal{P}_t \mathcal{C}_t + \delta K_t.$$

- Recall also, that we have the aggregation of output

$$Y_t = \frac{1}{1-\gamma(1-\alpha)} \widetilde{Y}_t$$
$$\widetilde{Y}_t = \sum_k \widetilde{y}_{kt}$$

- The steady-state Euler equation of the government is

$$1 = \beta [R + (1-\delta)]$$

- **Result C** - *This corresponds to the household Euler equation from type-k household optimality in a competitive equilibrium with lump sum transfers T_k.*
  – This follows immediately from our derivation in the competitive equilibrium, and the fact that the presence of lump sum transfers in household k 's budget constraint do not affect any such derivations.

G.4.7 Aggregating labor demand and output

- From **Result A** above, the labor supply system for type k labor from the solution to the government's primal (allocation) problem corresponds to the labor supply system in the competitive equilibrium.
- Firm optimality conditions will therefore be the same as in the competitive equilibrium, and the aggregation results derived earlier hold up to the type-k level.
- Recall that these results yielded the following. For type-k, output, the shadow wage index, labor supply are as follows, where the third line is the new solution to the government's supply of type-k labor

$$\widetilde{y}_k = \omega_k \widetilde{Z} \widetilde{\xi}_k \widetilde{z}_k n_k^{\widetilde{\alpha}}$$
$$\widetilde{w}_k = \widetilde{\mu}_k \widetilde{\alpha} \widetilde{Z} \widetilde{z}_k \widetilde{\xi}_k n_k^{\widetilde{\alpha}-1}$$
$$n_k = \phi_k \left(\frac{\widetilde{w}_k}{\widetilde{\mathcal{W}}} \right)^{\varphi} \mathcal{N}$$

where $\phi_k = \pi_k^{1+\varphi}\left(\frac{\overline{\varphi}_k}{\widehat{\varphi}}\right)\psi_k^{-\varphi}$, where $\{\omega_k, \widetilde{\mu}_k, \widetilde{z}_k\}$ are as in the competitive equilibrium, derived above.

- We also have two aggregation conditions, where we have substituted in

$$\widetilde{Y} = \sum_k \widetilde{y}_k$$
$$\widetilde{\mathcal{W}} = \left[\sum_k \phi_k \widetilde{w}_{kt}^{1+\varphi}\right]^{\frac{1}{1+\varphi}}$$

- In the same way as we used the sets of 5 conditions to aggregate output and labor demand in the competitive equilibrium we can also use the same approach on this set of conditions.
- The result is that aggregate output and the aggregate shadow wage can be expressed using aggregate shadow markdown, productivity and misallocation wedges:

$$\widetilde{Y} = \omega \widetilde{Z}\widetilde{z}\mathcal{N}^{\alpha}$$
$$\widetilde{\mathcal{W}} = \widetilde{\mu}\alpha\widetilde{Z}\widetilde{z}\mathcal{N}^{\alpha-1}$$

where

$$\widetilde{z} = \left[\sum_k (\xi_k \widetilde{z}_k)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}}\right]^{\frac{1+\varphi(1-\alpha)}{1+\varphi}}$$
$$\widetilde{\mu} = \left[\sum_k \left(\frac{\xi_k \widetilde{z}_k}{\widetilde{z}}\right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \widetilde{\mu}_k^{\frac{1+\varphi}{1+\varphi(1-\alpha)}}\right]^{\frac{1+\varphi(1-\alpha)}{1+\varphi}}$$
$$\omega = \sum_k \left(\frac{\xi_k \widetilde{z}_k}{\widetilde{z}}\right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \left(\frac{\widetilde{\mu}_k}{\widetilde{\mu}}\right)^{\frac{\varphi\alpha}{1+\varphi(1-\alpha)}} \omega_k$$

- Capital demand is as in the competitive equilibrium:

$$Rk_{ijk} = \alpha(1-\gamma)\, y_{ijk}$$

which when aggregated yeilds

$$RK = \alpha(1-\gamma)\, Y$$

G.4.8 Full set of conditions for the solution of government allocation problem and competitive equilibrium

- Equilibrium under the government allocation problem, can therefore be summarized in the following conditions, given the wedges $\{z, \widetilde{\mu}, \omega\}$:

$$\widetilde{Y} = \omega \widetilde{Z} \widetilde{z} \mathcal{N}^{\alpha}$$
$$\mathcal{W} = \widetilde{\mu} \alpha \widetilde{Z} \widetilde{z} \mathcal{N}^{\alpha-1}$$
$$\mathcal{N} = \widehat{\varphi} \left(\frac{\widetilde{\mathcal{W}}}{\mathcal{P}} \right)^{\varphi} \mathcal{C}^{-\varphi\sigma}$$
$$Y = \mathcal{P}\mathcal{C} + \delta K$$
$$Y = \frac{1}{1-\gamma(1-\alpha)} \widetilde{Y}$$
$$\mathcal{P} = \left[\sum_k \psi_k^{\frac{1}{\sigma}} \pi_k^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}}$$
$$1 = \beta \left[R + (1-\delta) \right]$$
$$RK = \alpha (1-\gamma) Y$$

- This system of 8 equations in 8 unknowns $\left\{ \widetilde{Y}, Y, \mathcal{N}, \mathcal{C}, \mathcal{P}, R, K, \mathcal{W} \right\}$ can be solved in closed form.
- Once solved, and given $\{\widetilde{w}_k\}_{k=1}^{K}$, household type variables can be determined from the government's first order conditions:

$$n_k = \phi_k \left(\frac{\widetilde{w}_k}{\widetilde{\mathcal{W}}} \right)^{\varphi} \mathcal{N}$$
$$c_k = \pi_k \left(\frac{\psi_k}{\pi_k} \right)^{\frac{1}{\sigma}} \left(\frac{1}{\mathcal{P}} \right)^{-\frac{1}{\sigma}} \mathcal{C}$$

- The allocation of labor n_k to firms is then determined by the labor supply system:

$$n_{ijk} = \left(\frac{\widetilde{w}_{ijk}}{\widetilde{w}_{jk}} \right)^{\eta} \left(\frac{\widetilde{w}_{jk}}{\widetilde{w}_k} \right)^{\theta} n_k$$

G.4.9 Implementation with lump-sum taxes

- **Results A, B, C** above imply that the government's optimality conditions of its allocation problem coincide with those of households in a competitive equilibrium with lump sum transfers T_k.
- Therefore the government can choose arbitrary lump sum transfers and yield the same set of optimality conditions.
- The only thing that is left is to determine the lump sum transfers themselves.

- These can simply be read off of the household's budget constraints, which in steady state are:

$$c_k = \int \sum_{i \in j} w_{ijk} n_{ijk} \, dj + \kappa_k \left[(R - \delta) K + \Pi \right] + T_k$$

$$T_k = c_k - \int \sum_{i \in j} w_{ijk} n_{ijk} \, dj - \kappa_k \left[(R - \delta) K + \Pi \right]$$

- Transfers clearly sum to zero since summing the household budget constraint yeilds the government budget constraint if and only if $\sum_k T_k = 0$.

G.5 Leveraging the government solution to solve the competitive equilibrium

- In practice we leverage the government problem described above to solve the competitive equilibrium of the economy.
- We do this in the trandition of the *Negishi algorithm.*
- The above section described how we can first *fix social welfare weights*, then solve the government problem, then determine the required lump-sum transfers.
- The competitive equilibrium can be solved under *guessing of social welfare weights*, then solve the government problem, then determine the required lump-sum transfers, and then *iterating on the guess of social welfare weights*, until the implied lump sum transfers are all equal to zero.
- Under the social welfare weights that deliver zero lump sum transfers, the competitive equilibrium budget constraints of all households hold by construction:

$$c_k = \int \sum_{i \in j} w_{ijk} n_{ijk} \, dj + \kappa_k \left[(R - \delta) K + \Pi \right]$$

  and all remaining competitive equilibrium conditions also hold (i.e. each household's Euler equation, labor supply system to firms, household labor supply curve, resource constraint, capital and labor demand).
- The solution of the government problem, which can be achieved largely in closed form, is therefore a key part of our computational strategy.

H Search model

In this appendix, we show how the simple monopsony model can be recast in terms of search effort. In this environment, firms and workers take equilibrium meeting rates as given.

Household problem. The household takes matching rates $\{p_i\}_{i \in [0,1]}$, wages $\{w_i\}_{i \in [0,1]}$ and profits Π as given. The household chooses search effort $\{s_i\}_{i \in [0,1]}$ sent to each firm. The household's problem is:

$$\max_{\{s_i\}_{i \in [0,1]}} u(C, S) \quad , \quad S = \left[\int s_i^{\frac{1+\eta}{\eta}} \, di \right]^{\frac{\eta}{1+\eta}}$$

subject to its budget constraint, which captures that only s_i units of search effort yield a match at firm i:

$$C = \int w_i p_i s_i \, di + \Pi$$

The first order conditions for s_i and C yields the optimality condition for household search effort at firm, which we refer to as the *inverse labor supply schedule:*

$$w_i = \frac{1}{p_i} \left(\frac{s_i}{S} \right)^{\frac{1}{\eta}} \left(-\frac{U_S}{U_C} \right)$$

Firm problem. Firm i takes as given the aggregate search index S and the matching rate p_i, and chooses its (i) wage w_i, and (ii) employment $n_i = p_i s_i$ to maximize profits. The firm is constrained by (a) the minimum wage $w_i \geq \underline{w}$, and (b) the inverse search supply curve of households. Therefore the firm problem is given by:

$$\max_{s_i} z_i \left(p_i s_i \right)^{\alpha} - w\left(s_i, p_i, S \right) p_i s_i$$

$$\text{subject to} \quad w\left(s_i, p_i, S \right) \geq \underline{w}$$

$$w\left(s_i, p_i, S \right) = \frac{1}{p_i} \left(\frac{s_i}{S} \right)^{\frac{1}{\eta}} \left(-\frac{U_S}{U_C} \right)$$

Under our assumption of Cournot competition, the firm understands $\frac{\partial w(s_i, p_i, S)}{\partial s_i} \neq 0$, yielding monopsony power.

Matching technology. We assume a constant returns to scale matching function given by the *short-side matching function*. This gives $p_i = \min \left\{ s_i, n_i \right\} / s_i$ from the household's perspective. If $s_i < n_i$, then $p_i = 1$ and all units of search are converted into a job. If $s_i > n_i$, then $p_i = n_i / s_i < 1$ and only some of the units of search effort lead to a job.

Comparison. A comparison of the two economies is then as follows

- In the monopsony model, aggregation can be written in terms of

$$Y = \omega Z N^{\alpha} \quad , \quad \widetilde{W} = \widetilde{\mu} \alpha Z N^{\alpha - 1} \quad , \quad N = \overline{\varphi} \widetilde{W}^{\varphi} C^{-\varphi\sigma} \quad , \quad C = Y$$

- In the search model, aggregation can be written in terms of

$$Y = \omega Z S^{\alpha} \quad , \quad \widetilde{W} = \widetilde{\mu} \alpha Z S^{\alpha - 1} \quad , \quad S = \overline{\varphi} \widetilde{W}^{\varphi} C^{-\varphi\sigma} \quad , \quad C = Y$$

- **If** $\{Z, \omega, \widetilde{\mu}\}$ are identical across these two economies, then it must be that $\left\{ Y, \widetilde{W}, C \right\}$ are the same and $N = S$, and hence welfare $U(C, N) = U(C, S)$ is the same. **But**, the appropriate measure of aggregate productivity Z is, however, lower in the search economy. In the search economy $y_i = p_i^{\alpha} \times z_i s_i^{\alpha}$,and hence

$$Z^S = \left[\int \left(p_i^{\alpha} z_i \right)^{\frac{1}{1+\eta(1-\alpha)}} di \right]^{\frac{1}{1+\eta(1-\alpha)}} < \left[\int z_i^{\frac{1}{1+\eta(1-\alpha)}} di \right]^{\frac{1}{1+\eta(1-\alpha)}} = Z^M.$$

Table H1: Comparison of monopsony and search models

	Monopsony	**Search**
Preferences	$U(C,N)$	$U(C,S)$
Disutility	*... of labor*	*... of search effort*
	$N=\left[\int n_i^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$	$S=\left[\int s_i^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$
Budget constraint	$C=\int w_i n_i di+\Pi$	$C=\int p_i w_i s_i+\Pi$ $=\widetilde{W}S+\Pi$
Optimality condition	$p_i w_i=\left(\frac{n_i}{N}\right)^{\frac{1}{\eta}}N^{\frac{1}{\varphi}}$	$p_i w_i=\left(\frac{s_i}{S}\right)^{\frac{1}{\eta}}S^{\frac{1}{\varphi}}$
Wedge	*Rationing multiplier* $p_i\in(0,1]$ if $n_i=\overline{n}_i$	*Job finding rate* $p_i<1$ if $s_i>n_i$
Wage determinative of n_i	*Shadow wage* $\widetilde{w}_i=p_i w_i$	*Expected wage* $\widetilde{w}_i=p_i w_i$
Aggregation	$\widetilde{W}=\left[\int\widetilde{w}_i^{1+\eta}\right]^{\frac{1}{1+\eta}}$	$\widetilde{W}=\left[\int\widetilde{w}_i^{1+\eta}\right]^{\frac{1}{1+\eta}}$
Supply	$\widetilde{W}=-U_S/U_C$	$\widetilde{W}=-U_N/U_C$
Production	$y_i=z_i n_i^{\alpha}$	$y_i=z_i\left(p_i s_i\right)^{\alpha}$
Optimality	$\widetilde{w}_i=\widetilde{\mu}_i\alpha z_i n_i^{\alpha-1}$	$\widetilde{w}_i=\widetilde{\mu}_i\alpha z_i s_i^{\alpha-1}$
Shadow-markdown	$\widetilde{\mu}_i=\begin{cases}\mu_i & \text{Region I}\\ p_i & \text{Region III}\end{cases}$	$\widetilde{\mu}_i=\begin{cases}\mu_i & \text{Region I}\\ p_i^{\alpha} & \text{Region III}\end{cases}$

The search economy has lower productivity due to frictional non-employment $p_i<0$ at firms in Region III.

Discussion. The main difference between our benchmark economy and the search economy is that, in the presence of a minimum wage, there may be *frictional* rationing of workers in the search economy which leads to search effort expended at some firms where $n_i<s_i$ matches are generated. For such firms in Region III, s_i is positive. Absent firm heterogeneity this frictional rationing is aptly thought of as *unemployment*, but with firm heterogeneity the search framework imposes additional welfare costs by having large volumes of costly search effort at firms that do not seek to employ it. We show below that the meeting rate p_i is declining in productivity in Region III. For example, a corner-store in Region III is characterized by $(n_i,s_i,p_i)=(1,5,0.20)$, while a super-market is characterized by $(n_i,s_i,p_i)=(9,12,0.75)$. The four units of excess search effort incur utility costs, while providing no additional output.

From a normative perspective, we view this as problematic for welfare computations. Our approach of rationing constraints implies that there is no costly search effort being wasted in equilibrium. The corner-store has a sign for $\overline{n}_i=1$ worker, and 1 worker applies. Firms stop recruiting when full and households do not send excess labor in equilibrium. Hence we remove a welfare cost of minimum wages that would appear in the search model. This places the focus of inefficiencies on minimum wages forcing firms in Region III to shrink below their efficient size, rather than the frictional non-employment this creates.

From an empirical perspective, the primary challenge to estimating the search model is calibrating the labor disutility parameter(s) η (and θ with nested-CES preferences over search effort). Search effort is not observed, nor are plant level matching rates. A richer model would also introduce parameters via the matching function if we deviate from short-sided matching. Hence we believe it would be difficult to make progress on this model. The rationing constraint model introduces no new parameters.

With the above caveats, the above mapping leads us to expect very similar results in such a model for values of η and θ close to our benchmark economy. Since search effort contributes additional welfare costs without any additional benefits we view the welfare gains from our rationing constraint model as an upper bound on welfare gains in the corresponding search model.

Comparative statics. We can compute comparative statics of $\{p_i, \widetilde{w}_i, s_i\}$ in each of the three regions as $\{z_i, \underline{w}\}$ vary, *ceterus parabus.*

1. **Region I** - We have

$$\Delta \log p_i = 0$$
$$\Delta \log \widetilde{w}_i = \frac{1}{1+\eta(1-\alpha)} \Delta \log z_i$$
$$\Delta \log s_i = \frac{\eta}{1+\eta(1-\alpha)} \Delta \log z_i$$

2. **Region II** - We have

$$\Delta \log p_i = 0$$
$$\Delta \log \widetilde{w}_i = \Delta \log \underline{w}$$
$$\Delta \log s_i = \eta \Delta \log \underline{w}$$

3. **Region III -** Supply and demand are:

$$\widetilde{w}_i = p_i \underline{w} = \left(\frac{s_i}{S}\right)^{\frac{1}{\eta}} \widetilde{W}^{\frac{1}{\varphi}}$$
$$s_i = \frac{1}{p_i} \left(\frac{\alpha z_i}{\underline{w}}\right)^{\frac{1}{1-\alpha}}$$

   Then holding $\left\{S, \widetilde{W}\right\}$ fixed

$$\Delta \log p_i = \frac{1}{(1-\alpha)(1+\eta)} \Delta \log z_i - \underbrace{\frac{1+(1-\alpha)\eta}{(1-\alpha)(1+\eta)}}_{>1} \Delta \log \underline{w}$$
$$\Delta \log \widetilde{w}_i = \frac{1}{(1-\alpha)(1+\eta)} \Delta \log z_i - \frac{\alpha}{(1-\alpha)(1+\eta)} \Delta \log \underline{w}$$
$$\Delta \log s_i = \frac{\eta}{[1+\eta(1-\alpha)]-\alpha} \Delta \log z_i - \frac{\eta\alpha}{(1-\alpha)(1+\eta)} \Delta \log \underline{w}$$

   First, at firm i, as $\underline{w}$ increases, the matching rate p_i declines monotonically from 1 to 0 and declines sufficiently quickly that the expected wage $\widetilde{w}_i = p_i \underline{w}$ declines. With the expected wage falling, the household allocates lower search effort.

   Second, holding $\underline{w}$ fixed and comparing firms i and j in Region III with $z_i < z_j$, then the matching rate $p_i < p_j$, hence the expected wage is lower at firm i, and search effort is also lower.

Combined, search effort is increasing quickly in z_i in Region III, then flat in Region II, and increasing at a slower rate in Region III. The matching rate is constant and 1 in Region I and II and then as productivity falls,

p_i falls in Region III. Hence *lower productivity firms* are characterized by (i) shorter queues, (ii) lower meeting rates, (iii) lower employment.