Unbundling Labor

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Data - Residual wage inequality (CPS)

Red = High skill occupations, Blue = Low skill occupations

- 3 digit occupations - Rolling classification

- Annual earnings $y_{it}$ residualized using

$$X_{i,t} = \left[ Year_t, NAICS_{1it}, Ed_{it}, Race_{it}, Sex_{it}, FirmSize_{it}, Exp_{it}, Exp_{it}^2, Hours_{it} \right]$$
In low skill occupations, how might changes in technology across occupations generate declining within occupation inequality?

Have changes in technology made workers more ‘substituteable’?
Other motivating facts that could suggest workers more ‘substitutable’

1. Experience premium: High skill, ↓ Low skill
2. Hours premium: High skill, ↓ Low skill
3. Occupation switching: High skill, ↑ Low skill
Environment

- Workers $i \in [0, 1]$ endowed with two skills $k \in \{A, B\}$
  \[ l(i) = \left(l_A(i), l_B(i)\right), \quad \overline{L}_k := \int l_k(i) \, di \]

- Final good
  \[ U(Y_1, Y_2) \]

- Task $j$ technology - $\alpha_1 = (1 - \alpha_2) > 0.5$
  \[ Y_j = F_j\left(L_{jA}, L_{jB}\right) = Z_j \left[ \alpha_j L_{jA}^\sigma + (1 - \alpha_j) L_{jB}^\sigma \right]^{\frac{1}{\sigma}}, \quad \sigma < 1 \]
Environment

- Workers $i \in [0, 1]$ endowed with two skills $k \in \{A, B\}$
  \[ l(i) = \left( l_A(i), l_B(i) \right) , \quad \overline{L}_k := \int l_k(i) \, di \]

- Final good
  \[ U \left( Y_1, Y_2 \right) \]

- Task $j$ technology - $\alpha_1 = (1 - \alpha_2) > 0.5$
  \[ Y_j = F_j \left( L_{jA}, L_{jB} \right) = Z_j \left[ \alpha_j L_{jA}^\sigma + (1 - \alpha_j) L_{jB}^\sigma \right]^{\frac{1}{\sigma}} , \quad \sigma < 1 \]
  \[ L_{jA} = \int \phi(i) l_A(i) \, di , \quad L_{jB} = \int \phi(i) l_B(i) \, di \]

1. **Bundled** - Worker $i$ must allocate $\left( l_A(i), l_B(i) \right)$ to the same task

Rosen (1983), Heckman Schenkman (1987) + GE + Solution
Environment

- Workers \( i \in [0, 1] \) endowed with two skills \( k \in \{A, B\} \)

\[
l(i) = \begin{pmatrix} l_A(i) \cr l_B(i) \end{pmatrix}, \quad \overline{L}_k := \int l_k(i) \, di
\]

- Final good

\[
U\left(Y_1, Y_2\right)
\]

- Task \( j \) technology - \( \alpha_1 = (1 - \alpha_2) > 0.5 \)

\[
Y_j = F_j\left(L_{jA}, L_{jB}\right) = Z_j \left[ \psi_{jA} \alpha_j L_{jA}^\sigma + \psi_{jB} \left(1 - \alpha_j\right) L_{jB}^\sigma \right]^{1/\sigma}, \quad \sigma < 1
\]

\[
L_{jA} = \int \phi(i) l_A(i) \, di, \quad L_{jB} = \int \phi(i) l_B(i) \, di
\]

2. Technology choice - Firms choose task level technologies \( \left(\psi_{jA}, \psi_{jB}\right) \)

\[
\left[\psi_{jA}^\rho + \psi_{jB}^\rho\right]^{1/\rho} = \overline{A}_j, \quad \rho > 1 \quad \text{Caselli Coleman (2006)}
\]
Efficient allocation

$$\max_{\phi(i) \in \{0,1\}, L_{sk}} U\left( F_1(L_{1A}, L_{1B}), F_2(L_{2A}, L_{2B}) \right)$$

subject to

$$L_{1A} = \int \phi(i) \, l_A(i) \, di$$
$$L_{1B} = \int \phi(i) \, l_B(i) \, di$$
$$L_{2A} = \int \left[ 1 - \phi(i) \right] \, l_A(i) \, di$$
$$L_{2B} = \int \left[ 1 - \phi(i) \right] \, l_B(i) \, di$$
Efficient allocation

\[
\max_{\phi(i) \in \{0,1\}, L_{sk}} U\left(F_1(L_{1A}, L_{1B}), F_2(L_{2A}, L_{2B})\right)
\]

subject to

\[
\begin{align*}
L_{1A} &= \int \phi(i) \ l_A(i) \ di \quad \rightarrow \quad \omega_{1A} = U_1 F_{1A} \\
L_{1B} &= \int \phi(i) \ l_B(i) \ di \quad \rightarrow \quad \omega_{1B} = U_1 F_{1B} \\
L_{2A} &= \int \left[ 1 - \phi(i) \right] \ l_A(i) \ di \quad \rightarrow \quad \omega_{2A} = U_2 F_{2A} \\
L_{2B} &= \int \left[ 1 - \phi(i) \right] \ l_B(i) \ di \quad \rightarrow \quad \omega_{2B} = U_2 F_{2B}
\end{align*}
\]
Efficient allocation

\[
\max_{\phi(i) \in \{0,1\}, L_{sk}} U\left(F_1(L_{1A}, L_{1B}), F_2(L_{2A}, L_{2B})\right)
\]

subject to

\[
L_{1A} = \int \phi(i) l_A(i) \, di \quad \rightarrow \quad \omega_{1A} = U_1 F_{1A}
\]

\[
L_{1B} = \int \phi(i) l_B(i) \, di \quad \rightarrow \quad \omega_{1B} = U_1 F_{1B}
\]

\[
L_{2A} = \int \left[1 - \phi(i)\right] l_A(i) \, di \quad \rightarrow \quad \omega_{2A} = U_2 F_{2A}
\]

\[
L_{2B} = \int \left[1 - \phi(i)\right] l_B(i) \, di \quad \rightarrow \quad \omega_{2B} = U_2 F_{2B}
\]

If $\omega_{1A} > \omega_{1B}$ and $\omega_{2B} > \omega_{2A}$ then

1. **Sorting** - Occupation 1 chosen by individuals with high $l_A/l_B$

2. **Inequality** - Possibly large rents to inframarginal workers if $\uparrow \omega_{1A}/\omega_{1B}$
Divisible economy

\[
\max_{\phi_A(i) \in \{0,1\}, \phi_B(i) \in \{0,1\}, L_{sk}} U\left(F_1(L_{1A}, L_{1B}), F_2(L_{2A}, L_{2B})\right)
\]

subject to

\[
\begin{align*}
L_{1A} &= \int \phi_A(i) \, l_A(i) \, di \quad \rightarrow \quad \omega_{1A} = U_1 F_{1A} \\
L_{1B} &= \int \phi_B(i) \, l_B(i) \, di \quad \rightarrow \quad \omega_{1B} = U_1 F_{1B} \\
L_{2A} &= \int \left[1 - \phi_A(i) \right] \, l_A(i) \, di \quad \rightarrow \quad \omega_{2A} = U_2 F_{2A} \\
L_{2B} &= \int \left[1 - \phi_B(i) \right] \, l_B(i) \, di \quad \rightarrow \quad \omega_{2B} = U_2 F_{2B}
\end{align*}
\]
Divisible economy

\[
\max_{\phi_A(i) \in \{0,1\}, \phi_B(i) \in \{0,1\}, L_{sk}} U \left( F_1(L_{1A}, L_{1B}), F_2(L_{2A}, L_{2B}) \right)
\]

subject to

\[
L_{1A} = \int \phi_A(i) l_A(i) \, di \quad \rightarrow \quad \omega_{1A} = U_1 F_{1A}
\]

\[
L_{1B} = \int \phi_B(i) l_B(i) \, di \quad \rightarrow \quad \omega_{1B} = U_1 F_{1B}
\]

\[
L_{2A} = \int \left[ 1 - \phi_A(i) \right] l_A(i) \, di \quad \rightarrow \quad \omega_{2A} = U_2 F_{2A}
\]

\[
L_{2B} = \int \left[ 1 - \phi_B(i) \right] l_B(i) \, di \quad \rightarrow \quad \omega_{2B} = U_2 F_{2B}
\]

Unbundled allocation

\[
\phi_A(i) = \begin{cases} 
1 & \text{, if } \omega_{1A} > \omega_{2A} \\
0 & \text{, if } \omega_{1A} < \omega_{2A} \\
\in [0,1] & \text{, if } \omega_{1A} = \omega_{2A}
\end{cases}
\]
Divisible economy

\[
\max_{\phi_A(i) \in \{0,1\}, \phi_B(i) \in \{0,1\}, L_{sk}} U\left(F_1(L_{1A}, L_{1B}), F_2(L_{2A}, L_{2B})\right)
\]

subject to

\[
\begin{align*}
L_{1A} &= \int \phi_A(i) l_A(i) \, di \quad \rightarrow \quad \omega_{1A} = U_1 F_{1A} \\
L_{1B} &= \int \phi_B(i) l_B(i) \, di \quad \rightarrow \quad \omega_{1B} = U_1 F_{1B} \\
L_{2A} &= \int \left[1 - \phi_A(i)\right] l_A(i) \, di \quad \rightarrow \quad \omega_{2A} = U_2 F_{2A} \\
L_{2B} &= \int \left[1 - \phi_B(i)\right] l_B(i) \, di \quad \rightarrow \quad \omega_{2B} = U_2 F_{2B}
\end{align*}
\]

Unbundled allocation

\[
\phi_A(i) = \begin{cases} 
\in [0, 1] & , \text{ if } \omega_{1A} = \omega_{2A} \\
\end{cases}
\]

\underline{Shadow prices of skills equalized}

\[
\begin{align*}
\omega_{1A} &= \omega_{2A} \\
\omega_{1B} &= \omega_{2B}
\end{align*}
\]
Indivisible economy

$$\max_{\phi(i) \in \{0,1\}, L_{sk}} U\left(F_1(L_{1A}, L_{1B}), F_2(L_{2A}, L_{2B})\right)$$

subject to

$$L_{1A} = \int \phi(i) \, l_A(i) \, di \quad \rightarrow \quad \omega_{1A} = U_1 F_{1A}$$

$$L_{1B} = \int \phi(i) \, l_B(i) \, di \quad \rightarrow \quad \omega_{1B} = U_1 F_{1B}$$

$$L_{2A} = \int \left[1 - \phi(i)\right] \, l_A(i) \, di \quad \rightarrow \quad \omega_{2A} = U_2 F_{2A}$$

$$L_{2B} = \int \left[1 - \phi(i)\right] \, l_B(i) \, di \quad \rightarrow \quad \omega_{2B} = U_2 F_{2B}$$
Indivisible economy

$$\max_{\phi(i) \in \{0,1\}, L_{sk}} U \left( F_1(L_{1A}, L_{1B}), F_2(L_{2A}, L_{2B}) \right)$$

subject to

$$L_{1A} = \int \phi(i) l_A(i) \, di \quad \rightarrow \quad \omega_{1A} = U_1 F_{1A}$$

$$L_{1B} = \int \phi(i) l_B(i) \, di \quad \rightarrow \quad \omega_{1B} = U_1 F_{1B}$$

$$L_{2A} = \int [1 - \phi(i)] l_A(i) \, di \quad \rightarrow \quad \omega_{2A} = U_2 F_{2A}$$

$$L_{2B} = \int [1 - \phi(i)] l_B(i) \, di \quad \rightarrow \quad \omega_{2B} = U_2 F_{2B}$$

Allocation

$$\phi(i) = \begin{cases} 
1 & \text{if } \omega_{1A} l_A(i) + \omega_{1B} l_B(i) > \omega_{2A} l_A(i) + \omega_{2B} l_B(i) \\
0 & \text{if } \omega_{1A} l_A(i) + \omega_{1B} l_B(i) < \omega_{2A} l_A(i) + \omega_{2B} l_B(i) \\
\in [0, 1] & \text{if } \omega_{1A} l_A(i) + \omega_{1B} l_B(i) = \omega_{2A} l_A(i) + \omega_{2B} l_B(i) 
\end{cases}$$
Indivisible economy

\[
\max_{\phi(i) \in \{0,1\}, L_{sk}} U\left( F_1(L_{1A}, L_{1B}), F_2(L_{2A}, L_{2B}) \right)
\]

subject to

\[
L_{1A} = \int \phi(i) l_A(i) \, di \quad \rightarrow \quad \omega_{1A} = U_1 F_{1A}
\]

\[
L_{1B} = \int \phi(i) l_B(i) \, di \quad \rightarrow \quad \omega_{1B} = U_1 F_{1B}
\]

\[
L_{2A} = \int \left[ 1 - \phi(i) \right] l_A(i) \, di \quad \rightarrow \quad \omega_{2A} = U_2 F_{2A}
\]

\[
L_{2B} = \int \left[ 1 - \phi(i) \right] l_B(i) \, di \quad \rightarrow \quad \omega_{2B} = U_2 F_{2B}
\]

Allocation

\[
\phi(i) = \begin{cases} 
1 & \text{if } (l_A(i)/l_B(i)) > (\omega_{2B} - \omega_{1B})/(\omega_{1A} - \omega_{2A}) \\
0 & \text{if } \omega_{1A} l_A(i) + \omega_{1B} l_B(i) < \omega_{2A} l_A(i) + \omega_{2B} l_B(i) \\
\in [0, 1] & \text{if } \omega_{1A} l_A(i) + \omega_{1B} l_B(i) = \omega_{2A} l_A(i) + \omega_{2B} l_B(i)
\end{cases}
\]
Indivisible economy

\[
\max_{\phi(i) \in \{0, 1\}, L_{sk}} \quad U \left( F_1(L_{1A}, L_{1B}), F_2(L_{2A}, L_{2B}) \right)
\]

subject to

\[
L_{1A} = \int \phi(i) \, l_A(i) \, di \quad \rightarrow \quad \omega_{1A} = U_1 F_{1A}
\]

\[
L_{1B} = \int \phi(i) \, l_B(i) \, di \quad \rightarrow \quad \omega_{1B} = U_1 F_{1B}
\]

\[
L_{2A} = \int \left[ 1 - \phi(i) \right] \, l_A(i) \, di \quad \rightarrow \quad \omega_{2A} = U_2 F_{2A}
\]

\[
L_{2B} = \int \left[ 1 - \phi(i) \right] \, l_B(i) \, di \quad \rightarrow \quad \omega_{2B} = U_2 F_{2B}
\]

Allocation

\[
\phi(i) = \begin{cases} 
1 & \text{, if } \frac{l_A(i)}{l_B(i)} > \frac{(\omega_{2B} - \omega_{1B})}{(\omega_{1A} - \omega_{2A})} \\
0 & \text{, if } \omega_{1A} l_A(i) + \omega_{1B} l_B(i) < \omega_{2A} l_A(i) + \omega_{2B} l_B(i) \\
\in [0, 1] & \text{, if } \text{equal} \text{ or } \{\omega_{1A} = \omega_{2A} \text{ and } \omega_{1B} = \omega_{2B}\}
\end{cases}
\]
Feasible Allocations

**Bundling constraint:** \( L_{1B} \in [\mathcal{B}(L_{1A}), \bar{\mathcal{B}}(L_{1A})] \)

- Given some \( L_{1A} \) what is the *minimum* \( L_{1B} \) bundled with it?
- Construct \( L_{1A} \) using workers with highest \( \frac{l_A(i)}{l_B(i)} \) first
Feasible Allocations

**Bundling constraint:** \( L_{1B} \in \left[ B(L_{1A}) , \overline{B}(L_{1A}) \right] \)

- Given some \( L_{1A} \) what is the *minimum* \( L_{1B} \) bundled with it?

- Construct \( L_{1A} \) using workers with highest \( \frac{l_A(i)}{l_B(i)} \) first

- In Occupation 1, a binding constraint, creates a
  - Skill A shortage, due to \( l_A \) bundled in Occupation 2 workers: \( \uparrow \omega_{1A} \)
  - Skill B surplus, due to \( l_B \) bundled in Occupation 1 workers: \( \downarrow \omega_{1B} \)
Feasible Allocations

**Bundling constraint:** $L_{1B} \in \left[ B(L_{1A}), \bar{B}(L_{1A}) \right]$

- Given some $L_{1A}$ what is the minimum $L_{1B}$ bundled with it?
- Construct $L_{1A}$ using workers with highest $l_A(i)/l_B(i)$ first
- In Occupation 1, a binding constraint, creates a
  - Skill $A$ shortage, due to $l_A$ bundled in Occupation 2 workers: $\uparrow \omega_{1A}$
  - Skill $B$ surplus, due to $l_B$ bundled in Occupation 1 workers: $\downarrow \omega_{1B}$
- Example: If $l_k(i) \sim Fréchet(\theta)$ for each skill $k$

$$B(L_{1A}) = \left( 1 - \left( 1 - \left( \frac{L_{1A}}{L_{1A}} \right)^{\frac{\theta}{\theta-1}} \right)^{\frac{\theta-1}{\theta}} \right) \bar{L}_B$$
Feasible Allocations

Feasible allocations must satisfy aggregate *bundling constraints* \( L_{1B} \in [B(L_{1A}), \overline{B}(L_{1A})] \). Determined by distribution of skill endowments only. Example: \( l_k(i) \sim Fréchet(\theta) \).
Efficient allocation

\[
\max_{L_{1A}, L_{1B}} \ U \left( F_1 \left( L_{1A}, L_{1B} \right), F_2 \left( \bar{L}_A - L_{1A}, \bar{L}_B - L_{1B} \right) \right)
\]

subject to

\[
L_{1B} \geq \text{Maximize} \ (L_{1A}) \quad \underbrace{\text{Multiplier: } \beta}_{\text{Multiplier: } \beta}
\]

\[
L_{1B} \leq \text{Maximize} \ (L_{1A}) \quad \underbrace{\text{Multiplier: } \beta}_{\text{Multiplier: } \beta}
\]
**Efficient allocation**

\[
\max_{L_{1A}, L_{1B}} U\left(F_1\left(L_{1A}, L_{1B}\right), F_2\left(\bar{L}_A - L_{1A}, \bar{L}_B - L_{1B}\right)\right)
\]

subject to

\[
L_{1B} \geq B(L_{1A})
\]

Multiplier: \(\beta\)
**Efficient allocation**

\[
\max_{L_{1A}, L_{1B}} U\left(F_1\left(L_{1A}, L_{1B}\right), F_2\left(\bar{L}_A - L_{1A}, \bar{L}_B - L_{1B}\right)\right)
\]

subject to

\[
L_{1B} \geq B(L_{1A})
\]

Multiplier: \(\beta\)

First order conditions:

\[
L_{1A} : \quad \omega_{1A} = \omega_{2A} + \beta B'(L_{1A})
\]

\[
L_{1B} : \quad \omega_{1B} = \omega_{2B} - \beta
\]

- Cost of \(L_{1A}\): Shadow value in occupation 2, and *tighten* constraint
- Cost of \(L_{1B}\): Shadow value in occupation 2, and *loosen* constraint
Unbundled allocation

Across occupation skill price differences when $\beta = 0$

\[
\omega_{1A} - \omega_{2A} = 0 \\
\omega_{2B} - \omega_{1B} = 0
\]

**Unbundled allocation** - Allocation when $\beta = 0$

(i) Across occupations – No sorting by comparative advantage

(ii) Within occupations – No wage inequality due to skill price differences

$\checkmark$ $\omega_{1A} > \omega_{1B}$, $\omega_{2B} > \omega_{2A}$ $\times$
Unbundled Allocation

‘Contract curve’ equates marginal rates of technical substitution: $F_{1A}/F_{1B} = F_{2A}/F_{2B}$. 
Unbundled allocation equates $U_1/U_2$ to marginal rate of transformation $F_{2k}/F_{1k}$. 
Bundled allocation

Across occupation skill price differences:

\[
\begin{align*}
\omega_{1A} &= \omega_{2A} + \frac{\beta B'(L_{1A})}{\omega_{2B} - \omega_{2B}} \\
\omega_{2B} &= \omega_{2A} + \beta
\end{align*}
\]

\[
\frac{\omega_{1A} - \omega_{2A}}{\omega_{2B} - \omega_{2B}} = B'(L_{1A}) = \left(\frac{l_B}{l_A}\right)^*
\]

**Bundled Allocation** - Allocation when \( \beta > 0 \)

(i) Across occupations – Sorting by comparative advantage

(ii) Within occupations – Wage inequality due to skill price differences

\[\checkmark \quad \omega_{1A} > \omega_{1B} \quad , \quad \omega_{2B} > \omega_{2A} \quad \checkmark\]
Bundled allocation

Across occupation skill price differences:

\[
\begin{align*}
\omega_{1A} &= \omega_{2A} + \beta B'(L_{1A}) \\
\omega_{2B} &= \omega_{2A} + \beta
\end{align*}
\]

\[
\frac{\omega_{1A} - \omega_{2A}}{\omega_{2B} - \omega_{2B}} = B'(L_{1A}) = \left(\frac{l_B}{l_A}\right)^*
\]

Bundled allocation - Allocation when $\beta > 0$

(i) Across occupations – Sorting by comparative advantage

(ii) Within occupations – Wage inequality due to skill price differences

\[\checkmark \quad \omega_{1A} > \omega_{1B} \quad , \quad \omega_{2B} > \omega_{2A} \quad \checkmark\]

\[
U_1\left[ F_{1A} + B'(L_{1A})F_{1B} \right] = U_2\left[ F_{2A} + B'(L_{1A})F_{2B} \right]
\]
Bundled Allocation

Bundling constraint binds. Cannot ‘break open’ workers to get at underlying skill content.

\[ U_1 \left[ F_{1A} + B'(L_{1A}) F_{1B} \right] = U_2 \left[ F_{2A} + B'(L_{1A}) F_{2B} \right] \]
Competitive equilibrium

\[
\Pi_1 = \max_{L_{1A}, L_{1B}} P_1 F_1(L_{1A}, L_{1B}) - C_1(L_{1A}, L_{1B})
\]

\[
C_1(L_{1A}, L_{1B}) = \min_{\tilde{\phi}_1(i)} \int \tilde{\phi}_1(i) w_1(l_A, l_B) \, di
\]

subject to

\[
L_{1A} = \int \tilde{\phi}_1(i) \, l_A \, di \quad \rightarrow \quad \omega_1 A = P_1 F_1 A
\]

\[
L_{1B} = \int \tilde{\phi}_1(i) \, l_B \, di \quad \rightarrow \quad \omega_1 B = P_1 F_1 B
\]

Labor demand for each type

\[
\tilde{\phi}_1(i) = \begin{cases} 
1 & \text{, if } \omega_1 A l_A(i) + \omega_1 B l_B(i) > w_1(l_A, l_B) \\
0 & \text{, if } \omega_1 A l_A(i) + \omega_1 B l_B(i) < w_1(l_A, l_B) \\
\in (0, 1) & \text{, if } \omega_1 A l_A(i) + \omega_1 B l_B(i) = w_1(l_A, l_B)
\end{cases}
\]
Competitive equilibrium

- Prices per efficiency unit of skill

\[ w_j(l_A, l_B) = \omega_jA l_A + \omega_jB l_B \]
\[ \omega_{jk} = P_j F_{jk} \]

- Worker \((l_A, l_B)\) chooses occupation \(j = 1\) only if

\[ w_1(l_A, l_B) > w_2(l_A, l_B) \]

- Cutoff worker indifferent

\[ \frac{\omega_{1A} - \omega_{2A}}{\omega_{2B} - \omega_{1B}} = \left( \frac{l_B}{l_A} \right)^* = B'(L_{1A}) \]

- Benefit of \(j = 1\)
- Relative skill in \(j = 2\)
Competitive equilibrium

- **Bundled equilibrium**: Sorting premia are increasing in $\beta$

\[
\begin{align*}
\omega_{1A} - \omega_{2A} &= \beta B'(L_{1A}) \\
\omega_{2B} - \omega_{1B} &= \beta
\end{align*}
\]

- Inframarginal workers earn rents due to comparative advantage, determined by sorting premia.
- Additional source of within-occupation wage inequality
Competitive equilibrium

• **Bundled equilibrium:** Sorting premia are increasing in $\beta$

\[
\begin{align*}
\omega_1^A - \omega_2^A &= \beta B'(L_{1A}) \\
\omega_2^B - \omega_1^B &= \beta
\end{align*}
\]

- Inframarginal workers earn rents due to comparative advantage, determined by sorting premia.
- Additional source of within-occupation wage inequality

• **Unbundled equilibrium:** Sorting premia are zero, indeterminate sorting

\[
\begin{align*}
\omega_1^A - \omega_2^A &= 0 \\
\omega_2^B - \omega_1^B &= 0
\end{align*}
\]

All workers are marginal. No rents due to comparative advantage.
When is the equilibrium bundled?

Illustrate with two nested cases: Roy Model $\alpha_j \to 1$ and Canonical Model $\theta \to 1$

1. Complete factor bias ⇒ Always bundled

\[ Y_1 = Z_1 L_{1A}, \quad Y_2 = Z_2 L_{2B} \]

One positive price for each skill: $\omega_{1A}, \omega_{2B}$

\[ \text{var} \left( \log w(i) \left| j \right. \right) = \text{var} \left( \log l_A(i) \left| i < i^* \right. \right) \]
When is the equilibrium bundled?

Illustrate with two nested cases: **Roy Model** and **Canonical Model**

\[
\alpha_j \rightarrow 1 \quad \text{and} \quad \theta \rightarrow 1
\]

1. **Complete factor bias** ⇒ Always bundled

\[
Y_1 = Z_1 L_{1A} \quad , \quad Y_2 = Z_2 L_{2B}
\]

One positive price for each skill: \( \omega_{1A} , \omega_{2B} \)

\[
\text{var} \left( \log w(i) \mid j \right) = \text{var} \left( \log l_A(i) \mid i < i^* \right)
\]

2. **‘Complete’ skill supply** ⇒ Always unbundled

\[
l \in \{ (l_A, 0) , (0, l_B) \}
\]

Law of one price for each skill: \( \omega_A , \omega_B \)

\[
\text{var} \left( \log w(i) \mid j \right) = \text{var} \left( \log (l_A(i) + l_B(i)) \right)
\]
When is the equilibrium bundled?

Illustrate with two nested cases: Roy Model and Canonical Model

$\alpha_j \to 1$  \hspace{2cm} $\theta \to 1$

1. **Complete factor bias $\Rightarrow$ Always bundled**

   \[ Y_1 = Z_1 L_{1A}, \quad Y_2 = Z_2 L_{2B} \]

   One positive price for each skill: $\omega_{1A}, \omega_{2B}$

   \[ \text{var} \left( \log w(i) \bigg| j \right) = \text{var} \left( \log l_A(i) \bigg| i < i^* \right) \]

2. **‘Complete’ skill supply $\Rightarrow$ Always unbundled**

   \[ l \in \left\{ \left(l_A, 0\right), \left(0, l_B\right) \right\} \]

   Law of one price for each skill: $\omega_A, \omega_B$

   \[ \text{var} \left( \log w(i) \bigg| j \right) = \text{var} \left( \log (l_A(i) + l_B(i)) \right) \]

   *Silent on relationship between technology and within-occupation inequality*
1. Roy Model: $Y_1 = Z_1 L_{1A}$

Equilibrium always bundled. Workers sorted by comparative advantage. Skill prices $\omega_1 A / \omega_2 B$ pinned down by relative skills of marginal worker, $x^*$. 

![Diagram showing the relationship between skills in occupation 1 and occupation 2, with labeled axes: Skill A in occupation 1 - $L_{1A}$, Skill B in occupation 1 - $L_B$, and the shaded area representing the equilibrium.]
2. Canonical Model: \( l(i) \in \{(l_A(i), 0), (0, l_B(i))\} \)

Entire set feasible. Equilibrium always unbundled, regardless of technology. Workers not sorted. All workers indifferent. No rents due to comparative advantage.
Comparative Statics

Three comparative statics to illustrate the model

1. *Symmetric increase in factor bias* - $\uparrow \alpha_j$

2. *Task-biased change* - $\uparrow Z_1$

3. *Skill-biased change* - $\uparrow \psi_A$

\[
U\left(Y_1, Y_2\right) = \left[ \eta Y_1^{\frac{\phi-1}{\phi}} + \left(1 - \eta\right) Y_2^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} \quad \phi > 1
\]

\[
Y_1 = Z_1 \left[ \psi_A \alpha_1 L_{sA}^\sigma + \left(1 - \alpha_1\right) L_{sB}^\sigma \right]^{\frac{1}{\sigma}}
\]

\[
Y_2 = \left[ \psi_A \left(1 - \alpha_2\right) L_{sA}^\sigma + \alpha_2 L_{sB}^\sigma \right]^{\frac{1}{\sigma}}
\]
1. Symmetric Increase Factor Bias

Increase factor bias $\uparrow \alpha_j$. Unbundled: $\omega_{1A} = \omega_{2A}$, $\omega_{1B} = \omega_{2B}$. Bundled: $\omega_{1A} > \omega_{2A}$, $\omega_{1B} > \omega_{2B}$. Economy shifts from unbundled equilibrium to bundled equilibrium as $\uparrow \beta$.

Other parameters: $\sigma = 0.20$, $\phi = 1$, $\theta = 2$, $\bar{L}_A = \bar{L}_B = 1$, $Z_1 = 1$. 
1. Symmetric Increase Factor Bias

Increase factor bias $\uparrow \alpha_j$. Unbundled: $\omega_{1A} = \omega_{2A}$, $\omega_{1B} = \omega_{2B}$. Bundled: $\omega_{1A} > \omega_{2A}$, $\omega_{1B} > \omega_{2B}$. Economy shifts from unbundled equilibrium to bundled equilibrium as $\uparrow \beta$.

Wage: $w(i) = \omega_{1A} l_A(i) + \omega_{1B} l_B(i)$
1. Symmetric Increase Factor Bias

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Wage: $w(i) = \omega_{1A}l_A(i) + \omega_{1B}l_B(i)$
2. Task-Biased Change

Exogenous $\uparrow Z_1$, with $\phi > 1$: $\uparrow Y_1$, $\downarrow Y_2$.
Marginal worker has more Skill $B$, pushes up $\omega_{1A}/\omega_{1B}$. Opposite for task 2.

Other parameters: $\alpha_{1A} = \alpha_{2B} = 0.80$, $\sigma = 0.20$, $\theta = 2$, $L_1 = L_2 = 1$, $Z_2 = 1$. 
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Other parameters: $\alpha_{1A} = \alpha_{2B} = 0.80$, $\sigma = 0.20$, $\theta = 2$, $L_1 = L_2 = 1$, $Z_1 = Z_2 = 1$. 
Recap

• Can generate opposite trends in within-occupation wage inequality from single shock (e.g., task-biased or skill-biased technical change)

• Increasing factor bias tightens bundling constraints
  \[ \uparrow \text{return to comparative advantage, } \uparrow \text{ sorting} \]
  \[ \uparrow \text{within-occupation wage inequality} \]

• Decreasing factor bias relaxes bundling constraints
  \[ \downarrow \text{return to comparative advantage, } \downarrow \text{ sorting} \]
  \[ \downarrow \text{within-occupation wage inequality} \]
Recap

• Can generate opposite trends in within-occupation wage inequality from single shock (e.g., task-biased or skill-biased technical change)

• Increasing factor bias tightens bundling constraints
  ↑ returns to comparative advantage, ↑ sorting
  ↑ within-occupation wage inequality

• Decreasing factor bias relaxes bundling constraints
  ↓ returns to comparative advantage, ↓ sorting
  ↓ within-occupation wage inequality

*What happens to factor bias when firms can choose their technology?*
Bachelor Farmer vs. Starbucks

\[ \uparrow \text{Skill bias} \rightarrow \text{Bundled / Sorted equilibrium} \rightarrow \uparrow \text{Inequality} \]

\[ \downarrow \text{Skill bias} \rightarrow \text{Unbundled / Unsorted equilibrium} \rightarrow \downarrow \text{Inequality} \]
Bachelor Farmer vs. Starbucks

$\uparrow$ Skill bias $\rightarrow$ Bundled / Sorted equilibrium $\rightarrow$ $\uparrow$ Inequality

$\downarrow$ Skill bias $\rightarrow$ Unbundled / Unsorted equilibrium $\rightarrow$ $\downarrow$ Inequality

- Consistent? $\downarrow$ Hours premium, $\downarrow$ Experience premium, $\uparrow$ Switching
Endogenous Technology

What is the effect of an increase in the set of available technologies?
What is the effect of an increase in the set of available technologies?
Environment

What is the effect of an increase in the set of available technologies?

1. Production given technology coefficients: $\mathbf{a}_j = (a_{jA}, a_{jB})$

$$Y_j = Z_j \left[ \alpha_j \left( a_{jA}L_{jA} \right)^\sigma + (1 - \alpha_j) \left( a_{jB}L_{jB} \right)^\sigma \right]^{1/\sigma}, \quad \sigma < 1$$
What is the effect of an increase in the set of available technologies?

1. Production given technology coefficients:  \( \mathbf{a}_j = (a_{jA}, a_{jB}) \)

\[
Y_j = Z_j \left[ \alpha_j \left( a_{jA} L_{jA} \right)^\sigma + (1 - \alpha_j) \left( a_{jB} L_{jB} \right)^\sigma \right]^{1/\sigma}, \quad \sigma < 1
\]

2. Minimize marginal cost subject to available technologies

\[
\min_{a_{jA}, a_{jB}} \frac{1}{Z_j} \left[ \left( \frac{\omega_{jA}}{\alpha_j^{1/\sigma} a_{jA}} \right)^{\frac{\sigma}{\sigma-1}} + \left( \frac{\omega_{jB}}{(1 - \alpha_j)^{1/\sigma} a_{jB}} \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\sigma-1}{\sigma}}
\]

s.t.  \[
\left[ a_{jA}^\rho + a_{jB}^\rho \right]^{1/\rho} = \overline{A}_j, \quad \rho > 1
\]
Available Technologies

Technology frontier \( [a_{jA}^\rho + a_{jB}^\rho]^{1/\rho} = \overline{A}_j \). As \( \rho \searrow 1 \), technology coefficients \( a_{jA}, a_{jB} \) ‘more substitutable’. Can reach more combinations of \( a_{jA}, a_{jB} \) for given \( \overline{A}_j \).
Equilibrium

• Skill prices determine technology adoption

\[ \omega_{jk} \implies a_{jk}^* \]

Caselli-Coleman (2006)

• Technology determines sorting and skill premia

\[ a_{jk}^* \implies \beta \geq 0 \implies \omega_{jk} \]

Rosen (1983)
Example

• Symmetric sectors

• Innate skill bias $\alpha_j = 0.8$

• Short-run $\rho = \infty$ $\implies$ $a_{jk} = 1$

• Long-run $\rho = 1$, choose technologies

• Production function CES

  $\sigma > 0$ skills are substitutes $\rightarrow$ bundling

  $\sigma < 0$ skills are complements $\rightarrow$ unbundling
Bundling Labor: $\sigma > 0$

Skills are substitutes, $\sigma > 0$. 

---

**A. Labor market equilibrium**

**B. Distribution of relative wages**

\[ \sigma > 0 \]
Bundling Labor: $\sigma > 0$

Skills are substitutes, $\sigma > 0$. Choose technology more skill biased. *Endogenously more ‘Roy-like’. Bundling constraints tighter. Wage gains polarized. Increasing inequality.*
Unbundling Labor: $\sigma < 0$

Skills are complements, $\sigma < 0$. 

A. Labor market equilibrium 

B. Distribution of relative wages

Exog. technology

$\text{var} (\log w_i) = 0.33$
Unbundling Labor: $\sigma < 0$

Skills are complements, $\sigma < 0$. Choose technology less skill biased. *Bundling constraints slack. Wage gains for generalists. Wage losses for specialists. Decreasing inequality.*
Unbundling Labor: $\downarrow \rho, \sigma < 0$

Technology frontier $[a_{jA}^\rho + a_{jB}^\rho]^{1/\rho} = \bar{A}_j$. As $\rho \searrow 1$, technology coefficients $a_{jA}, a_{jB}$ ‘more substitutable’. Can reach more combinations of $a_{jA}, a_{jA}$ for given $\bar{A}_j$. Intensity: $\alpha_{jk} a_{jk}^{\sigma_k}$
Unbundling Labor: $\downarrow \rho, \sigma < 0$

As $\rho$ falls, technologies become ‘more substitutable’. If $\sigma < 0$, firms undo existing skill bias, bundling constraints loosen, skill premia fall, wage gains for generalists. $p_A = \omega_{1A} - \omega_{2A}$
Absolute vs. Comparative Advantage

- Worker types

\[(l_1, l_2) = (\psi, \psi x) \quad , \quad (\psi, x) \sim H(\psi, x)\]

+ fixed utility of being out of the labor market

- Selection on \(x\) margin (occupation) and on \(\psi\) margin (participation)

- **Result:** Competitive equilibrium allocation is efficient
Absolute vs. Comparative Advantage

- Worker types
  \[(l_1, l_2) = (\psi, \psi x), \quad (\psi, x) \sim H(\psi, x)\]
  + fixed utility of being out of the labor market

- Selection on \(x\) margin (occupation) and on \(\psi\) margin (participation)

- **Result**: Competitive equilibrium allocation is efficient

- What are the effects of adding a mass of *low-productivity unspecialized workers* (\(\downarrow \psi, x \approx 1\))?

  (sr) wages and allocations for fixed technology
  (lr) wages and allocations for endogenous technology
Empirics

- Consider sectors \( s \) independently, and occupations \( j \in s \)
- O*NET - Two-dimensional skill intensity for each sector-occupation
  
  For each sector \( s \), obtain \( (a^s_{Ajt}, a^s_{Bjt}) \) for each occupation \( j \in s \)
- OEUS - Sector-occupation wages
  
  For each sector \( s \), obtain \( w^s_{jt} \) for each occupation \( j \in s \)
- Compare sectors

  How are \( \Delta a^s_{kjt} \) and \( \Delta w^s_{jt} \) related?

- Question  Within sectors, across occupations, do \( a^s_{jkt} \) converge?

- Question  In sectors where technologies became similar across occupations, what happened to inequality within occupations?

- Validate  How do these relate to measures of automation, ICT etc?
Empirics

- Consider sectors $s$ independently, and occupations $j \in s$

- Similarity of occupational skill requirements for each $j \in s$ \((O^*NET)\)

\[
\sigma_{jt}^s = \mathbb{E} \left[ \left( a_{jkt}^s - \bar{a}_{kt}^s \right)^2 \right] , \quad \bar{a}_{kt}^s = \mathbb{E} \left[ a_{jkt}^s \right]
\]

- Wage inequality within and between $j \in s$ \((CPS)\)

\[
\underbrace{\nabla \left[ \log w_{ijt}^s \right]}_{\text{Total}_t^s} , \quad \underbrace{\mathbb{E}_j \left[ \nabla \left[ \log w_{ijt}^s | i \in j \right] \right]}_{\text{Within}_t^s} , \quad \underbrace{\nabla_j \left[ \mathbb{E} \left[ \log w_{ijt}^s | i \in j \right] \right]}_{\text{Between}_t^s}
\]

**Question** In which industries has $\sigma_t^s$ changed over time? Are these changes correlated with use of IT capital \((BEA)\)?

**Question** In sectors $s$ in which $\downarrow \sigma_t^s$ declined, did $\downarrow Within_t^s$ decline as well, consistent with the theory?
Conclusions

• Deviations from law of one price for skills if either
  
  (i) technologies sufficiently factor biased, or
  (ii) weak pattern of comparative advantage in skills

• Can generate opposite trends in within-occupation wage inequality from single shock (e.g., task-biased or skill-biased technical change)

• If skill substitutes, technology adoption tightens bundling constraints
  \[\uparrow\text{returns to comparative advantage, } \uparrow\text{sorting}\]
  \[\uparrow\text{within-occupation wage inequality}\]

• If skills are complements, technology adoption can cause unbundling
  \[\downarrow\text{returns to comparative advantage, } \downarrow\text{sorting}\]
  \[\downarrow\text{within-occupation wage inequality}\]
Appendix
Empirics - Details

- All data based on March CPS ‘last year’ questions

- Occupation, Industry - Dorn’s 1990 harmonized cross-walk
  - Drop military
  - Occupation skill = Fraction of workers with high-school or less
  - Occupations sorted on occupation skill

- Use HPV (RED, 2010)
  - Earnings = Wage income + \((2/3)\times\) Self employment income
  - Annual hours = Weeks worked last year \(\times\) Usual hours worked per week
  - Wage = Earnings / Annual hours
  - Age 25-65, Wage > 0.5\(\times\) Federal minimum wage, Hours > One month of 8hr days

- Regression controls for residualized wage:
  - Worker education (3 levels), Industry (1 digit), Experience, Experience\(^2\)
  - Race, Log hours,
  - Experience = \((\text{age} - \max(\text{years in school},12)) - 6\)
Empirics - Regressions

1. Workers in low skill occupations getting paid more ‘similarly’.
   - Reduced form empirical evidence from the CPS
     \[
     \log Earnings_{i,t} = \gamma_t + \delta_{Occ}^{\text{period}} + \beta'_{\text{period}} X_{i,t} + \varepsilon_{i,t}
     \]
     \[X_{i,t} = \begin{bmatrix} Year_t, NAICS1_{it}, Ed_{it}, Race_{it}, Sex_{it}, FirmSize_{it}, Exp_{it}, Exp^2_{it}, Hours_{it} \end{bmatrix}\]
   - Low skill: Decline in $\downarrow \hat{\beta}_{\text{period}}$ for (i) experience, (ii) hours, (iii) large firm
   - High skill: No change

2. Anecdotal evidence from US labor market
   - Hard to explain declining level of ‘attachment’ of working age men
Data - Wage inequality

A. Total variance

\[ \nabla_t [\log \tilde{y}_{ijt}] = \sum_j \omega_{jt} \nabla_{jt} [\log \tilde{y}_{ijt}] + \sum_j \omega_{jt} \left( \mathbb{E}_{jt} [\log \tilde{y}_{ijt}] - \mathbb{E}_t [\log \tilde{y}_{ijt}] \right)^2 \]

B. Within occupation

C. Between occupation

- Red = High skill occupations, Blue = Low skill occupations
- 3 digit occupations - **Classified in 2010**

\[ X_{i,t} = [Year_t, NAICS1_{it}, Ed_{it}, Race_{it}, Sex_{it}, FirmSize_{it}, Exp_{it}, Exp^2_{it}, Hours_{it}] \]
Data - Wage inequality

A. Total variance

\[ \nabla_t \left[ \log \tilde{y}_{ijt} \right] = \sum_j \omega_{jt} \nabla_j \left[ \log \tilde{y}_{ijt} \right] + \sum_j \omega_{jt} \left( \mathbb{E}_j \left[ \log \tilde{y}_{ijt} \right] - \mathbb{E}_t \left[ \log \tilde{y}_{ijt} \right] \right)^2 \]

B. Within occupation

C. Between occupation

- Red = High skill occupations, Blue = Low skill occupations
- 3 digit occupations - **Classified in 1980**

\[ \mathbf{X}_{i,t} = \left[ \text{Year}_t, \text{NAICS1}_{it}, \text{Ed}_{it}, \text{Race}_{it}, \text{Sex}_{it}, \text{FirmSize}_{it}, \text{Exp}_{it}, \text{Exp}^2_{it}, \text{Hours}_{it} \right] \]
Decreasing hours premium in low skill occ

\( (= 1) \): wage independent of hours, \(( \geq 1) \): wage increasing in hours

\[
\log Inc_{it} = \alpha + \beta^T_{Hours} \log Hours_{it} + \beta^T_{Exp} Exp_{it} + \beta^T_{Exp^2} Exp^2_{it} + \beta^T_{Size} Size_{it} \ldots \\
+ \beta^T_X [Year_t, Race_{it}, NAICS1_{it}, Ed_{it}, Sex_{it}]
\]

Back - Motivating empirics
Decreasing experience premium in low skill occ

One extra year experience associated with 2 to 3 percent higher wage

\[ \log Inc_{it} = \alpha + \beta_{Hours} \log Hours_{it} + \beta_{Exp} Exp_{it} + \beta_{Exp^2} Exp_{it}^2 + \beta_{Size} Size_{it} \ldots + \beta_X [Year_{it}, Race_{it}, NAICS1_{it}, Ed_{it}, Sex_{it}] \]
Decreasing size premium in low skill occ

1000+ employee firms associated with a 10 to 15 percent premium

\[
\log Inc_{it} = \alpha + \beta^\tau_{Hours} \log Hours_{it} + \beta^\tau_{Exp} Exp_{it} + \beta^\tau_{Exp^2} Exp^2_{it} + \beta^\tau_{Size} Size_{it} \ldots \\
+ \beta^\tau_X [Year_t, Race_{it}, NAICS1_{it}, Ed_{it}, Sex_{it}]
\]
Fraction of **male** workers experiencing \( \left\{ E_{March}, \ldots, U_m, \ldots, E_{March'} \right\} \)

that swap **1-digit** occupations across \( \left\{ E_{March}, E_{March'} \right\} \)
Increasing switching in low skill occ

Fraction of **male** workers experiencing $\left\{ E_{March}, \ldots, U_m, \ldots, E_{March'} \right\}$

that swap **3-digit** occupations across $\left\{ E_{March}, E_{March'} \right\}$
Increasing **switching in low skill occ**

Fraction of **male** workers experiencing $\left\{ E_{Month}, E_{Month+1} \right\}$
that swap **1-digit** occupations across $\left\{ E_{Month}, E_{Month+1} \right\}$
Increasing switching in low skill occ

Fraction of male workers experiencing $\left\{ E_{Month}, E_{Month+1} \right\}$ that swap 3-digit occupations across $\left\{ E_{Month}, E_{Month+1} \right\}$