FIRM AND WORKER DYNAMICS IN A FRICTIONAL LABOR MARKET

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Princeton Macro Lunch

INTRODUCTION

- Reallocation of labor in the economy is key to understand:
  - aggregate productivity growth
  - how aggregate shocks propagate across firms
  - how worker/firm-level policies affect the macroeconomy

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  - job flows: firm dynamics → entry, growth, exit
  - worker flows: labor market histories → job ladder

- Churn → labor market frictions hinder this adjustment process

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- Two sides of labor reallocation:
  - job flows: firm dynamics → entry, growth, exit
  - worker flows: labor market histories → job ladder
- Churn → labor market frictions hinder this adjustment process
- Tons of micro data, but no unifying framework

THIS PAPER

A theory of joint firm & worker dynamics in a frictional LM

- Firms produce using DRS: enter, receive shocks, hire/fire, exit
- Workers search at random off and on the job

Challenge: value sharing with DRS

Solution

- Allocation characterized by joint value of firm + employees
- Joint value depends only on productivity + size

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- Allocation characterized by joint value of firm + employees
- Joint value depends only on productivity + size
- Caveat: need more structure to determine wages

LITERATURE

	Firm dynamics			Worker dynamics		
	Entry	Idio. shocks	Size	EU	EE	Flows by size
Competitive labor markets						
HR (1993)	✓	✓	✓			
Competitive search						
MS (2011)				✓	✓	
KK (2013)	✓	✓	✓	✓		
Schaal (2018)	✓	✓	✓	✓	✓	
Random search						
BM (1998)			✓	✓	✓	✓
PVR (2002)				✓	✓	✓
EM (2013)		✓	✓	✓		
This paper	✓	✓	✓	✓	✓	✓

Roadmap

Environment

Static example

Value functions

Coalition value

Illustration of firm dynamics

ENVIRONMENT

WORKERS

Demographics and preferences

- Unit continuum of ∞-lived, ex-ante homogenous workers
- Risk neutral with discount rate ρ
- Supply one unit of labor

Unemployed

- Produce b of the final good at home
- Meet with firms at rate λ^U

Employed

- Receive wage payment w
- Meet firms at rate $\lambda^E = \xi\lambda^U$
- Lose job endogenously, or exogenously at rate δ

Decisions: What offers to accept and when to quit

FIRMS

Production

- Endogenous mass `m` of firms, with entry cost c_e
- Subject to idiosyncratic productivity z_t

$$dz_t = \mu(z_t)dt + \sigma(z_t)dW_t$$

- Employ finitely many workers n
- Produce output of final good using only labor, $y(z, n)$

Hiring

- Hire workers by posting vacancies v
- Cost $c(v; z, n)$ includes fixed operating cost
- Vacancies meet workers at rate λ^F

Decisions: entry, vacancies/hire, fire, and exit

RANDOM SEARCH

- Single labor market
- Matches given by CRS function, $m(\mathtt{s}, \mathtt{v})$
- $\mathtt{s} = \mathtt{u} + \xi(1 - \mathtt{u})$ is total search effort of workers
- Meeting rates $\{\lambda^U, \lambda^E, \lambda^F\}$ depend only on tightness $\theta = \mathtt{v}/\mathtt{s}$
- $\phi = \mathtt{u}/\mathtt{s}$ is the probability a searching worker is unemployed

STATES

- x = firm state $(\{z, n, \{w_i\}_{i=1}^{n}, ...\})$
- i = indicator that identifies worker i
- $F(x), G\,(x)$ = vacancy/employment-weighted distribution of x

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Challenge: obtain result under plausible, **incomplete contracts**

Distributions and aggregates

CONTRACTS

Contract

- A fixed wage payment in exchange for labor services
- No collective action

Two-sided limited commitment

- Firms can fire workers, workers can quit at will

Mutual consent

- Contracts can be renegotiated only by mutual agreement
- Both parties agree to renegotiate when one has a credible threat
- A credible threat arises when one party is better off dissolving the match

Bargaining

- Firms make take-it-or-leave-it offers to workers
- Attempt to poach resolved by sequential auction (as in PV-R, 2002)

Transfers

- Transfers are allowed between workers and firms

STATIC EXAMPLE: HIRES FROM UNEMPLOYMENT

- Consider a firm with productivity z and one worker paid w_0
- Firm has met an unemployed worker → take-leave offer b

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- Technology $y(z,n) = zn^{\alpha}$
- Four cases:

$$\begin{aligned}
\text{Hire w/o renegotiation :} \quad & \boldsymbol{\pi} = z \cdot 2^{\alpha} - w_0 - b \\
\text{Hire w/ renegotiation :} \quad & \boldsymbol{\pi} = z \cdot 2^{\alpha} - b - b \\
\text{No hire w/o renegotiation :} \quad & \boldsymbol{\pi} = z \cdot 1 - w_0 \\
\text{No hire w/ renegotiation :} \quad & \boldsymbol{\pi} = z \cdot 1 - b
\end{aligned}$$

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- Technology $y(z,n) = z \cdot n^\alpha$
- Three cases, since $w_0 \geq b$ and firing threat always credible:

Hire w/o renegotiation : $\boldsymbol{\pi} = z \cdot 2^\alpha - w_0 - b$

Hire w/ renegotiation : $\boldsymbol{\pi} = z \cdot 2^\alpha - b - b$

~~No hire w/o renegotiation~~ : $\boldsymbol{\pi} = z \cdot 1 - w_0$

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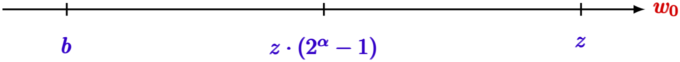
- If firm has met an employed worker, that case remains relevant

STATIC EXAMPLE: HIRES FROM UNEMPLOYMENT



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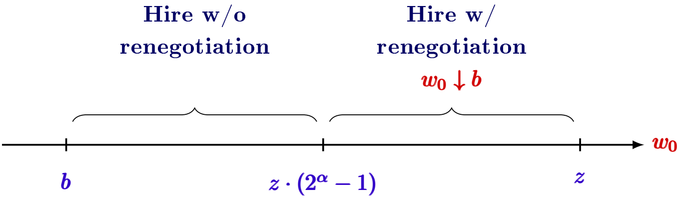


Hire w/o renegotiation

Hire w/ renegotiation

$w_0 \downarrow b$

b

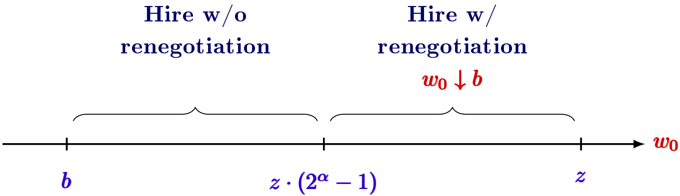
$z \cdot (2^\alpha - 1)$

z

w_0

STATIC EXAMPLE: HIRES FROM UNEMPLOYMENT

Firm hires if and only if coalition better off: $z \cdot (2^{\alpha} - 1) > b$



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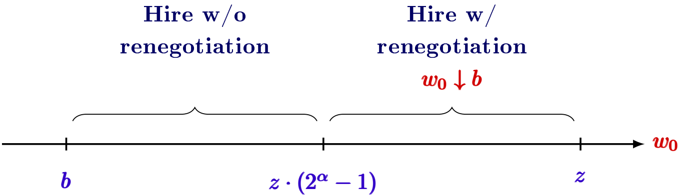

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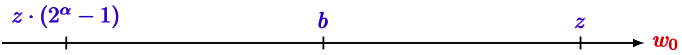

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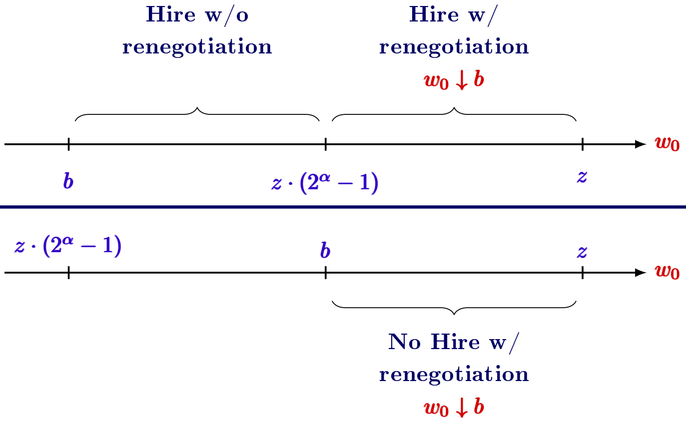

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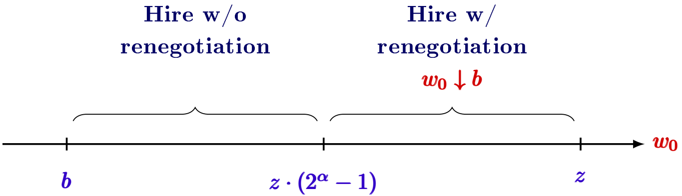
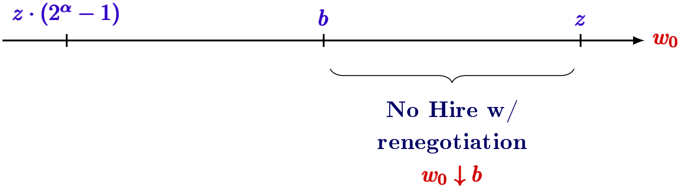
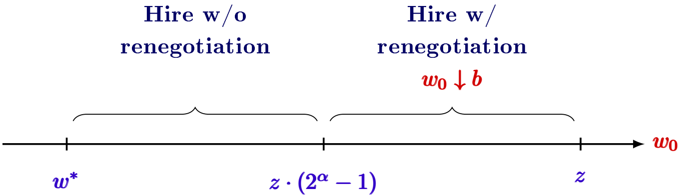




Firm does not hire if and only if coalition worse off: $z \cdot (2^{\alpha} - 1) < b$

Static example: poach from firm $(z' < z, n' = 1)$

Firm hires if and only if coalition better off: $z \cdot (2^{\alpha} - 1) > w^*$



Hire w/o renegotiation

Hire w/ renegotiation

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w^*

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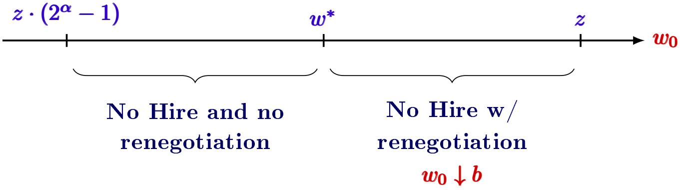
w_0



$z \cdot (2^{\alpha} - 1)$

w^*

z

w_0

No Hire and no renegotiation

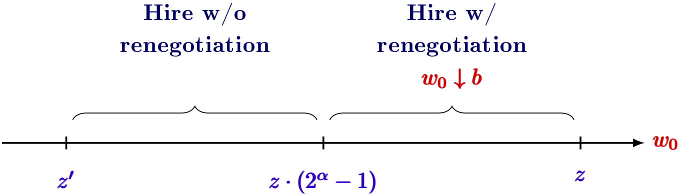
No Hire w/ renegotiation

$w_0 \downarrow b$

Firm does not hire if and only if coalition worse off: $z \cdot (2^{\alpha} - 1) < w^*$

STATIC EXAMPLE: POACH FROM FIRM $(z' < z, n' = 1)$

Firm poaches if and only if its MPL is higher: $z \cdot (2^{\alpha} - 1) > z'$



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z'

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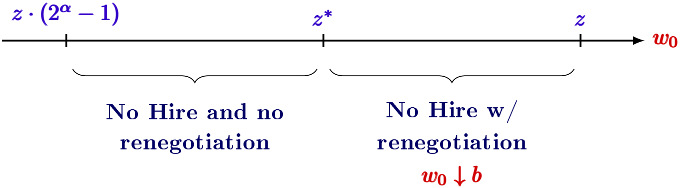

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z^*

z

w_0

No Hire and no renegotiation

No Hire w/ renegotiation

$w_0 \downarrow b$

Firm does not poach if and only if its MPL is lower: $z \cdot (2^{\alpha} - 1) < z'$

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- Now firm must pay vacancy cost $\boldsymbol{c}$ to meet unemployed worker
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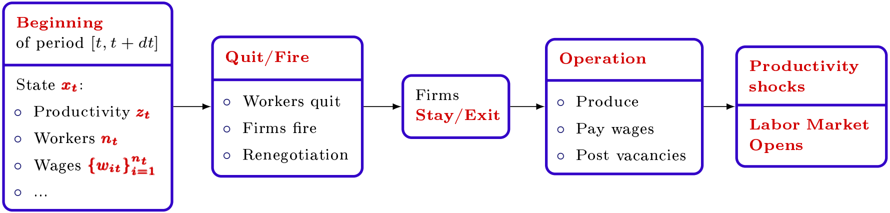
⟶ **Vacancy and hiring decisions depend only on (z,n)**

TIMING OF DECISIONS



Beginning
of period $[t, t+dt]$

State x_t:
- Productivity z_t
- Workers n_t
- Wages $\{w_{it}\}_{i=1}^{n_t}$
- ...

Quit/Fire
- Workers quit
- Firms fire
- Renegotiation

Firms
Stay/Exit

Operation
- Produce
- Pay wages
- Post vacancies

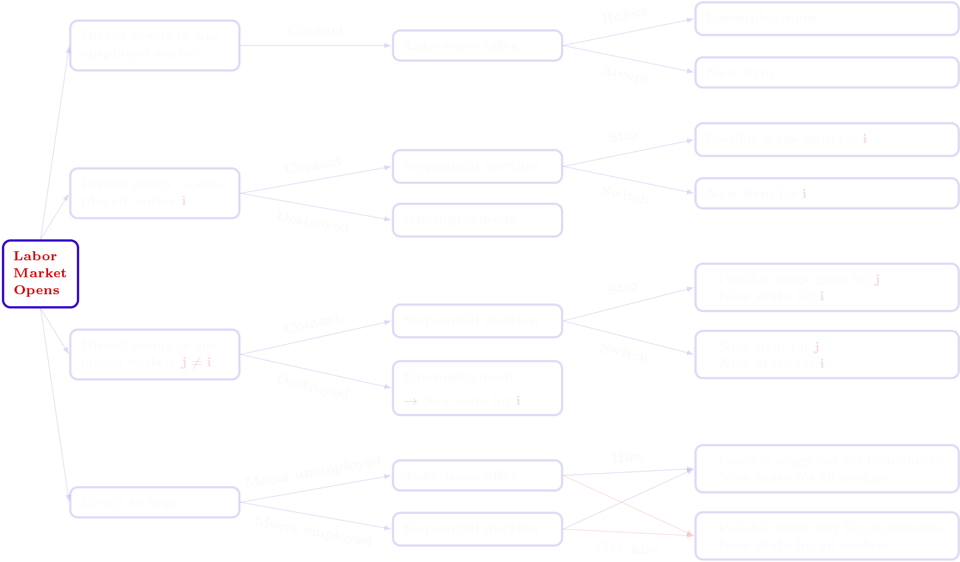
Productivity shocks

Labor Market Opens

Continuous time ⇒ labor market events are **mutually exclusive**

- At any instant a firm is hit by productivity shocks, and:
  - either one worker separates exogenously
  - or one worker is contacted by another firm
  - or the firm meets one worker
  - or none of the above

Labor market events



Labor Market Opens

Direct events to unemployed worker

Contact

Take-leave offer

Reject

Unemployment

Accept

New firm

Direct events to employed worker **i**

Contact

Sequential auction

Stay

Possible wage gain for **i**

Switch

New firm for **i**

Destroyed

Unemployment

Direct events to employed workers **j** ≠ **i**

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Sequential auction

Stay

- Possible wage gain for **j**
- New state for **i**

Switch

- New firm for **j**
- New state for **i**

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Unemployment

→ New state for **i**

Events to firm

Meets unemployed

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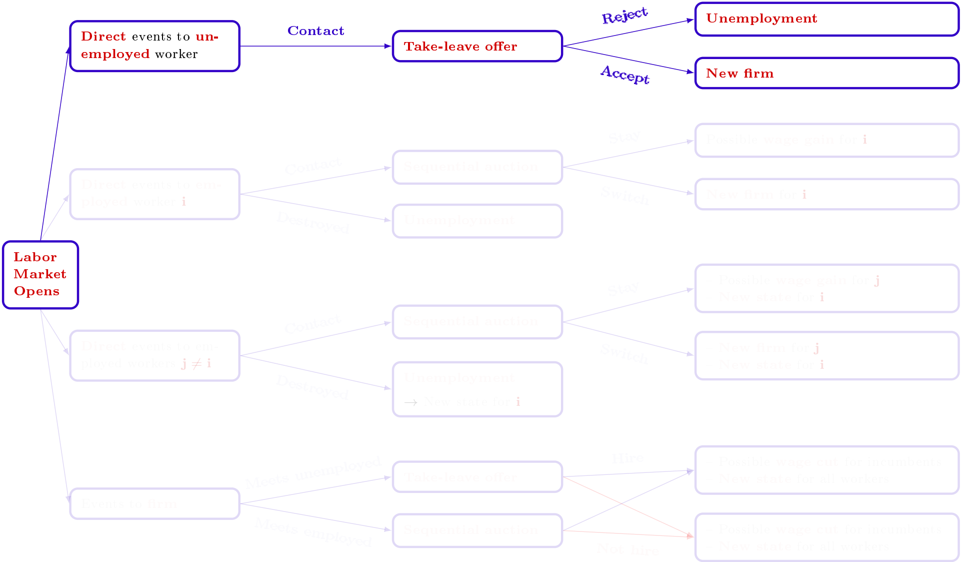
LABOR MARKET EVENTS



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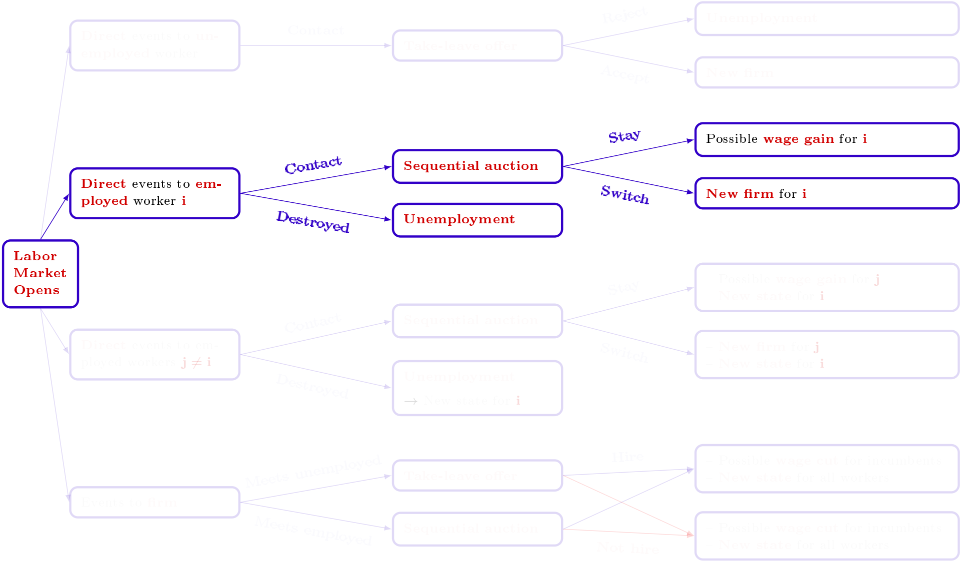

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Direct events to employed workers $\mathbf{j} \neq \mathbf{i}$

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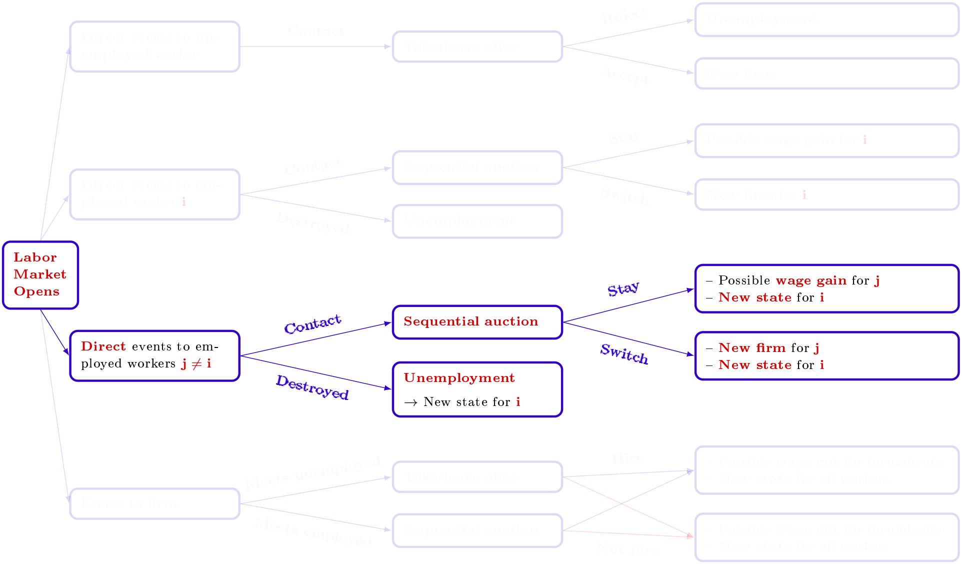

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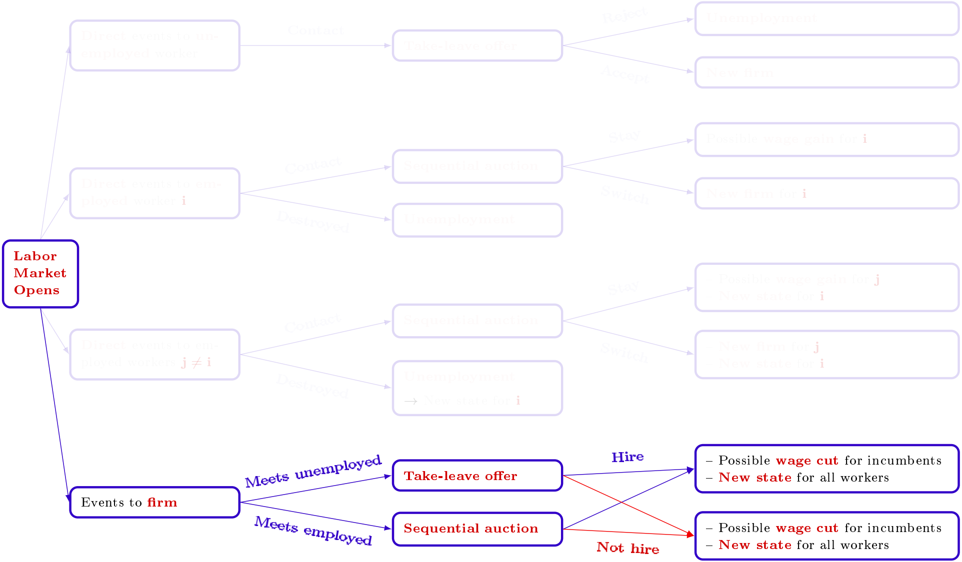

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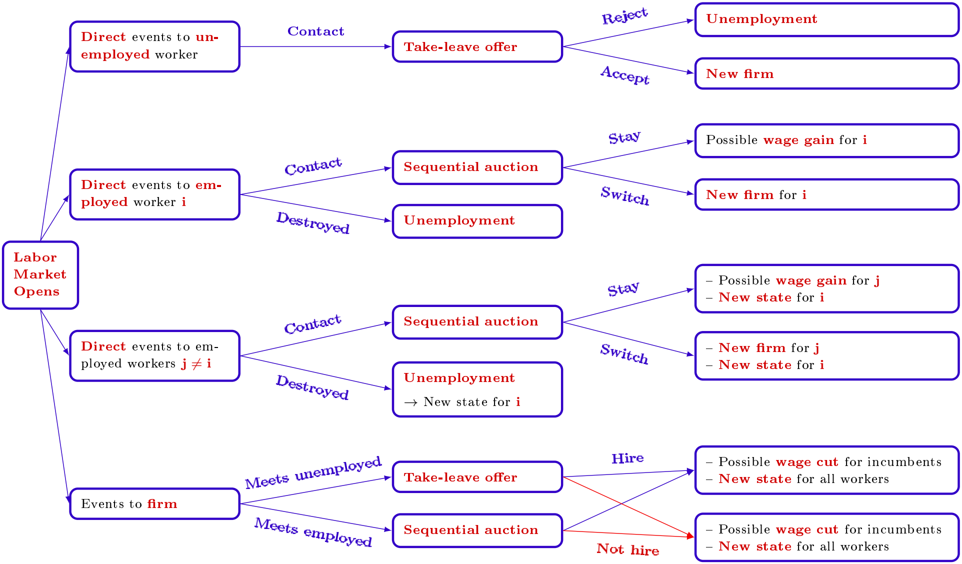

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VALUE FUNCTIONS

UNEMPLOYED WORKER

Definitions

- U = value of unemployment
- $\mathbf{V}(x, i)$ = value of employment before exit and layoff decisions

Value of unemployment

$$\rho U = b + \lambda^U \int_{x \in \mathcal{A}} \Big[\mathbf{V}(h_U(x), i) - U \Big] dF(x)$$

where

- $\mathcal{A}$ contains firms x (i) willing to hire (ii) acceptable to worker
- $h_U\,(x)$ updates x when firm hires from unemployment:

$$h_U\Big(\ z\ ,\ n\ ,\ \{\ w_i\ \}_{i=1}^{n}\ ,\ ...\ \Big)\ =\ \Big(\ z\ ,\ n+1\ ,\ \{\ w_i^*\ \}_{i=1}^{n+1}\ ,\ ...\ \Big)$$

EMPLOYED WORKER

Definition

- $V(x,i) \geq U$, value of employment after exit and layoff decisions

Value of employment

$$\rho V(x,i) = \underbrace{w(x,i)}_{\text{wage}} + \underbrace{\rho V_D(x,i)}_{\substack{\text{labor market}\\ \text{shocks to } i}} + \underbrace{\rho V_I(x,i)}_{\substack{\text{labor market}\\ \text{shocks to } j \neq i}} + \underbrace{\rho V_F(x,i)}_{\substack{\text{hiring and product-}\\ \text{ivity shocks to firm}}}$$

Details

LABOR MARKET SHOCKS DIRECTLY TO WORKER i

$$
\begin{aligned}
\rho V_D(x,i) \quad = & \quad \underbrace{\delta\left[U - V(x,i)\right]}_{\text{exogenous destruction}} \\
+ & \quad \underbrace{\lambda^E \int_{x' \in \mathcal{Q}^E(x,i)} \Big[\mathbf{V}(h_E(x,i,x'),i) - V(x,i)\Big] dF(x')}_{\text{EE transition to better firms}} \\
+ & \quad \underbrace{\lambda^E \int_{x' \notin \mathcal{Q}^E(x,i)} \Big[\mathbf{V}(r(x,i,x'),i) - V(x,i)\Big] dF(x')}_{\text{rebargaining with current employer}}
\end{aligned}
$$

where

- $\mathcal{Q}^E(x,i)$ is the set of firms x' such that worker i at x quits to x'
- $h_E(x,i,x')$ updates x' when i is hired from x
- $r(x,i,x')$ updates x when i is retained from x'

LABOR MARKET SHOCKS TO CO-WORKERS $j \neq i$

$$\begin{aligned}
\rho V_I(x, \boldsymbol{i}) \quad &= \quad \underbrace{\delta \sum_{j \neq i} \Big[\mathbf{V}(d(x,j), \boldsymbol{i}) - V(x,i) \Big]}_{\text{exogenous destructions}} \\
&+ \quad \underbrace{\lambda^E \sum_{j \neq i} \int_{x' \in \mathcal{Q}^E(x,j)} \Big[\mathbf{V}(q_E(x,j,x'), \boldsymbol{i}) - V(x,i) \Big] dF(x')}_{\text{voluntary quits to better firms}} \\
&+ \quad \underbrace{\lambda^E \sum_{j \neq i} \int_{x' \notin \mathcal{Q}^E(x,j)} \Big[\mathbf{V}(r(x,j,x'), \boldsymbol{i}) - V(x,i) \Big] dF(x')}_{\text{rebargaining with current employer}}
\end{aligned}$$

where

- $d(x, j)$ updates x when j is exogenously separated to U
- $q_E(x, i, x')$ updates x when i is hired by x'

HIRING AND PRODUCTIVITY SHOCKS TO FIRM

$$\rho V_F(x,i) \quad = \quad \underbrace{\lambda^F v(x)\phi\left[\mathbf{V}(h_U(x),i) - V(x,i)\right]\mathbb{I}_{x\in\mathcal{A}}}_{\text{hiring from U}}$$

HIRING AND PRODUCTIVITY SHOCKS TO FIRM

$$\begin{aligned}\rho V_F(x,i) &= \underbrace{\lambda^F v(x)\phi\left[\mathbf{V}(h_U(x),i) - V(x,i)\right]\mathbb{I}_{x\in\mathcal{A}}}_{\text{hiring from U}} \\ &+ \underbrace{\lambda^F v(x)\phi\left[\mathbf{V}(t_U(x),i) - V(x,i)\right]\mathbb{I}_{x\notin\mathcal{A}}}_{\text{threat of hiring from U}}\end{aligned}$$

HIRING AND PRODUCTIVITY SHOCKS TO FIRM

$$
\begin{aligned}
\rho V_F(x,i) &= \underbrace{\lambda^F v(x)\phi \left[\mathbf{V}(h_U(x),i) - V(x,i)\right]\mathbb{I}_{x\in\mathcal{A}}}_{\text{hiring from U}} \\
&+ \underbrace{\lambda^F v(x)\phi \left[\mathbf{V}(t_U(x),i) - V(x,i)\right]\mathbb{I}_{x\notin\mathcal{A}}}_{\text{threat of hiring from U}} \\
&+ \underbrace{\lambda^F v(x)(1-\phi) \int_{x\in\mathcal{Q}^E(x',i')} \left[\mathbf{V}(h_E(x',i',x),i) - V(x,i)\right] dG(x',i')}_{\text{hiring from E}}
\end{aligned}
$$

HIRING AND PRODUCTIVITY SHOCKS TO FIRM

$$
\begin{aligned}
\rho V_F(x,i) \quad = \quad & \underbrace{\lambda^F v(x)\phi\,\Big[\mathbf{V}(h_U(x),i) - V(x,i)\Big]\mathbb{I}_{x\in\mathcal{A}}}_{\text{hiring from U}} \\
+ \quad & \underbrace{\lambda^F v(x)\phi\,\Big[\mathbf{V}(t_U(x),i) - V(x,i)\Big]\mathbb{I}_{x\notin\mathcal{A}}}_{\text{threat of hiring from U}} \\
+ \quad & \underbrace{\lambda^F v(x)(1-\phi)\int_{x\in\mathcal{Q}^E(x',i')}\Big[\mathbf{V}(h_E(x',i',x),i) - V(x,i)\Big]dG(x',i')}_{\text{hiring from E}} \\
+ \quad & \underbrace{\lambda^F v(x)(1-\phi)\int_{x\notin\mathcal{Q}^E(x',i')}\Big[\mathbf{V}(t_E(x',i',x),i) - V(x,i)\Big]dG(x',i')}_{\text{threat of hiring from E}}
\end{aligned}
$$

HIRING AND PRODUCTIVITY SHOCKS TO FIRM

$$\begin{aligned}
\rho V_F(x,i) &= \underbrace{\lambda^F v(x)\phi\left[\mathbf{V}(h_U(x),i) - V(x,i)\right]\mathbb{I}_{x\in\mathcal{A}}}_{\text{hiring from U}} \\
&+ \underbrace{\lambda^F v(x)\phi\left[\mathbf{V}(t_U(x),i) - V(x,i)\right]\mathbb{I}_{x\notin\mathcal{A}}}_{\text{threat of hiring from U}} \\
&+ \underbrace{\lambda^F v(x)(1-\phi)\int\limits_{x\in\mathcal{Q}^E(x',i')}\left[\mathbf{V}(h_E(x',i',x),i) - V(x,i)\right]dG(x',i')}_{\text{hiring from E}} \\
&+ \underbrace{\lambda^F v(x)(1-\phi)\int\limits_{x\notin\mathcal{Q}^E(x',i')}\left[\mathbf{V}(t_E(x',i',x),i) - V(x,i)\right]dG(x',i')}_{\text{threat of hiring from E}} \\
&+ \underbrace{\mu(z)\frac{\partial V}{\partial z}(x,i) + \frac{\sigma^2(z)}{2}\frac{\partial^2 V}{\partial z^2}(x,i)}_{\text{shocks to firm productivity}}
\end{aligned}$$

FIRM'S PROBLEM

Definitions

- $\mathbf{J}(x)$ = value to firm prior to exit and layoff decisions
- $J \geq 0$, value to firm after these decisions
- $v(x)$ = optimal vacancy decision

Value of the firm

$$\rho J(x) = \overbrace{y(z,n) - \sum_{i=1}^{n(x)} w(x,i) - c(v(x), z, n)}^{\text{net profit}}$$

$$+ \underbrace{\rho J_I(x)}_{\text{incumbents}} + \underbrace{\rho J_F(x)}_{\text{new workers}} + \underbrace{\mu(z)\frac{\partial J}{\partial z}(x) + \frac{\sigma^2(z)}{2}\frac{\partial^2 J}{\partial z^2}(x)}_{\text{shocks to firm productivity}}$$

COALITION: DEFINITION

Definitions

- A **coalition** is composed of
  - The firm, owner of the technology
  - The workers that operate the technology at a given point in time

- Joint value of the coalition before layoffs/exit

$$\mathbf{\Omega}(x) := \mathbf{J}(x) + \sum_{i=1}^{n(x)} \mathbf{V}(x,i)$$

- Joint value of the coalition after layoffs/exit

$$\Omega(x) := J(x) + \sum_{i=1}^{n(x)} V(x,i)$$

COALITION VALUE

$$
\begin{aligned}
\rho\Omega(x) \quad &= \quad y(z,n) - c(v(x),z,n) \\
&+ \quad \delta \sum_{i=1}^{n(x)} \Big[U - (\Omega(x) - \boldsymbol{\Omega}(d(x,i))) \Big] \\
&+ \quad \lambda^E \sum_{i=1}^{n(x)} \int_{x' \notin \mathcal{Q}^E(x,i)} \Big[\boldsymbol{\Omega}(r(x,i,x')) - \Omega(x) \Big] dF(x') \\
&+ \quad \lambda^E \sum_{i=1}^{n(x)} \int_{x' \in \mathcal{Q}^E(x,i)} \Big[\mathbf{V}(h_E(x,i,x'),i) - (\Omega(x) - \boldsymbol{\Omega}(q_E(x,i,x'))) \Big] dF(x') \\
&+ \quad \lambda^F v(x) \phi \Big\{ \Big[\boldsymbol{\Omega}(h_U(x)) - \Omega(x) - \mathbf{V}(h_U(x),i) \Big] \mathbb{I}_{x \in \mathcal{A}} + \Big[\boldsymbol{\Omega}(t_U(x)) - \Omega(x) \Big] \mathbb{I}_{x \notin \mathcal{A}} \Big\} \\
&+ \quad \lambda^F v(x)(1-\phi) \int_{x \in \mathcal{Q}^E(x',i')} \Big[\boldsymbol{\Omega}(h_E(x',i',x)) - \Omega(x) - \mathbf{V}(h_E(x',i',x),i') \Big] dG(x',i') \\
&+ \quad \lambda^F v(x)(1-\phi) \int_{x \notin \mathcal{Q}^E(x',i')} \Big[\boldsymbol{\Omega}(t_E(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
&+ \quad \mu(z) \frac{\partial \Omega}{\partial z}(x) + \frac{\sigma^2(z)}{2} \frac{\partial^2 \Omega}{\partial z^2}(x)
\end{aligned}
$$

Reducing the state space

VALUE SHARING: HIRES FROM UNEMPLOYMENT

Proposition

- Firms' offer to unemployed workers

$$\mathbf{V}(h_U(x,i)) = U$$

- Firm's x acceptance set

$$\mathcal{A} = \left\{ x \;\middle|\; \boldsymbol{\Omega}(h_U(x)) - \Omega(x) \geq U \right\}$$

⟶ Firm x hires from U if marginal coalition value above U

VALUE SHARING: HIRES FROM UNEMPLOYMENT

Proof

- Timing: internal negotiation before external one
- Denote values with:
  - "$*$" after internal negotiation with incumbents
  - "$**$" after external negotiation with job seeker

VALUE SHARING: HIRES FROM UNEMPLOYMENT

Proof

- Timing: internal negotiation before external one
- Denote values with:
  - "$*$" after internal negotiation with incumbents
  - "$**$" after external negotiation with job seeker
- Firm x who has met a jobseeker hires him iff:

$$\begin{aligned} J^{**} &> J^{*} \\ J^{**} + \sum_{i=1}^{n} V_i^{*} + U &> J^{*} + \sum_{i=1}^{n} V_i^{*} + U \\ J^{**} + \sum_{i=1}^{n} V_i^{**} + U &> J^{*} + \sum_{i=1}^{n} V_i^{*} + U \\ \Omega^{**} &> \Omega^{*} + U \\ \Omega(h_U(x)) &> \Omega(x) + U \end{aligned}$$

VALUE SHARING: HIRES FROM EMPLOYMENT

Proposition

- Offers to employed workers

$$\mathbf{V}(h_E(x',i',x),i') = \Omega(x') - \boldsymbol{\Omega}(q_E(x',i',x))$$

⟶ Poached workers receive marginal coalition value of origin firm

- Employed worker (x',i') mobility set

$$\mathcal{Q}^E(x',i') = \left\{ x \;\middle|\; \boldsymbol{\Omega}(h_E(x',i',x)) - \Omega(x) \geq \Omega(x') - \boldsymbol{\Omega}(q_E(x',i',x)) \right\}$$

⟶ Workers move to firms with higher marginal coalition value

COALITION VALUE

$$
\begin{aligned}
\rho\Omega(z,n,\ldots) \quad = \quad & y(z,n) - c(v(x),z,n) \\
+ \quad & \delta \sum_{i=1}^{n(x)} \Big[U - \big(\Omega(x) - \boldsymbol{\Omega}(d(x,i))\big) \Big] \\
+ \quad & \lambda^{E} \sum_{i=1}^{n(x)} \int_{x' \notin \mathcal{Q}^{E}(x,i)} \Big[\boldsymbol{\Omega}(r(x,i,x')) - \Omega(x) \Big] dF(x') \\
+ \quad & \lambda^{E} \sum_{i=1}^{n(x)} \int_{x' \in \mathcal{Q}^{E}(x,i)} \Big[\mathbf{V}(h_{E}(x,i,x'),i) - \big(\Omega(x) - \boldsymbol{\Omega}(q_{E}(x,i,x'))\big) \Big] dF(x') \\
+ \quad & \lambda^{F} v(x)\phi \Big\{ \Big[\boldsymbol{\Omega}(h_{U}(x)) - \Omega(x) - \mathbf{V}(h_{U}(x),i) \Big] \mathbb{I}_{x \in \mathcal{A}} + \Big[\boldsymbol{\Omega}(t_{U}(x)) - \Omega(x) \Big] \mathbb{I}_{x \notin \mathcal{A}} \Big\} \\
+ \quad & \lambda^{F} v(x)(1-\phi) \int_{x \in \mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(h_{E}(x',i',x)) - \Omega(x) - \mathbf{V}(h_{E}(x',i',x),i') \Big] dG(x',i') \\
+ \quad & \lambda^{F} v(x)(1-\phi) \int_{x \notin \mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(t_{E}(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
+ \quad & \mu(z) \frac{\partial \Omega}{\partial z}(x) + \frac{\sigma^{2}(z)}{2} \frac{\partial^{2} \Omega}{\partial z^{2}}(x)
\end{aligned}
$$

COALITION VALUE: EXOGENOUS DESTRUCTIONS

$$
\begin{aligned}
\rho\Omega(z, n, \ldots) \quad = \quad & y(z, n) - c(v(x), z, n) \\
+ \quad & \delta \sum_{i=1}^{n(x)} \Big[U - (\Omega(x) - \boldsymbol{\Omega}(d(x, i))) \Big] \\
+ \quad & \lambda^E \sum_{i=1}^{n(x)} \int_{x' \notin \mathcal{Q}^E(x,i)} \Big[\boldsymbol{\Omega}(r(x, i, x')) - \Omega(x) \Big] dF(x') \\
+ \quad & \lambda^E \sum_{i=1}^{n(x)} \int_{x' \in \mathcal{Q}^E(x,i)} \Big[\mathbf{V}(h_E(x, i, x'), i) - \big(\Omega(x) - \boldsymbol{\Omega}(q_E(x, i, x'))\big) \Big] dF(x') \\
+ \quad & \lambda^F v(x) \phi \Big\{ \Big[\boldsymbol{\Omega}(h_U(x)) - \Omega(x) - \mathbf{V}(h_U(x), i) \Big] \mathbb{I}_{x \in \mathcal{A}} + \Big[\boldsymbol{\Omega}(t_U(x)) - \Omega(x) \Big] \mathbb{I}_{x \notin \mathcal{A}} \Big\} \\
+ \quad & \lambda^F v(x)(1 - \phi) \int_{x \in \mathcal{Q}^E(x', i')} \Big[\boldsymbol{\Omega}(h_E(x', i', x)) - \Omega(x) - \mathbf{V}(h_E(x', i', x), i') \Big] dG(x', i') \\
+ \quad & \lambda^F v(x)(1 - \phi) \int_{x \notin \mathcal{Q}^E(x', i')} \Big[\boldsymbol{\Omega}(t_E(x', i', x)) - \Omega(x) \Big] dG(x', i') \\
+ \quad & \mu(z) \frac{\partial \Omega}{\partial z}(x) + \frac{\sigma^2(z)}{2} \frac{\partial^2 \Omega}{\partial z^2}(x)
\end{aligned}
$$

COALITION VALUE: EXOGENOUS DESTRUCTIONS

$$
\begin{aligned}
\rho\Omega(z,n,\ldots) \quad = & \quad y(z,n) - c(v(x),z,n) \\
+ & \quad \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\ldots) - \boldsymbol{\Omega}(z,n-1,\ldots)) \Big] \\
+ & \quad \lambda^{E} \sum_{i=1}^{n(x)} \int_{x' \notin \mathcal{Q}^{E}(x,i)} \Big[\boldsymbol{\Omega}(r(x,i,x')) - \Omega(x) \Big] dF(x') \\
+ & \quad \lambda^{E} \sum_{i=1}^{n(x)} \int_{x' \in \mathcal{Q}^{E}(x,i)} \Big[\mathbf{V}(h_E(x,i,x'),i) - \big(\Omega(x) - \boldsymbol{\Omega}(q_E(x,i,x')) \big) \Big] dF(x') \\
+ & \quad \lambda^{F} v(x) \phi \Big\{ \Big[\boldsymbol{\Omega}(h_U(x)) - \Omega(x) - \mathbf{V}(h_U(x),i) \Big] \mathbb{I}_{x \in \mathcal{A}} + \Big[\boldsymbol{\Omega}(t_U(x)) - \Omega(x) \Big] \mathbb{I}_{x \notin \mathcal{A}} \Big\} \\
+ & \quad \lambda^{F} v(x)(1-\phi) \int_{x \in \mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(h_E(x',i',x)) - \Omega(x) - \mathbf{V}(h_E(x',i',x),i') \Big] dG(x',i') \\
+ & \quad \lambda^{F} v(x)(1-\phi) \int_{x \notin \mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(t_E(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
+ & \quad \mu(z) \frac{\partial \Omega}{\partial z}(x) + \frac{\sigma^2(z)}{2} \frac{\partial^2 \Omega}{\partial z^2}(x)
\end{aligned}
$$

COALITION VALUE: RENEGOTIATIONS

$$
\begin{aligned}
\rho\Omega(z,n,\ldots) \quad = \quad & y(z,n) - c(v(x),z,n) \\
+ \quad & \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\ldots) - \boldsymbol{\Omega}(z,n-1,\ldots)) \Big] \\
+ \quad & \lambda^{E} \sum_{i=1}^{n(x)} \int_{x' \notin \mathcal{Q}^{E}(x,i)} \Big[\boldsymbol{\Omega}(r(x,i,x')) - \Omega(x) \Big] dF(x') \\
+ \quad & \lambda^{E} \sum_{i=1}^{n(x)} \int_{x' \in \mathcal{Q}^{E}(x,i)} \Big[\mathbf{V}(h_E(x,i,x'),i) - \big(\Omega(x) - \boldsymbol{\Omega}(q_E(x,i,x'))\big) \Big] dF(x') \\
+ \quad & \lambda^{F} v(x)\phi \Big\{ \Big[\boldsymbol{\Omega}(h_U(x)) - \Omega(x) - \mathbf{V}(h_U(x),i) \Big] \mathbb{I}_{x\in\mathcal{A}} + \Big[\boldsymbol{\Omega}(t_U(x)) - \Omega(x) \Big] \mathbb{I}_{x\notin\mathcal{A}} \Big\} \\
+ \quad & \lambda^{F} v(x)(1-\phi) \int_{x \in \mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(h_E(x',i',x)) - \Omega(x) - \mathbf{V}(h_E(x',i',x),i') \Big] dG(x',i') \\
+ \quad & \lambda^{F} v(x)(1-\phi) \int_{x \notin \mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(t_E(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
+ \quad & \mu(z) \frac{\partial \Omega}{\partial z}(x) + \frac{\sigma^2(z)}{2} \frac{\partial^2 \Omega}{\partial z^2}(x)
\end{aligned}
$$

COALITION VALUE: RENEGOTIATIONS

$$
\begin{aligned}
\rho\Omega(z,n,\dots) &= y(z,n) - c(v(x),z,n) \\
&+ \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\dots) - \boldsymbol{\Omega}(z,n-1,\dots))\Big] \\
&+ \lambda^{E} \sum_{i=1}^{n(x)} \int_{x' \notin \mathcal{Q}^{E}(x,i)} \Big[\Omega(x) - \Omega(x)\Big] dF(x') \\
&+ \lambda^{E} \sum_{i=1}^{n(x)} \int_{x' \in \mathcal{Q}^{E}(x,i)} \Big[\mathbf{V}(h_E(x,i,x'),i) - \big(\Omega(x) - \boldsymbol{\Omega}(q_E(x,i,x'))\big)\Big] dF(x') \\
&+ \lambda^{F} v(x)\phi \Big\{ \Big[\boldsymbol{\Omega}(h_U(x)) - \Omega(x) - \mathbf{V}(h_U(x),i)\Big]\mathbb{I}_{x\in\mathcal{A}} + \Big[\boldsymbol{\Omega}(t_U(x)) - \Omega(x)\Big]\mathbb{I}_{x\notin\mathcal{A}} \Big\} \\
&+ \lambda^{F} v(x)(1-\phi) \int_{x \in \mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(h_E(x',i',x)) - \Omega(x) - \mathbf{V}(h_E(x',i',x),i')\Big] dG(x',i') \\
&+ \lambda^{F} v(x)(1-\phi) \int_{x \notin \mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(t_E(x',i',x)) - \Omega(x)\Big] dG(x',i') \\
&+ \mu(z)\frac{\partial\Omega}{\partial z}(x) + \frac{\sigma^2(z)}{2}\frac{\partial^2\Omega}{\partial z^2}(x)
\end{aligned}
$$

COALITION VALUE: RENEGOTIATIONS

$$
\begin{aligned}
\rho\Omega(z, n, \ldots) &= y(z,n) - c(v(x), z, n) \\
&+ \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\ldots) - \mathbf{\Omega}(z, n-1, \ldots)) \Big] \\
&+ \mathbf{0} \\
&+ \lambda^{E} \sum_{i=1}^{n(x)} \int_{x' \in \mathcal{Q}^{E}(x,i)} \Big[\mathbf{V}(h_E(x,i,x'),i) - \big(\Omega(x) - \mathbf{\Omega}(q_E(x,i,x'))\big) \Big] dF(x') \\
&+ \lambda^{F} v(x)\phi \Big\{ \Big[\mathbf{\Omega}(h_U(x)) - \Omega(x) - \mathbf{V}(h_U(x),i) \Big] \mathbb{I}_{x \in \mathcal{A}} + \Big[\mathbf{\Omega}(t_U(x)) - \Omega(x) \Big] \mathbb{I}_{x \notin \mathcal{A}} \Big\} \\
&+ \lambda^{F} v(x)(1-\phi) \int_{x \in \mathcal{Q}^{E}(x',i')} \Big[\mathbf{\Omega}(h_E(x',i',x)) - \Omega(x) - \mathbf{V}(h_E(x',i',x),i') \Big] dG(x',i') \\
&+ \lambda^{F} v(x)(1-\phi) \int_{x \notin \mathcal{Q}^{E}(x',i')} \Big[\mathbf{\Omega}(t_E(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
&+ \mu(z) \frac{\partial \Omega}{\partial z}(x) + \frac{\sigma^2(z)}{2} \frac{\partial^2 \Omega}{\partial z^2}(x)
\end{aligned}
$$

Coalition value: quits to employment

$$
\begin{aligned}
\rho\Omega(z,n,\ldots) &= y(z,n) - c(v(x),z,n) \\
&+ \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\ldots) - \mathbf{\Omega}(z,n-1,\ldots)) \Big] \\
&+ 0 \\
&+ \lambda^{E} \sum_{i=1}^{n(x)} \int_{x' \in \mathcal{Q}^{E}(x,i)} \Big[\mathbf{V}(h_E(x,i,x'),i) - (\Omega(x) - \mathbf{\Omega}(q_E(x,i,x'))) \Big] dF(x') \\
&+ \lambda^{F} v(x) \phi \Big\{ \Big[\mathbf{\Omega}(h_U(x)) - \Omega(x) - \mathbf{V}(h_U(x),i) \Big] \mathbb{I}_{x \in \mathcal{A}} + \Big[\mathbf{\Omega}(t_U(x)) - \Omega(x) \Big] \mathbb{I}_{x \notin \mathcal{A}} \Big\} \\
&+ \lambda^{F} v(x)(1-\phi) \int_{x \in \mathcal{Q}^{E}(x',i')} \Big[\mathbf{\Omega}(h_E(x',i',x)) - \Omega(x) - \mathbf{V}(h_E(x',i',x),i') \Big] dG(x',i') \\
&+ \lambda^{F} v(x)(1-\phi) \int_{x \notin \mathcal{Q}^{E}(x',i')} \Big[\mathbf{\Omega}(t_E(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
&+ \mu(z) \frac{\partial \Omega}{\partial z}(x) + \frac{\sigma^2(z)}{2} \frac{\partial^2 \Omega}{\partial z^2}(x)
\end{aligned}
$$

Coalition value: quits to employment

$$
\begin{aligned}
\rho\Omega(z,n,\ldots) \quad &= \quad y(z,n) - c(v(x),z,n) \\
&+ \quad \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\ldots) - \boldsymbol{\Omega}(z,n-1,\ldots)) \Big] \\
&+ \quad 0 \\
+ \ \lambda^{E} \sum_{i=1}^{n(x)} \int_{x' \in \mathcal{Q}^{E}(x,i)} & \Big[\big(\Omega(x) - \boldsymbol{\Omega}(q_E(x,i,x'))\big) - \big(\Omega(x) - \boldsymbol{\Omega}(q_E(x,i,x'))\big) \Big] dF(x') \\
&+ \quad \lambda^{F} v(x)\phi \Big\{ \Big[\boldsymbol{\Omega}(h_U(x)) - \Omega(x) - \mathbf{V}(h_U(x),i) \Big] \mathbb{I}_{x \in \mathcal{A}} + \Big[\boldsymbol{\Omega}(t_U(x)) - \Omega(x) \Big] \mathbb{I}_{x \notin \mathcal{A}} \Big\} \\
&+ \quad \lambda^{F} v(x)(1-\phi) \int_{x \in \mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(h_E(x',i',x)) - \Omega(x) - \mathbf{V}(h_E(x',i',x),i') \Big] dG(x',i') \\
&+ \quad \lambda^{F} v(x)(1-\phi) \int_{x \notin \mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(t_E(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
&+ \quad \mu(z) \frac{\partial \Omega}{\partial z}(x) + \frac{\sigma^2(z)}{2} \frac{\partial^2 \Omega}{\partial z^2}(x)
\end{aligned}
$$

COALITION VALUE: QUITS TO EMPLOYMENT

$$
\begin{aligned}
\rho\Omega(z,n,\ldots) \quad &= \quad y(z,n) - c(v(x),z,n) \\
&+ \quad \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\ldots) - \mathbf{\Omega}(z,n-1,\ldots)) \Big] \\
&+ \quad 0 \\
&+ \quad \mathbf{0} \\
&+ \quad \lambda^F v(x)\phi \Big\{ \Big[\mathbf{\Omega}(h_U(x)) - \Omega(x) - \mathbf{V}(h_U(x),i) \Big] \mathbb{I}_{x\in\mathcal{A}} + \Big[\mathbf{\Omega}(t_U(x)) - \Omega(x) \Big] \mathbb{I}_{x\notin\mathcal{A}} \Big\} \\
&+ \quad \lambda^F v(x)(1-\phi) \int_{x\in\mathcal{Q}^E(x',i')} \Big[\mathbf{\Omega}(h_E(x',i',x)) - \Omega(x) - \mathbf{V}(h_E(x',i',x),i') \Big] dG(x',i') \\
&+ \quad \lambda^F v(x)(1-\phi) \int_{x\notin\mathcal{Q}^E(x',i')} \Big[\mathbf{\Omega}(t_E(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
&+ \quad \mu(z)\frac{\partial\Omega}{\partial z}(x) + \frac{\sigma^2(z)}{2}\frac{\partial^2\Omega}{\partial z^2}(x)
\end{aligned}
$$

COALITION VALUE: HIRES FROM UNEMPLOYMENT

$$
\begin{aligned}
\rho\Omega(z,n,\ldots) \quad = \quad & y(z,n) - c(v(x),z,n) \\
+ \quad & \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\ldots) - \boldsymbol{\Omega}(z,n-1,\ldots)) \Big] \\
+ \quad & 0 \\
+ \quad & 0 \\
+ \quad & \lambda^{F} v(x)\phi \Big[\boldsymbol{\Omega}(h_U(x)) - \Omega(x) - \mathbf{V}(h_U(x), i) \Big] \mathbb{I}_{x \in \mathcal{A}} \\
+ \quad & \lambda^{F} v(x)\phi \Big[\boldsymbol{\Omega}(t_U(x)) - \Omega(x) \Big] \mathbb{I}_{x \notin \mathcal{A}} \\
+ \quad & \lambda^{F} v(x)(1-\phi) \int_{x \in \mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(h_E(x',i',x)) - \Omega(x) - \mathbf{V}(h_E(x',i',x),i') \Big] dG(x',i') \\
+ \quad & \lambda^{F} v(x)(1-\phi) \int_{x \notin \mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(t_E(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
+ \quad & \mu(z)\frac{\partial\Omega}{\partial z}(x) + \frac{\sigma^2(z)}{2}\frac{\partial^2\Omega}{\partial z^2}(x)
\end{aligned}
$$

COALITION VALUE: HIRES FROM UNEMPLOYMENT

$$
\begin{aligned}
\rho\Omega(z,n,\ldots) \quad = \quad & y(z,n) - c(v(x),z,n) \\
+ \quad & \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\ldots) - \boldsymbol{\Omega}(z,n-1,\ldots)) \Big] \\
+ \quad & 0 \\
+ \quad & 0 \\
+ \quad & \lambda^F v(z,n,\Omega(\cdot),\ldots)\phi \Big[\boldsymbol{\Omega}(z,n+1,\ldots) - \Omega(x) - U \Big] \mathbb{I}_{(z,n,\ldots)\in\mathcal{A}[z,n,\Omega(\cdot)]} \\
+ \quad & \lambda^F v(x)\phi \Big[\boldsymbol{\Omega}(t_U(x)) - \Omega(x) \Big] \mathbb{I}_{x\notin\mathcal{A}} \\
+ \quad & \lambda^F v(x)(1-\phi) \int_{x\in\mathcal{Q}^E(x',i')} \Big[\boldsymbol{\Omega}(h_E(x',i',x)) - \Omega(x) - \mathbf{V}(h_E(x',i',x),i') \Big] dG(x',i') \\
+ \quad & \lambda^F v(x)(1-\phi) \int_{x\notin\mathcal{Q}^E(x',i')} \Big[\boldsymbol{\Omega}(t_E(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
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COALITION VALUE: THREATS FROM UNEMPLOYMENT

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+ \quad & \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\dots) - \boldsymbol{\Omega}(z,n-1,\dots)) \Big] \\
+ \quad & 0 \\
+ \quad & 0 \\
+ \quad & \lambda^F v(z,n,\Omega(\cdot),\dots)\phi \Big[\boldsymbol{\Omega}(z,n+1,\dots) - \Omega(x) - U \Big] \mathbb{I}_{(z,n,\dots)\in\mathcal{A}[z,n,\Omega(\cdot)]} \\
+ \quad & \lambda^F v(x)\phi \Big[\boldsymbol{\Omega}(t_U(x)) - \Omega(x) \Big] \mathbb{I}_{x\notin\mathcal{A}} \\
+ \quad & \lambda^F v(x)(1-\phi) \int_{x\in\mathcal{Q}^E(x',i')} \Big[\boldsymbol{\Omega}(h_E(x',i',x)) - \Omega(x) - \mathbf{V}(h_E(x',i',x),i') \Big] dG(x',i') \\
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COALITION VALUE: THREATS FROM UNEMPLOYMENT

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\begin{aligned}
\rho\Omega(z, n, \ldots) \quad = \quad & y(z, n) - c(v(x), z, n) \\
+ \quad & \delta \sum_{i=1}^{n} \Big[U - (\Omega(z, n, \ldots) - \boldsymbol{\Omega}(z, n-1, \ldots)) \Big] \\
+ \quad & 0 \\
+ \quad & 0 \\
+ \quad & \lambda^F v(z, n, \Omega(\cdot), \ldots)\phi \Big[\boldsymbol{\Omega}(z, n+1, \ldots) - \Omega(x) - U \Big] \mathbb{I}_{(z,n,\ldots)\in\mathcal{A}[z,n,\Omega(\cdot)]} \\
+ \quad & \lambda^F v(x)\phi \Big[\Omega(x) - \Omega(x) \Big] \mathbb{I}_{x\notin\mathcal{A}} \\
+ \quad & \lambda^F v(x)(1-\phi) \int_{x\in\mathcal{Q}^E(x',i')} \Big[\boldsymbol{\Omega}(h_E(x', i', x)) - \Omega(x) - \mathbf{V}(h_E(x', i', x), i') \Big] dG(x', i') \\
+ \quad & \lambda^F v(x)(1-\phi) \int_{x\notin\mathcal{Q}^E(x',i')} \Big[\boldsymbol{\Omega}(t_E(x', i', x)) - \Omega(x) \Big] dG(x', i') \\
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Coalition value: threats from unemployment

$$
\begin{aligned}
\rho\Omega(z,n,\ldots) \quad = \quad & y(z,n) - c(v(x),z,n) \\
+ \quad & \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\ldots) - \boldsymbol{\Omega}(z,n-1,\ldots)) \Big] \\
+ \quad & 0 \\
+ \quad & 0 \\
+ \quad & \lambda^{F} v(z,n,\Omega(\cdot),\ldots)\phi \Big[\boldsymbol{\Omega}(z,n+1,\ldots) - \Omega(x) - U \Big] \mathbb{I}_{(z,n,\ldots)\in\mathcal{A}[z,n,\Omega(\cdot)]} \\
+ \quad & \mathbf{0} \\
+ \quad & \lambda^{F} v(x)(1-\phi) \int_{x\in\mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(h_E(x',i',x)) - \Omega(x) - \mathbf{V}(h_E(x',i',x),i') \Big] dG(x',i') \\
+ \quad & \lambda^{F} v(x)(1-\phi) \int_{x\notin\mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(t_E(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
+ \quad & \mu(z)\frac{\partial\Omega}{\partial z}(x) + \frac{\sigma^2(z)}{2}\frac{\partial^2\Omega}{\partial z^2}(x)
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COALITION VALUE: HIRES FROM EMPLOYMENT

$$
\begin{aligned}
\rho\Omega(z,n,\ldots) &= y(z,n) - c(v(x),z,n) \\
&+ \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\ldots) - \boldsymbol{\Omega}(z,n-1,\ldots)) \Big] \\
&+ 0 \\
&+ 0 \\
&+ \lambda^F v(z,n,\Omega(\cdot),\ldots)\phi \Big[\boldsymbol{\Omega}(z,n+1,\ldots) - \Omega(x) - U \Big] \mathbb{I}_{(z,n,\ldots)\in\mathcal{A}[z,n,\Omega(\cdot)]} \\
&+ 0 \\
&+ \lambda^F v(x)(1-\phi) \int_{x\in\mathcal{Q}^E(x',i')} \Big[\boldsymbol{\Omega}(h_E(x',i',x)) - \Omega(x) \\
&\qquad\qquad - \mathbf{V}(h_E(x',i',x),i') \Big] dG(x',i') \\
&+ \lambda^F v(x)(1-\phi) \int_{x\notin\mathcal{Q}^E(x',i')} \Big[\boldsymbol{\Omega}(t_E(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
&+ \mu(z)\frac{\partial\Omega}{\partial z}(x) + \frac{\sigma^2(z)}{2}\frac{\partial^2\Omega}{\partial z^2}(x)
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COALITION VALUE: HIRES FROM EMPLOYMENT

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\begin{aligned}
\rho\Omega(z,n,\dots) \quad = \quad & y(z,n) - c(v(x),z,n) \\
+ \quad & \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\dots) - \boldsymbol{\Omega}(z,n-1,\dots)) \Big] \\
+ \quad & 0 \\
+ \quad & 0 \\
+ \quad & \lambda^F v(z,n,\Omega(\cdot),\dots)\phi \Big[\boldsymbol{\Omega}(z,n+1,\dots) - \Omega(x) - U \Big] \mathbb{I}_{(z,n,\dots)\in\mathcal{A}[z,n,\Omega(\cdot)]} \\
+ \quad & 0 \\
+ \quad & \lambda^F v(x)(1-\phi) \int_{x\in\mathcal{Q}^E(x',i')} \Big[\boldsymbol{\Omega}(h_E(x',i',x)) - \Omega(x) \\
& \qquad\qquad - (\Omega(x') - \boldsymbol{\Omega}(q_E(x',i',x))) \Big] dG(x') \\
+ \quad & \lambda^F v(x)(1-\phi) \int_{x\notin\mathcal{Q}^E(x',i')} \Big[\boldsymbol{\Omega}(t_E(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
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& \qquad - \big(\Omega(z',n',\ldots) - \boldsymbol{\Omega}(z',n'-1,\ldots)\big) \Big] dG(z',n',\ldots \\
+ \quad & \lambda^{F} v(x)(1-\phi) \int_{x\notin\mathcal{Q}^{E}(x',i')} \Big[\boldsymbol{\Omega}(t_{E}(x',i',x)) - \Omega(x) \Big] dG(x',i') \\
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Coalition value: threats from employment

$$
\begin{aligned}
\rho\Omega(z,n,\ldots) &= y(z,n) - c(v(x),z,n) \\
&+ \delta\sum_{i=1}^{n}\Big[U - (\Omega(z,n,\ldots) - \boldsymbol{\Omega}(z,n-1,\ldots))\Big] \\
&+ 0 \\
&+ 0 \\
&+ \lambda^F v(z,n,\Omega(\cdot),\ldots)\phi\Big[\boldsymbol{\Omega}(z,n+1,\ldots) - \Omega(x) - U\Big]\mathbb{I}_{(z,n,\ldots)\in\mathcal{A}[z,n,\Omega(\cdot)]} \\
&+ 0 \\
&+ \lambda^F v(z,n,\Omega(\cdot),\ldots)(1-\phi)\int_{x\in\mathcal{Q}^E[z',n';\Omega'(\cdot)]}\Big[\boldsymbol{\Omega}(z,n+1,\ldots) - \Omega(z,n,\ldots) \\
&\qquad - (\Omega(z',n',\ldots) - \boldsymbol{\Omega}(z',n'-1,\ldots)\Big]dG(z',n',\ldots \\
&+ \lambda^F v(x)(1-\phi)\int_{x\notin\mathcal{Q}^E(x',i')}\Big[\boldsymbol{\Omega}(t_E(x',i',x)) - \Omega(x)\Big]dG(x',i') \\
&+ \mu(z)\frac{\partial\Omega}{\partial z}(x) + \frac{\sigma^2(z)}{2}\frac{\partial^2\Omega}{\partial z^2}(x)
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COALITION VALUE: THREATS FROM EMPLOYMENT

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\rho\Omega(z,n,\ldots) \quad = \quad & y(z,n) - c(v(x),z,n) \\
+ \quad & \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\ldots) - \boldsymbol{\Omega}(z,n-1,\ldots)) \Big] \\
+ \quad & 0 \\
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+ \quad & \lambda^F v(z,n,\Omega(\cdot),\ldots)\phi \Big[\boldsymbol{\Omega}(z,n+1,\ldots) - \Omega(x) - U \Big] \mathbb{I}_{(z,n,\ldots)\in\mathcal{A}[z,n,\Omega(\cdot)]} \\
+ \quad & 0 \\
+ \quad & \lambda^F v(z,n,\Omega(\cdot),\ldots)(1-\phi) \int_{x\in\mathcal{Q}^E[z',n';\Omega'(\cdot)]} \Big[\boldsymbol{\Omega}(z,n+1,\ldots) - \Omega(z,n,\ldots) \\
& \qquad - (\Omega(z',n',\ldots) - \boldsymbol{\Omega}(z',n'-1,\ldots)) \Big] dG(z',n',\ldots \\
+ \quad & \lambda^F v(x)(1-\phi) \int_{x\notin\mathcal{Q}^E(x',i')} \Big[\Omega(x) - \Omega(x) \Big] dG(x',i') \\
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COALITION VALUE: THREATS FROM EMPLOYMENT

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\rho\Omega(z,n,\ldots) \quad = \quad & y(z,n) - c(v(x),z,n) \\
+ \quad & \delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\ldots) - \boldsymbol{\Omega}(z,n-1,\ldots)) \Big] \\
+ \quad & 0 \\
+ \quad & 0 \\
+ \quad & \lambda^{F} v(z,n,\Omega(\cdot),\ldots)\phi \Big[\boldsymbol{\Omega}(z,n+1,\ldots) - \Omega(x) - U \Big] \mathbb{I}_{(z,n,\ldots)\in\mathcal{A}[z,n,\Omega(\cdot)]} \\
+ \quad & 0 \\
+ \quad & \lambda^{F} v(z,n,\Omega(\cdot),\ldots)(1-\phi) \int_{x\in\mathcal{Q}^{E}[z',n';\Omega'(\cdot)]} \Big[\boldsymbol{\Omega}(z,n+1,\ldots) - \Omega(z,n,\ldots) \\
& \qquad - \big(\Omega(z',n',\ldots) - \boldsymbol{\Omega}(z',n'-1,\ldots) \big) \Big] dG(z',n',\ldots \\
+ \quad & \mathbf{0} \\
+ \quad & \mu(z) \frac{\partial \Omega}{\partial z}(x) + \frac{\sigma^{2}(z)}{2} \frac{\partial^{2} \Omega}{\partial z^{2}}(x)
\end{aligned}
$$

Coalition value

$$
\begin{aligned}
\rho\Omega(z,n,\ldots) \quad = \quad & \underbrace{y(z,n) - c(v(z,n,\Omega(\cdot),\ldots,z,n)}_{\text{flow net output}} \\
+ \quad & \underbrace{\delta \sum_{i=1}^{n} \Big[U - (\Omega(z,n,\ldots) - \boldsymbol{\Omega}(z,n-1,\ldots)) \Big]}_{\text{exogenous destructions}} \\
+ \quad & \underbrace{\lambda^F v(z,n,\Omega(\cdot),\ldots)\phi \Big[\boldsymbol{\Omega}(z,n+1,\ldots) - \Omega(z,n,\ldots) - U \Big] \mathbb{I}_{(z,n,\ldots)\in\mathcal{A}[z,n,\Omega(\cdot)]}}_{\text{hires from U}} \\
+ \quad & \underbrace{\lambda^F v(z,n,\Omega(\cdot),\ldots)(1-\phi) \int_{(z,n,\ldots)\in\mathcal{Q}^E[z',n';\Omega'(\cdot)]} \Big[\boldsymbol{\Omega}(z,n+1,\ldots) - \Omega(z,n,\ldots) - (\Omega(z',n',\ldots) - \boldsymbol{\Omega}(z',n'-1,\ldots)) \Big] dG(z',n',\ldots)}_{\text{hires from E}} \\
+ \quad & \underbrace{\mu(z)\frac{\partial\Omega}{\partial z}(z,n,\ldots) + \frac{\sigma^2(z)}{2}\frac{\partial^2\Omega}{\partial z^2}(z,n,\ldots)}_{\text{productivity shocks}}
\end{aligned}
$$

COALITION VALUE

Proposition: (z, n) are the only **payoff-relevant** states

COALITION VALUE

Proposition: **(z, n)** are the only **payoff-relevant** states

$$
\begin{aligned}
\rho\Omega(z,n) &= y(z,n) - c(v(z,n), z, n) \\
&+ \delta \sum_{i=1}^{n} \Big[U - \left(\Omega(z,n) - \boldsymbol{\Omega}(z,n-1)\right) \Big] \\
&+ \lambda^F v(z,n) \phi \Big[\boldsymbol{\Omega}(z,n+1) - \Omega(z,n) - U \Big] \mathbb{I}_{(z,n) \in \mathcal{A}(z,n)} \\
&+ \lambda^F v(z,n) (1-\phi) \int_{(z,n) \in \mathcal{Q}^E(z',n')} \Big[\boldsymbol{\Omega}(z,n+1) - \Omega(z,n) \\
&\qquad\qquad - \big(\Omega(z',n') - \boldsymbol{\Omega}(z',n'-1) \big) \Big] dG(z',n') \\
&+ \mu(z) \frac{\partial \Omega}{\partial z}(z,n) + \frac{\sigma^2(z)}{2} \frac{\partial^2 \Omega}{\partial z^2}(z,n)
\end{aligned}
$$

COALITION VALUE: CONTINUOUS WORKFORCE

$$\begin{aligned}\rho\Omega\left(z,n\right)=\max_{v\geq0}\quad & \underbrace{y\left(z,n\right)-c\left(v;z,n\right)}_{\text{flow net output}}\\ + \quad & \underbrace{\left(\lambda^{F}\phi v-\delta n\right)\cdot\left[\Omega_{n}(z,n)-U\right]}_{\text{net hires from U}}\\ + \quad & \underbrace{\lambda^{F}v\left(1-\phi\right)\cdot\int_{U}^{\Omega_{n}\left(z,n\right)}\left[\Omega_{n}(z,n)-\Omega_{n}(z',n')\right]dG\left(z',n'\right)}_{\text{hires from E}}\\ + \quad & \underbrace{\mu(z)\Omega_{z}(z,n)+\frac{\sigma^{2}(z)}{2}\Omega_{zz}(z,n)}_{\text{productivity shocks}}\end{aligned}$$

subject to:

$$\underbrace{\mathbb{E}[\Omega(z,n_0)]\ \leq\ n_0U+c_e}_{\textbf{entry}\text{ if }=}\quad;\quad\underbrace{\Omega(z,n)\ \geq\ nU}_{\textbf{exit}\text{ if }=}\quad;\quad\underbrace{\Omega_n(z,n)\ \geq\ U}_{\textbf{endogenous separations}\text{ if }=}$$

Discrete workforce

PROPOSITION: COMPARATIVE STATICS

Assumptions

- GBM: $\mu(z) = \mu \times z, \ \sigma(z) = \sigma \times z$
- $c(v, z, n) = c_0 v^{1+\gamma}, \ \gamma > 0$
- $y_z > 0, \ y_n > 0$
- DRS: $y_{nn} < 0$
- Higher z raises the MPL: $y_{zn} > 0$

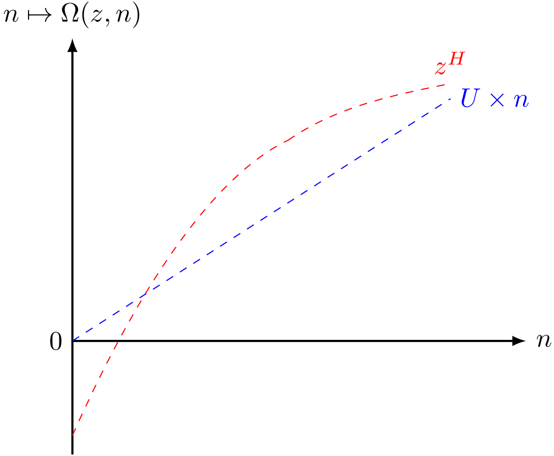
Coalition value

- Increasing and concave in employment: $\Omega_n > 0, \ \Omega_{nn} < 0$
- Increasing in productivity: $\Omega_z > 0$
- Higher z raises the marginal value of labor: $\Omega_{zn} > 0$

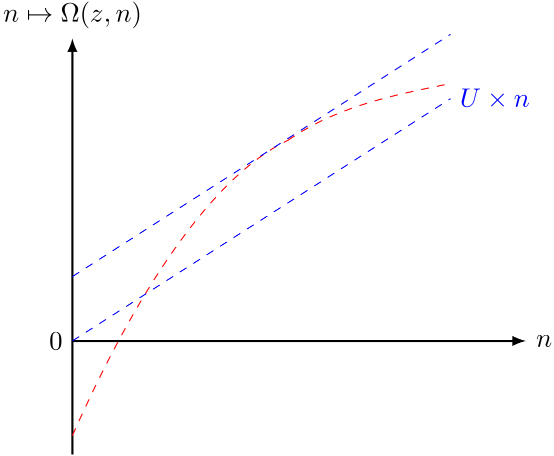
Hiring Net employment growth increases with z, decreases with n

ILLUSTRATION

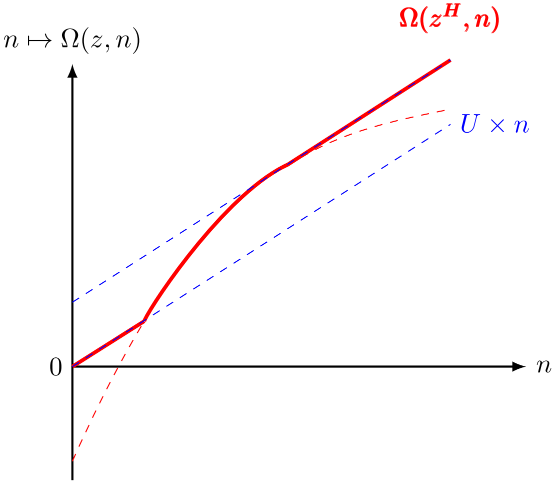
EXIT, HIRE AND SEPARATION REGIONS: HIGH z



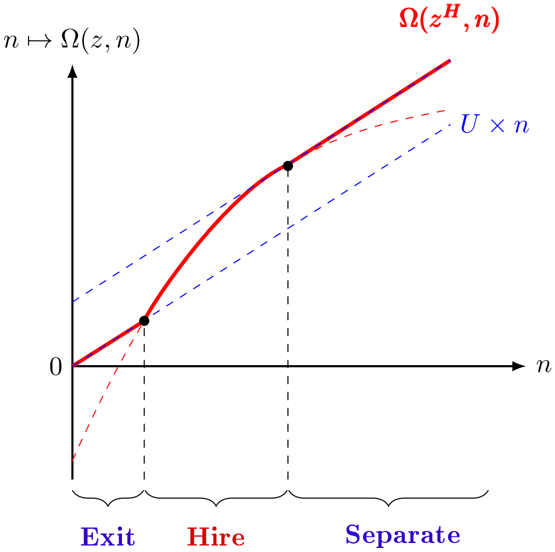
EXIT, HIRE AND SEPARATION REGIONS: HIGH z



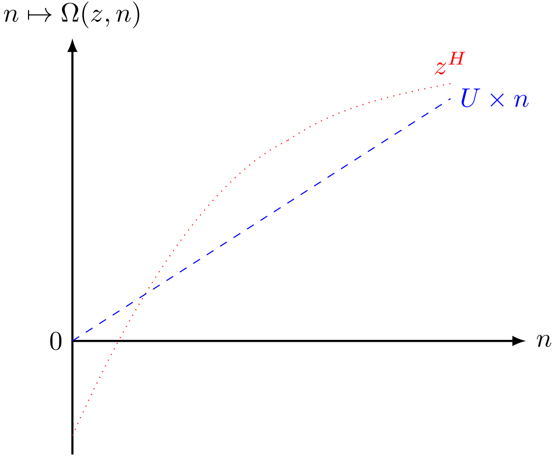
EXIT, HIRE AND SEPARATION REGIONS: HIGH z



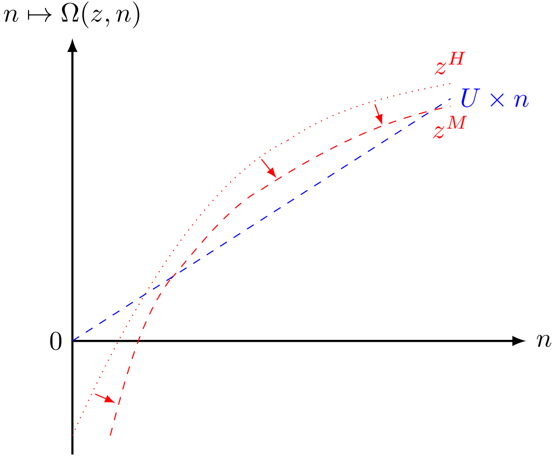
EXIT, HIRE AND SEPARATION REGIONS: HIGH z



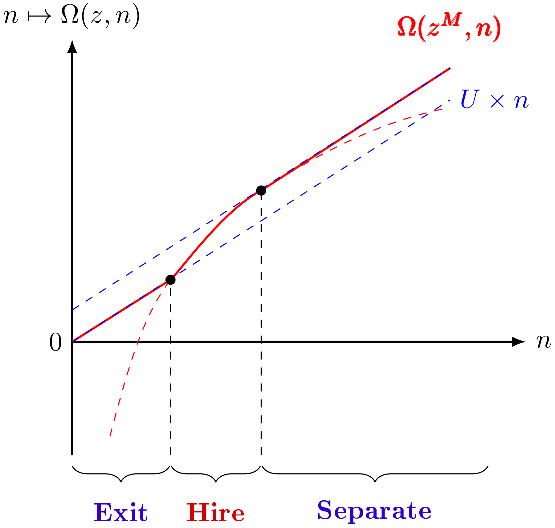
EXIT, HIRE AND SEPARATION REGIONS: MEDIUM z



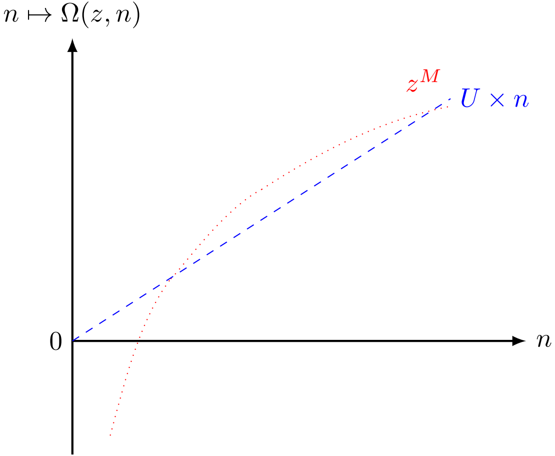
EXIT, HIRE AND SEPARATION REGIONS: MEDIUM z



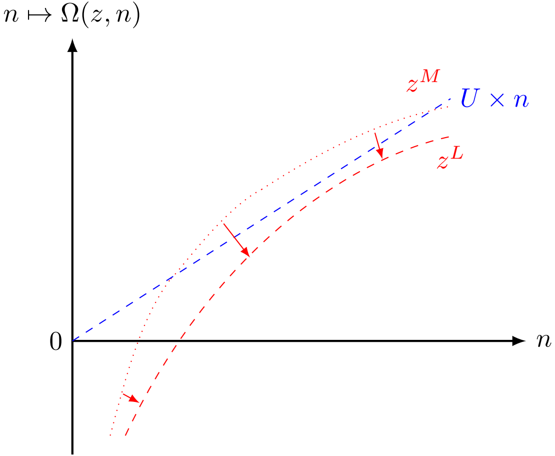
EXIT, HIRE AND SEPARATION REGIONS: MEDIUM z



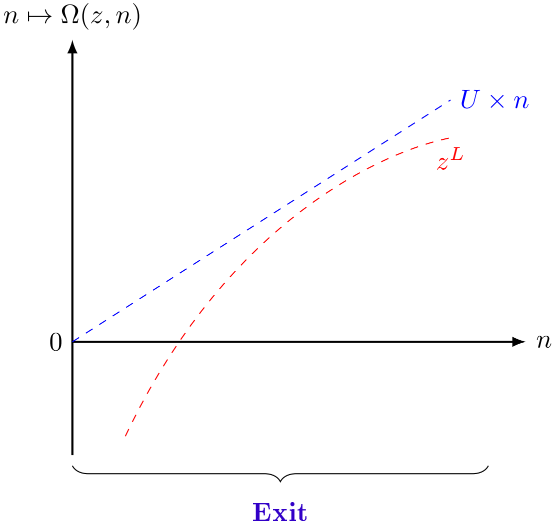
EXIT, HIRE AND SEPARATION REGIONS: LOW z



EXIT, HIRE AND SEPARATION REGIONS: LOW z

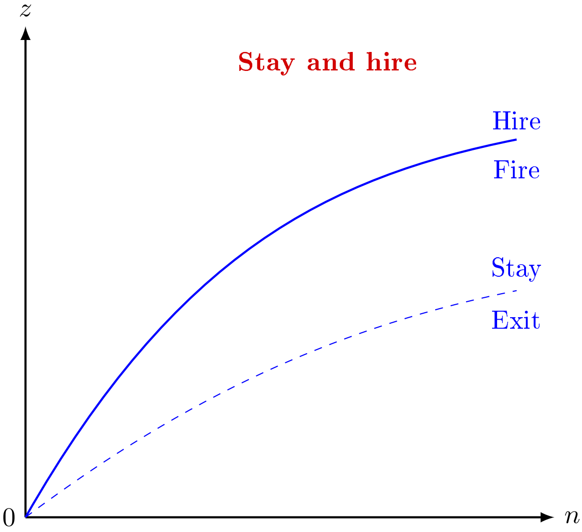


EXIT, HIRE AND SEPARATION REGIONS: LOW z



EXIT, HIRE AND SEPARATION REGIONS IN (n, z) SPACE

Suppose no fixed cost: $c_f = 0$.



EXIT, HIRE AND SEPARATION REGIONS IN (n, z) SPACE

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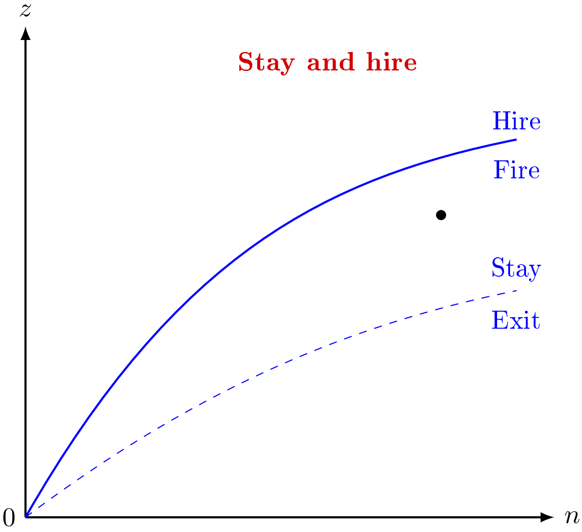

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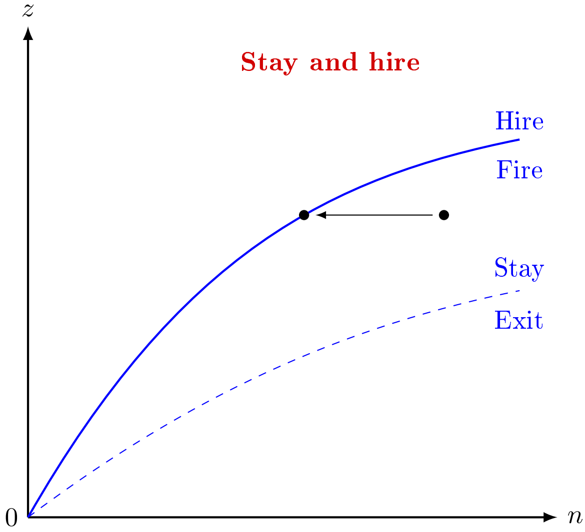

EXIT, HIRE AND SEPARATION REGIONS IN (n, z) SPACE

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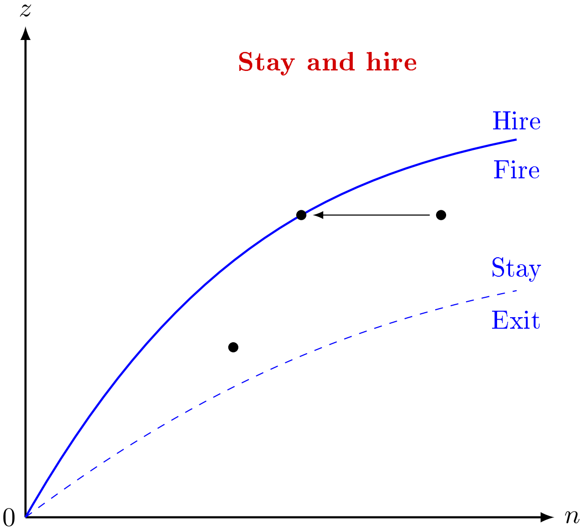

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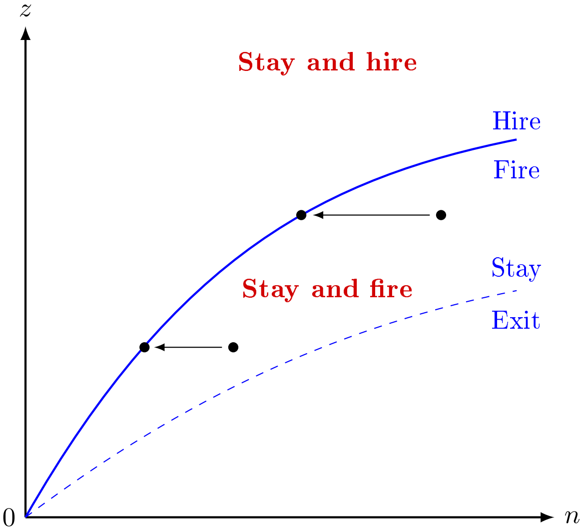
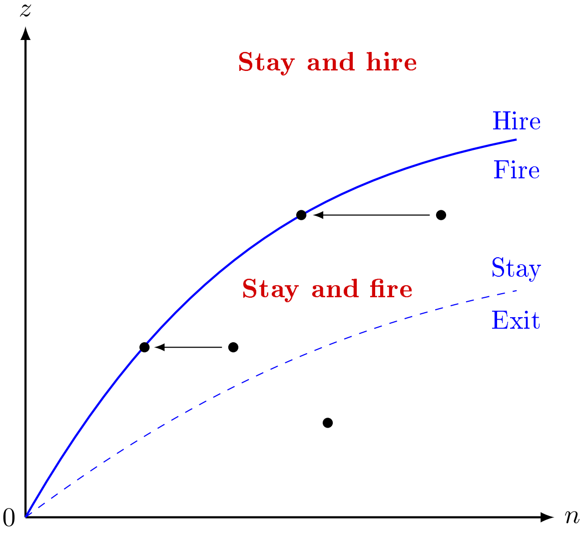


EXIT, HIRE AND SEPARATION REGIONS IN (n, z) SPACE

Suppose no fixed cost: $\boldsymbol{c_f = 0}$.



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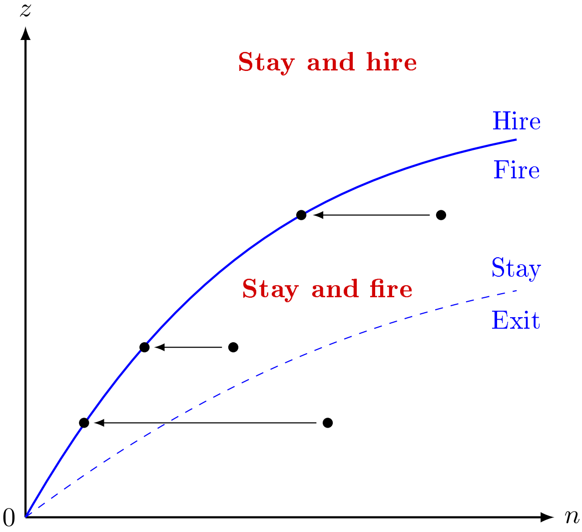

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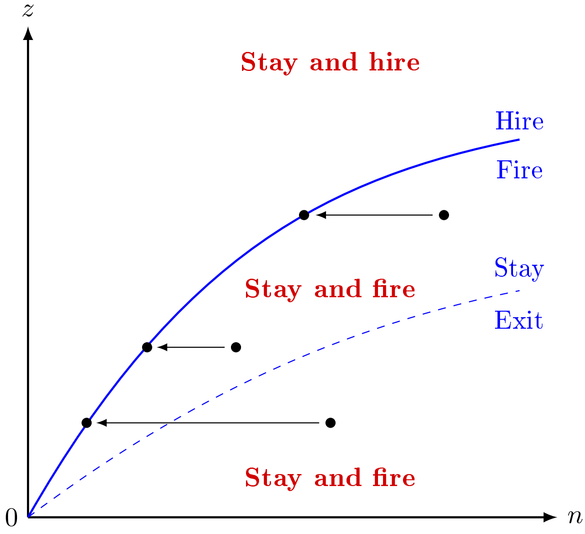

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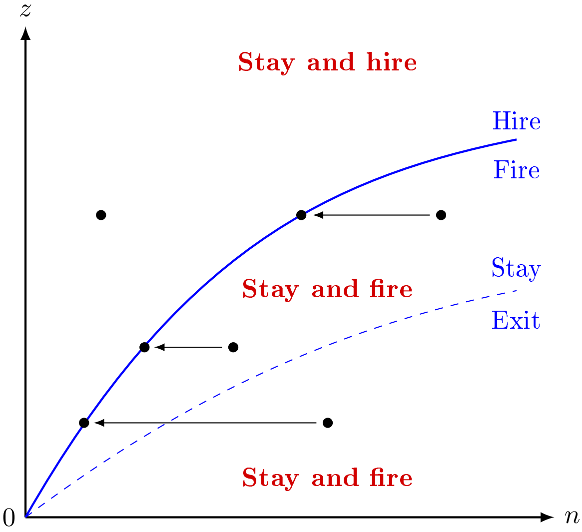

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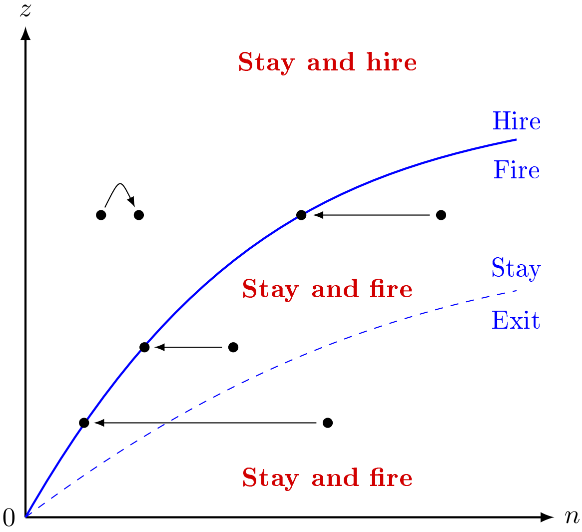

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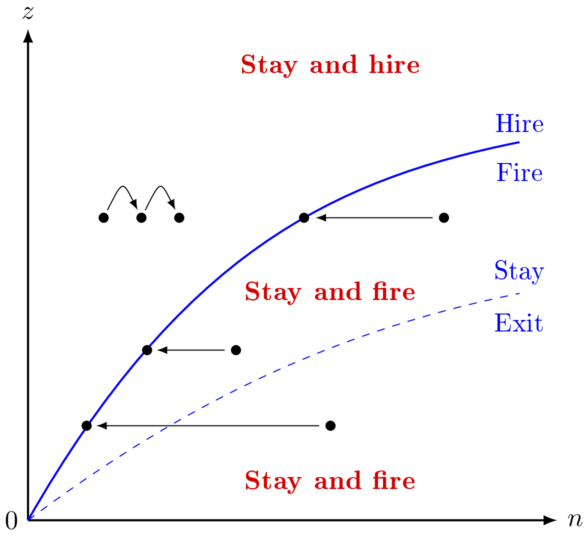

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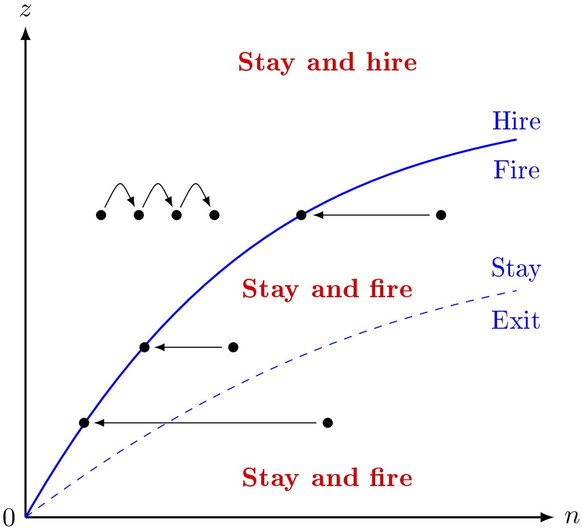

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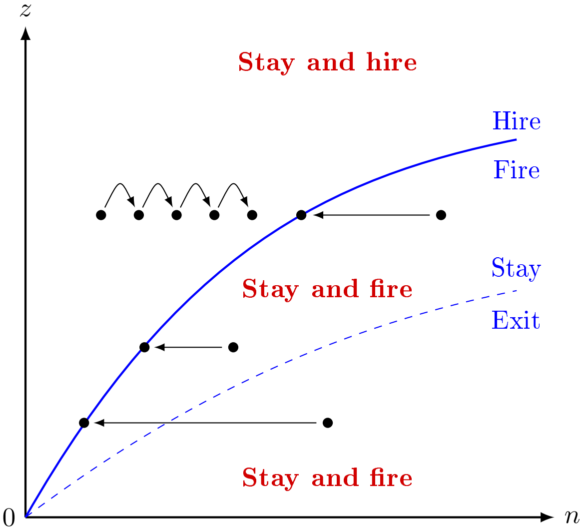

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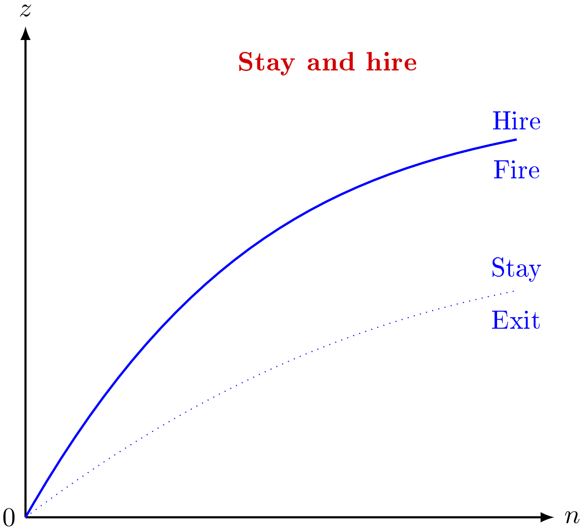

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Exit, hire and separation regions in (n, z) space

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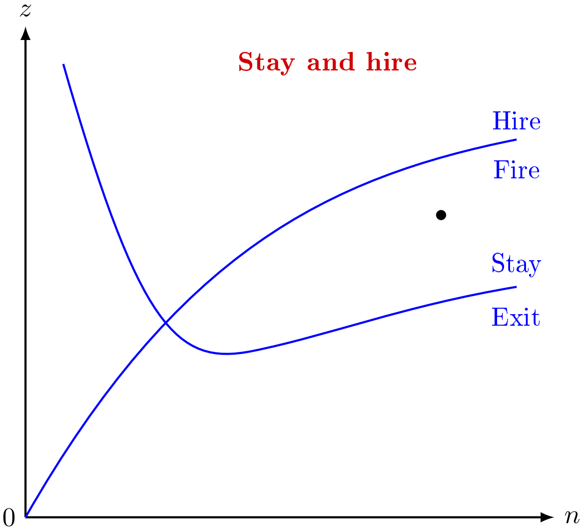

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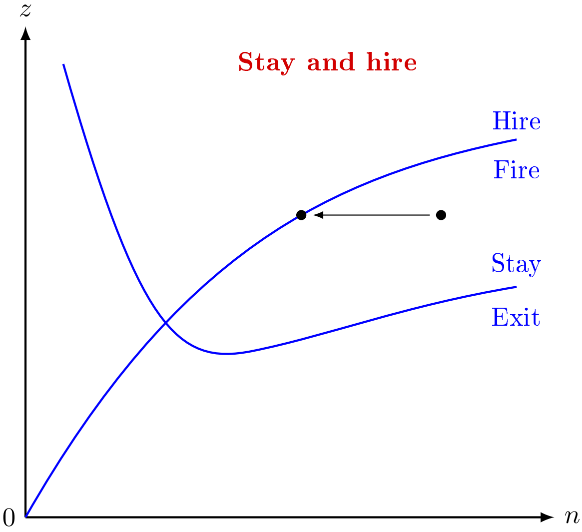

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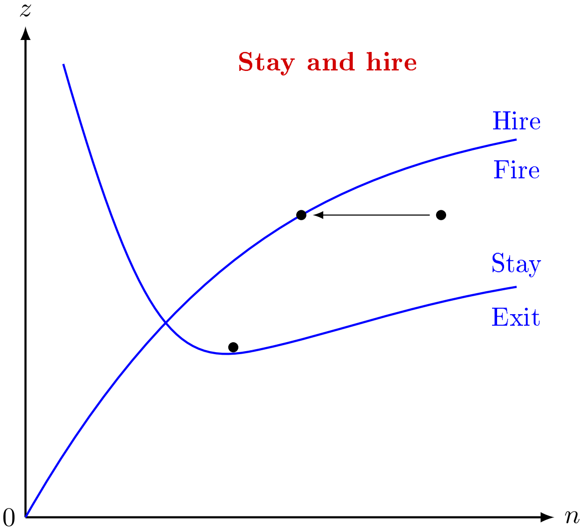

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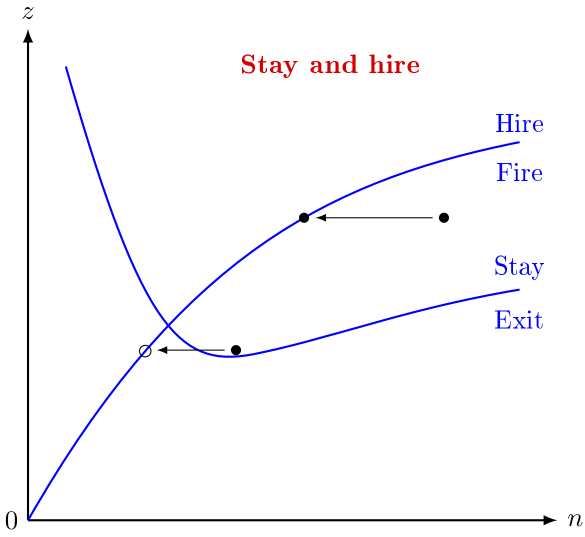

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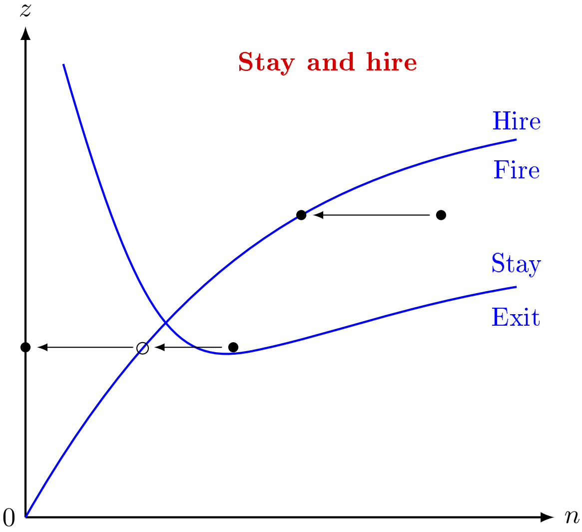
Suppose positive fixed cost: $c_f > 0$. Then some firms exit.



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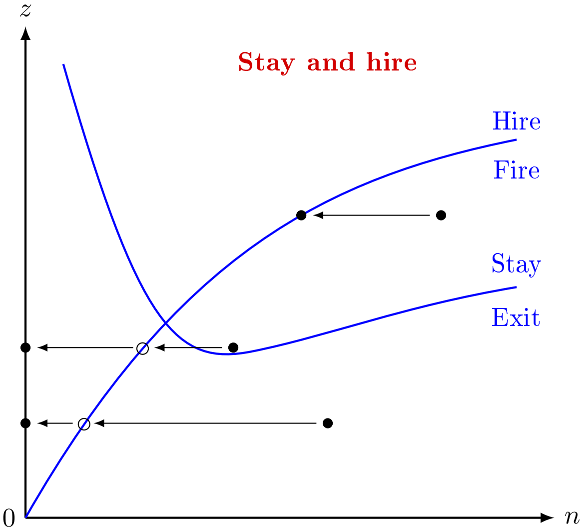

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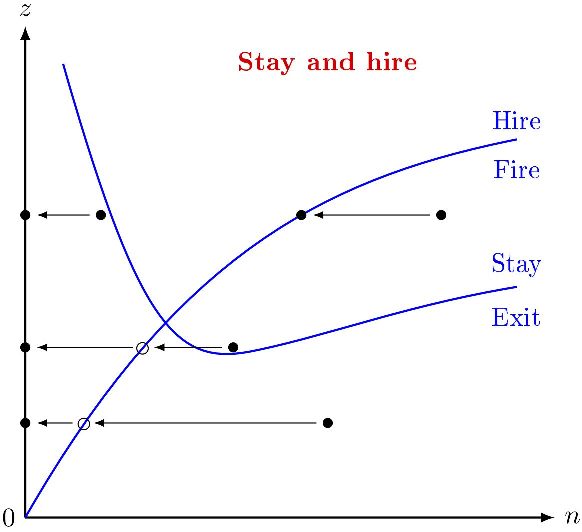

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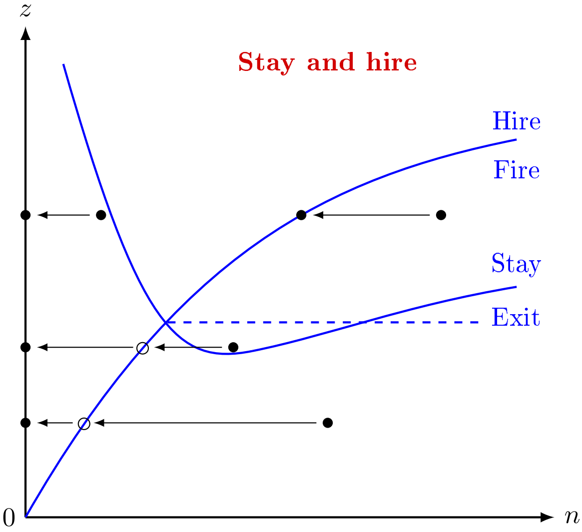

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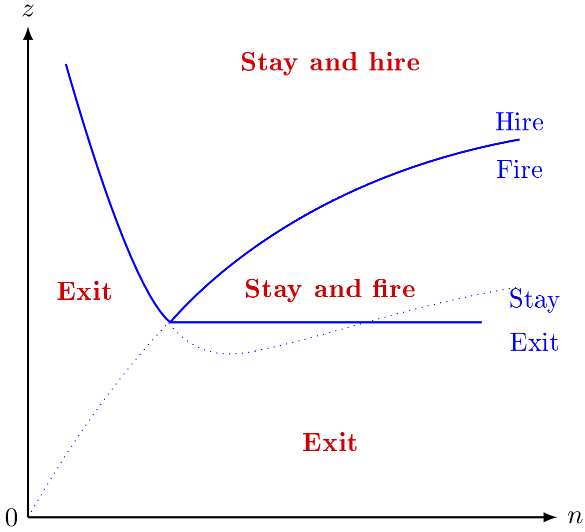

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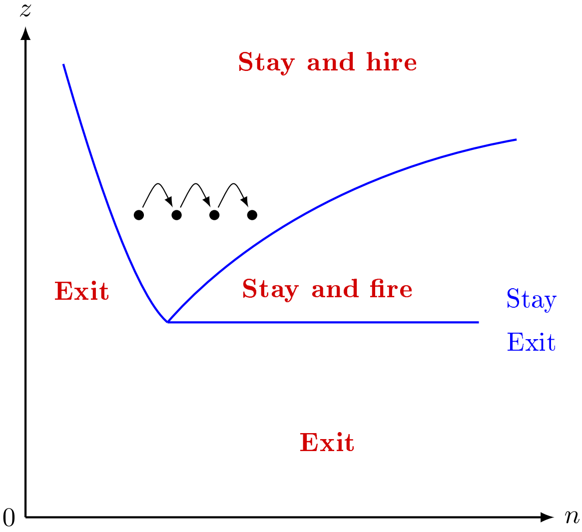

CONCLUSION

Framework that speaks to both firm & worker dynamics

- Equilibrium allocation characterized by joint coalition value
- Low-dimensional state space: productivity and size
- Life-cycle of firms and job ladder

Next steps

- **Theory**
  - Determine wages
  - Extensions: (i) end. growth, (ii) worker heter., (iii) aggr. shocks
- **Data**
  - Match job ladder by productivity, size and age of firm
  - MEE datasets for U.S., France, Sweden, and Brazil

APPENDIX

DISTRIBUTIONS AND AGGREGATES

- $H(x)$ = distribution of x, $v(x)$ = vacancies at x, $n(x)$ = size of x

$$\mathtt{v} = \mathtt{m} \int v(x)\, dH(x), \qquad \mathtt{n} = 1 - \mathtt{u} = \mathtt{m} \int n(x)\, dH(x)$$

- $f(x)$ = distr. of vacancies, $g(x)$ = distr. of employment

$$f(x) = \frac{\mathtt{m} v(x)}{\mathtt{v}}, \qquad g(x) = \frac{\mathtt{m} n(x)}{\mathtt{n}}$$

- $g(x,i) = g(x)/n(x)$ = distr. of workers employed in firm x

Back to States

EMPLOYED WORKER: DETAILS

Definition

- $\mathbf{V}(x,i)$ = value prior to exit, layoff and quit-to-U decisions
- $V(x,i)$ = value after these decisions
- $s\left[x,\kappa\left(x\right)\right]$ how x is updated after layoff/quit decision

Quit decision

$$\mathbf{V}(x,i) = \max\left\{ V\left(s[x,\kappa(x)],i\right) \ , \ U \right\}$$

Value of employment

$$\rho V(x,i) = \underbrace{w(x,i)}_{\text{wage}} + \underbrace{\rho V_D(x,i)}_{\substack{\text{labor market} \\ \text{shocks to } i}} + \underbrace{\rho V_I(x,i)}_{\substack{\text{labor market} \\ \text{shocks to } j \neq i}} + \underbrace{\rho V_F(x,i)}_{\substack{\text{hiring and product-} \\ \text{ivity shocks to firm}}}$$

Back to Employed worker

FIRM'S PROBLEM:DETAILS

Definitions

- $\mathbf{J}(x)$ = value to firm prior to exit, layoff and quit-to-U decisions
- $J(x)$ = value to firm after these decisions
- $v(x)$ = optimal vacancy decision

Exit decision

$$\mathbf{J}(x) = \max\left\{ \ J\big(s[x,\kappa(x)]\big) \ , \ 0 \ \right\}$$

Value of the firm

$$\begin{aligned} \rho J(x) \quad &= \quad \overbrace{y(z,n) - \sum_{i=1}^{n(x)} w(x,i) - c(v(x),z,n)}^{\text{net profit}} \\ &+ \quad \underbrace{\rho J_I(x)}_{\text{incumbents}} + \underbrace{\rho J_F(x)}_{\text{new workers}} + \underbrace{\mu(z)\frac{\partial J}{\partial z}(x) + \frac{\sigma(z)^2}{2}\frac{\partial^2 J}{\partial z^2}(x)}_{\text{shocks to firm productivity}} \end{aligned}$$

Back to Firm's problem

FIRM'S PROBLEM: INCUMBENTS

$$
\begin{aligned}
\rho J_I(x) &= \underbrace{\delta \sum_{i=1}^{n(x)} \Big[\mathbf{J}(d(x,i)) - J(x)\Big]}_{\text{exogenous destructions}} \\
&+ \underbrace{\lambda^E \sum_{i=1}^{n(x)} \int_{x' \in \mathcal{Q}^E(x,i)} \Big[\mathbf{J}(q_E(x,i,x')) - J(x)\Big] dF(x')}_{\text{voluntary quits to other firms}} \\
&+ \underbrace{\lambda^E \sum_{i=1}^{n(x)} \int_{x' \notin \mathcal{Q}^E(x,i)} \Big[\mathbf{J}(r(x,i,x')) - J(x)\Big] dF(x')}_{\text{rebargaining due to outside offers}}
\end{aligned}
$$

Back to Firm's problem

FIRM'S PROBLEM: NEW WORKERS

$$
\begin{aligned}
\rho J_F(x) &= \underbrace{\lambda^F v(x)\phi \left[\mathbf{J}(h_U(x)) - J(x)\right]\mathbb{I}_{x\in\mathcal{A}}}_{\text{hiring from U}} \\
&+ \underbrace{\lambda^F v(x)\phi \left[\mathbf{J}(t_U(x)) - J(x)\right]\mathbb{I}_{x\notin\mathcal{A}}}_{\text{threat of hiring from U}} \\
&+ \underbrace{\lambda^F v(x)(1-\phi) \int\limits_{x'\in\mathcal{Q}^E(x,i)} \left[\mathbf{J}(h_E(x,i,x')) - J(x)\right] dG(x',i')}_{\text{hiring from E}} \\
&+ \underbrace{\lambda^F v(x)(1-\phi) \int\limits_{x'\notin\mathcal{Q}^E(x,i)} \left[\mathbf{J}(q_E(x,i,x')) - J(x)\right] dG(x',i')}_{\text{threat of hiring from E}}
\end{aligned}
$$

Back to Firm's problem

FIRM'S PROBLEM: ENTRY

Entering firms:

- Pay a sunk entry cost c_e
- Draw productivity z from a c.d.f $\Phi_0(z)$
- Decide whether to stay or exit immediately
- Meet n_0 unemployed workers

Free entry:

$$c_e \geq \int_{z^*}^{\infty} J(z, n_0) d\Phi_0(z)$$

with equality iff there is positive entry: entry cutoff $z^* < \infty$.

Back to Firm's problem

BELLMAN EQUATION: DISCRETE WORKFORCE

Proposition: **(z,n)** are the only **payoff-relevant** states, and

$$\begin{aligned}\rho\Omega(z,n) = \max_{v\geq 0} \quad & y(z,n) - c(v;z,n) + \delta n\left[U - \left(\Omega(n,z) - \boldsymbol{\Omega}(n-1,z)\right)\right] \\ + \quad & \lambda^F v\phi\left\{\boldsymbol{\Omega}(n+1,z) - \Omega(n,z) - U\right\}^{+} \\ + \quad & \lambda^F v(1-\phi)\int\Big\{ \left[\boldsymbol{\Omega}(n+1,z) - \Omega(n,z)\right] \\ & \qquad -\left[\Omega(n',z') - \boldsymbol{\Omega}(n'-1,z')\right] \Big\}^{+} dG(z',n') \\ + \quad & \mu(z)\frac{\partial\Omega}{\partial z}(z,n) + \frac{\sigma(z)^2}{2}\frac{\partial^2\Omega}{\partial z^2}(z,n)\end{aligned}$$

where

$$\boldsymbol{\Omega}(n,z) = \max\left\{\underbrace{\max_{k\in\{0,\ldots,n\}}\Omega(k,z) + (n-k)U}_{\text{endogenous separations}},\ \underbrace{nU}_{\text{exit}}\right\}$$

Back to Continuous workforce