THE HAMMER AND THE DANCE:
EQUILIBRIUM AND OPTIMAL POLICY DURING A PANDEMIC CRISIS

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Covid, Search and Matching, Chicago, 05/28/2020
2 Specific features/challenges related to COVID-19:

1. **Fast propagation:** short horizon between infection and recovery/death
2. **Asymptomatic transmissions:** large fraction of transmissions/infections go undetected
ECONOMIC POLICY DURING A PANDEMIC CRISIS

• 2 Specific features/challenges related to COVID-19:
  1. **Fast propagation:** short horizon between infection and recovery/death
  2. **Asymptomatic transmissions:** large fraction of transmissions/infections go undetected

• Dynamic model linking economic interactions and infection risks
  ➔ study trade-offs between
  1. mortality costs induced by pandemic
  2. adverse economic costs of policy interventions

• Keep it at a fairly high level of generality
ECONOMIC POLICY DURING A PANDEMIC CRISIS

- Equilibrium and Central Planner’s allocations → “The Hammer and The Dance”

- The Hammer: Strong initial lockdown to bring infections under control

- The Dance: Slow gradual deconfinement towards herd immunity

- Results: interplay between static and dynamic externalities
  1. Static: instantaneous economic and infection risk spill-overs
  2. Dynamic: linked to dynamic immunization and infection externalities
     - Long-run: Strong infection externalities → Excess mortality in equilibrium
     - Short-run: Strong immunization externalities → too strong/weak equilibrium lockdown depending on welfare cost of initial infection peak
ECONOMIC POLICY DURING A PANDEMIC CRISIS

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  - **The Hammer**: Strong initial lockdown to bring infections under control
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Atkeson
Eichenbaum, Rebelo and Trabandt;
Farboodi, Jarosch, and Shimer;
Building on the Shoulders of Giants

- Alvarez, Argente and Lippi
- Atkeson
- Eichenbaum, Rebelo and Trabandt
- Farboodi, Jarosch, and Shimer
- Kaplan, Moll and Violante; Bethune and Korinek; Atkeson; Chang and Velasco; Garibaldi, Moen and Pissarides; Gonzalez-Eiras and Niepelt; Jones, Philippon and Venkateswaran; Krueger, Uhlig and Xie; Toxvaerd; Ritschl; Abaluck, Chevalier, Christakis, Forman, Kaplan, Ko and Vermund; Glover, Heathcote, Krueger and Ríos-Rull; Greenstone and Nigam; Cleevely, Susskind, Vines, Vines and Wills; Beck and Wagner; Rowthorn; von Thadden; Beenstock and Dai; Avery, Bossert, Clark, Ellison and Ellison; Berger, Herkenhoff and Mongey; Maloney and Taskin; Zhixian and Meissner; Victoria, Menzio and Wiczer, Forslid and Herzing, Moser and Yared, Hornstein, Bodenstein, Corsetti and Guerrieri, Deb, Furceri, Ostry and Tawk, and many others...
- Too numerous to mention
The Model

Time is discrete, Perfect foresight
Measure $\Lambda_t$ of Agents

Actions: $x \in X \subseteq \mathbb{R}^k$

$X$ Compact, Convex
Time is discrete, Perfect foresight Measure $\Lambda_t$ of Agents

**Actions:** $x \in X \subseteq \mathbb{R}^k$

$X$ Compact, Convex

**Economic Stage Game**

Instantaneous Payoffs: $U(x, X) \in [\underline{V}, \bar{V}]$, $\bar{V} > \underline{V} \geq 0$

Assumption:

$\exists X^* \in X, U(X^*, X^*) = \bar{V}$

$\rightarrow$ **Equilibrium Efficiency**

(2nd Welfare Thm)
**The Model**

**Confinement Stage Game**

Instant. Infection Risk: \( R\pi(i) \)
with \( R = R(x, X) \geq R \geq 0 \)

Assumption:
\[ \exists \bar{x} \in \mathcal{X}, R(\bar{x}, \bar{x}) = R \]

→ **Extreme Confinement Equil.**

**Economic Stage Game**

Time is discrete, Perfect foresight
Measure \( \Lambda_t \) of Agents

Actions: \( x \in \mathcal{X} \subseteq \mathbb{R}^k \)
\( \mathcal{X} \) Compact, Convex

**Instantaneous Payoffs:**
\[ U(x, X) \in [\underline{V}, \overline{V}], \quad \overline{V} > \underline{V} > 0 \]

Assumption:
\[ \exists x^* \in \mathcal{X}, U(x^*, x^*) = \overline{V} \]

→ **Equilibrium Efficiency**
(2\(^{nd}\) Welfare Thm)

---

**SIR Dynamics**

\[ R \cdot \pi(i) \]

\[ \gamma \delta \]

Susceptible \( \pi_t(s) \)

Infected \( \pi_t(i) \)

Recovered \( 1 - \pi_t(s) - \pi_t(i) \)

Dead \( 1 - \delta \pi_t(i) \)

\[ \Lambda_{t+1} = (1 - \delta \pi_t(i)) \Lambda_t \]
THE MODEL

Confinement Stage Game

Instant. Infection Risk: $R \pi(i)$
with $R = R(x, X) \geq R \geq 0$

Assumption:
$\exists \tilde{x} \in X, R(\tilde{x}, \tilde{x}) = R$
$\rightarrow$ Extreme Confinement Equil.

Economic Stage Game

Time is discrete, Perfect foresight
Measure $\Lambda_t$ of Agents
Actions: $x \in X \subseteq \mathbb{R}^k$
$X$ Compact, Convex

Instantaneous Payoffs:
$U(x, X) \in [V, \bar{V}], \bar{V} > V > 0$

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**The Model**

**Confinement Stage Game**

Instant. Infection Risk: $R \pi(i)$

with $R = R(x, \mathcal{X}) \geq R \geq 0$

Assumption:

$\exists \tilde{x} \in \mathcal{X}, R(\tilde{x}, \tilde{x}) = R$

$\rightarrow$ Extreme Confinement Equil.

**Economic Stage Game**

Instantaneous Payoffs:

$U(x, \mathcal{X}) \in [\underline{V}, \bar{V}], \quad \bar{V} > \underline{V} > 0$

Assumption:

$\exists x^* \in \mathcal{X}, U(x^*, x^*) = \bar{V}$

$\rightarrow$ Equilibrium Efficiency

(2nd Welfare Thm)

**SIR Dynamics**

\[
\begin{align*}
\text{Susceptible} & \quad \pi_t(s) \\
\quad \pi_t(i) & \quad R \cdot \pi(i) \\
\quad \pi_t(i) & \quad \gamma \\
\text{Recovered} & \quad 1 - \pi_t(s) - \pi_t(i) \\
\text{Dead} & \quad \Delta_{t+1} = (1 - \delta \pi_t(i)) \Lambda_t
\end{align*}
\]
**The Model**

**Confinement Stage Game**

Instant. Infection Risk: $R \pi(i)$ with $R = R(x, X) \geq R \geq 0$

Assumption:

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**Economic Stage Game**

Instantaneous Payoffs: $U(x, X) \in [V, \bar{V}], \bar{V} > V \geq 0$

Assumption:

\[ \exists x^* \in \mathcal{X}, U(x^*, x^*) = \bar{V} \]

→ Equilibrium Efficiency (2\textsuperscript{nd} Welfare Thm)

**SIR Dynamics**

- **Susceptible** $\pi_t(s)$
  - $R \cdot \pi(i)$

- **Infected** $\pi_t(i)$
  - $\gamma$
  - $\delta$

- **Recovered**
  - $1 - \pi_t(s) - \pi_t(i)$

- **Dead**
  - $\Lambda_{t+1} = (1 - \delta \pi_t(i))\Lambda_t$

Assumption (Asymptomatic Transmission)

Only Death is observable
**The Model**

**Confinement Stage Game**

Instant. Infection Risk: \( R\pi(i) \)

with \( R = R(x, \lambda) \geq R \geq 0 \)

Assumption:
\[ \exists \bar{x} \in \mathcal{X}, R(\bar{x}, \bar{x}) = R \]

\[ \rightarrow \text{Extreme Confinement Equil.} \]

**Economic Stage Game**

Instantaneous Payoffs:
\[ U(x, \lambda) \in [\underline{V}, \overline{V}], \quad \overline{V} > \underline{V} \geq 0 \]

Assumption:
\[ \exists x^* \in \mathcal{X}, U(x^*, x^*) = \overline{V} \]

\[ \rightarrow \text{Equilibrium Efficiency (2}^{\text{nd}}\text{ Welfare Thm)} \]

**SIR Dynamics**

- **Susceptible** \( \pi_t(s) \)
- **Infected** \( \pi_t(i) \)
- **Recovered** \( 1 - \pi_t(s) - \pi_t(i) \)
- **Dead** \( \Lambda_{t+1} = (1 - \delta \pi_t(i)) \Lambda_t \)

Assumption (Asymptomatic Transmission):
Only death is observable

\[ \gamma \]
\[ \delta \]
**The Model**

**Confinement Stage Game**

Instant. Infection Risk: \( R \pi(i) \)

with \( R = R(x, X) \geq R \geq 0 \)

Assumption:
\( \exists \bar{x} \in \mathcal{X}, R(\bar{x}, \bar{x}) = R \)

\[ \rightarrow \text{Extreme Confinement Equil.} \]

**Economic Stage Game**

Instantaneous Payoffs:
\( U(x, X) \in [\bar{V}, \bar{V}], \bar{V} > V > 0 \)

Assumption:
\( \exists x^* \in \mathcal{X}, U(x^*, x^*) = \bar{V} \)

\[ \rightarrow \text{Equilibrium Efficiency} \]

\( (2^{nd} \text{ Welfare Thm}) \)

**Dynamic Game Payoffs**

\[
(1 - \beta) \sum_{t=0}^{\infty} \beta^t \Lambda(x^{t-1}, x^{t-1}) U(x^{t-1}, x^{t-1})
\]

**SIR Dynamics**

- **Susceptible**
  \( \pi_t(s) \)

- **Infected**
  \( \pi_t(i) \)

- **Recovered**
  \( 1 - \pi_t(s) - \pi_t(i) \)

- **Dead**
  \( \Lambda_{t+1} = (1 - \delta \pi_t(i)) \Lambda_t \)

Assumption (Asymptomatic Transmission)

Only Death is observable
Key Steps

1. Summarize trade-off between economic activity and infection risks (future mortality) → *shadow price of infection risks*
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2. Break analysis into **static** and **dynamic** part:
   → Dynamic problem as a sequence of static reduced form interaction game: infection risk vs utility.
1. Summarize trade-off between economic activity and infection risks (future mortality) → shadow price of infection risks

2. Break analysis into **static** and **dynamic** part:
   → Dynamic problem as a sequence of static **reduced form interaction game**: infection risk vs utility.

   (i) **Static part**: which sectors to open or close?
   • Align private and social MRS between economic activity and infection risks to the shadow price.
1. Summarize trade-off between economic activity and infection risks (future mortality) → shadow price of infection risks

2. Break analysis into static and dynamic part:
   → Dynamic problem as a sequence of static reduced form interaction game: infection risk vs utility.

   (i) Static part: which sectors to open or close?
       • Align private and social MRS between economic activity and infection risks to the shadow price.

   (ii) Dynamic part: timing of interventions?
       • The dynamics of the shadow price of infection risks dictate the timing
Static Interaction Game

Confinement Stage Game
\[ R(x, X), \mathcal{R}(X) \equiv R(X, X) \]

Economic Stage Game
\[ U(x, X), V(X) \equiv U(X, X) \]
**Static Interaction Game**

**Period t**

**Hybrid Interaction Game**

\[ U(x, X) - \Phi^{eq} R(x, X) \]

\[ V(X) - \Phi^* R(X) \]

**Confinement Stage Game**

\[ R(x, X), \mathcal{R}(X) \equiv R(X, X) \]

**Economic Stage Game**

\[ U(x, X), V(X) \equiv U(X, X) \]
**Static Interaction Game**

- **Confinement Stage Game**
  \[ R(x, X), \mathcal{R}(X) \equiv R(X, X) \]

- **Economic Stage Game**
  \[ U(x, X), V(X) \equiv U(X, X) \]

- **Period t**
  **Hybrid Interaction Game**
  \[
  U(x, X) - \Phi^{\text{eq}} R(x, X) \\
  V(X) - \Phi^{\star} \mathcal{R}(X)
  \]

- **Shadow Price of Infection Risks**
Equate MRS to Shadow Cost of Infection Risk

For any activity $i$

$U_{x_i}(x, X) = \Phi^{eq} R_{x_i}(x, X)$
Confinement Stage Game
\( R(x, X), \mathcal{R}(X) \equiv R(X, X) \)

Economic Stage Game
\( U(x, X), V(X) \equiv U(X, X) \)

Period \( t \)
Hybrid Interaction Game
\[
\begin{align*}
U(x, X) &\equiv \Phi^{eq} R(x, X) \\
V(X) &\equiv \Phi^* \mathcal{R}(X)
\end{align*}
\]

Equate MRS to Shadow Cost of Infection Risk
\[
U_{x_j}(X, X) + U_{x_i}(X, X) = \Phi^* \left( R_{x_j}(X, X) + R_{x_i}(X, X) \right)
\]
For any activity \( i \)
Static Interaction Game

Confinement Stage Game
\( R(x, X), \mathcal{R}(X) \equiv R(X, X) \)

Economic Stage Game
\( U(x, X), V(X) \equiv U(X, X) \)

Period \( t \)

Hybrid Interaction Game
\[
\begin{align*}
U(x, X) &= \Phi^{eq} R(x, X) \\
V(X) &= \Phi^{*} \mathcal{R}(X)
\end{align*}
\]

Static Implementation Rule
\( x^{eq}(\Phi^{eq}), x^{*}(\Phi^{*}) \)

Equate MRS to Shadow Cost of Infection Risk
\[
U_{x_i}(X, X) + U_{X_i}(X, X) = \Phi^{*} (R_{x_i}(X, X) + R_{X_i}(X, X))
\]
For any activity \( i \)
**Static Interaction Game**

**Confinement Stage Game**
\[ R(x, X), \mathcal{R}(X) \equiv R(X, X) \]

**Economic Stage Game**
\[ U(x, X), V(X) \equiv U(X, X) \]

**Static Implementation Rule**
\[ x^{eq}(\Phi^{eq}), x^*(\Phi^*) \]

**Period t**

**Hybrid Interaction Game**
\[
U(x, X) - \Phi^{eq} R(x, X) \\
V(X) - \Phi^* \mathcal{R}(X)
\]

**Reduced form Game**
\[
\text{EQ: } U(r, R) - \Phi^{eq} r \\
\text{CP: } V(R) - \Phi^* R
\]

Equate MRS to Shadow Cost of Infection Risk
\[
U_{x_i}(X, X) + U_{X_i}(X, X) = \Phi^* (R_{x_i}(X, X) + R_{X_i}(X, X))
\]
For any activity \( i \)
EQUILIBRIUM AND PARETO FRONTIERS

Pareto Frontier
(Planner, \( \mathcal{V}^* (R) \))

Equilibrium Frontier
(\( \mathcal{V}^{eq} (R) \))
Static Efficiency Conditions → Optimal Policy

Focus policy on sectors with large absolute spill-overs.

- Protect/subsidize sectors if $Social\ MRS > Private\ MRS$.
- Restrict sectors if $Social\ MRS < Private\ MRS$.
- Milder (stronger) intervention on sectors with small (large) absolute spill-overs.

Economic Externalities

- $U_x(x,X) / U_x(x,X)$
- $R_x(x,X) / R_x(x,X)$

Infection Risk Externalities

- Protect/Subsidize
  - Essential, Non Critical:
    - Banking and finance
    - Pharmaceuticals
    - Basic research
    - Online services
  - Critical but Essential:
    - Groceries
    - Healthcare
    - Education
    - Essential travel

- Leave Alone
  - Neither critical Nor essential:
    - Home activities, Online video ...

- Restrict
  - Critical & Inessential:
    - Entertainment
    - Large Social Events
    - Restaurants
    - Non-essential travel

Manage Carefully
Static Efficiency Conditions → Optimal Policy

- Focus policy on sectors with large absolute spill-overs
- **Protect/subsidize** sectors if \( \text{Social MRS} > \text{Private MRS} \)
- **Restrict** sectors if \( \text{Social MRS} < \text{Private MRS} \)
- **Milder** (stronger) intervention on sectors with small (large) absolute spill-overs

Economic Externalities

- **Protect/Subsidize**
  - Essential, Non Critical:
    - Banking and finance
    - Pharmaceuticals
    - Basic research
    - Online services

- **Leave Alone**
  - Neither critical Nor essential:
    - Home activities,
    - Online video …

Infection Risk Externalities

- **Restrict**
  - Critical & Inessential:
    - Entertainment
    - Large Social Events
    - Restaurants
    - Non-essential travel

- **Manage Carefully**
  - Critical but Essential:
    - Groceries
    - Healthcare
    - Education
    - Essential travel
Dynamic Interaction Game

Confinement Stage Game
\( R(x_t, x_t), \mathcal{R}(x_t) \)

Static Implementation Rule
\( X^{eq}(R_t), X^*(R_t) \)

Economic Stage Game
\( U(x_t, x_t), V(x_t) \)

Reduced-Form Game
\( \begin{align*}
U(r_t, R_t) & - \Phi^{eq}_t R_t \\
V(R_t) & - \Phi^*_t R_t
\end{align*} \)
Dynamic Interaction Game

Confinement Stage Game
\( R(x_t, x_t), \mathcal{R}(x_t) \)

Static Implementation Rule
\( x^{eq}(r_t), x^*(r_t) \)

Economic Stage Game
\( U(x_t, x_t), V(x_t) \)

Reduced Form Policy Rule
\[
U_r(r_t, r_t) = \Phi^{eq}_t \rightarrow R^{eq}(\Phi^{eq}_t) \\
V'(r_t) = \Phi^{*}_t \rightarrow R^*(\Phi^{*}_t)
\]

Period \( t \) Reduced-Form Game
\[
U(r_t, r_t) - \Phi^{eq}_t r_t \\
V(r_t) - \Phi^{*}_t r_t
\]

Shadow Price of Infection Risk
\( \Phi^{eq}_t (\pi_t) \Phi^{*}_t (\pi_t) \)
**Dynamic Interaction Game**

- **Confinement Stage Game**
  \[ R(x_t, X_t), \mathcal{R}(X_t) \]

- **Static Implementation Rule**
  \[ x^{eq}(R_t), x^*(R_t) \]

- **Economic Stage Game**
  \[ U(x_t, X_t), V(X_t) \]

- **Reduced Form Policy Rule**
  \[ U_t(r_t, R_t) = \Phi^{eq}_t \rightarrow R^{eq}(\Phi^{eq}_t) \]
  \[ V'(R_t) = \Phi^*_t \rightarrow R^*(\Phi^*_t) \]

- **SIR Dynamics**
  \[ \pi_t = (\pi_t(s), \pi_t(i)) \]

- **Shadow Price of Infection Risk**
  \[ \Phi^{eq}_t(\pi_t) \]
  \[ \Phi^*_t(\pi_t) \]
**Dynamic Interaction Game**

**Confinement Stage Game**
\[ R(x_t, x_t), \mathcal{R}(x_t) \]

**Static Implementation Rule**
\[ X^{eq}(R_t), X^*(R_t) \]

**Economic Stage Game**
\[ U(x_t, x_t), V(x_t) \]

**Reduced Form Policy Rule**
\[
\begin{align*}
R^{eq}(\pi_t) &= R^{eq}(\Phi^{eq}(\pi_t)) \\
R^*(\pi_t) &= R^*(\Phi^*(\pi_t))
\end{align*}
\]

**Period t Reduced-Form Game**
\[
\begin{align*}
U(r_t, R_t) &= \Phi^{eq}_t r_t \\
V(R_t) &= \Phi^*_t R_t
\end{align*}
\]

**SIR Dynamics**
\[ \pi_t = (\pi_t(s), \pi_t(i)) \]

**Shadow Price of Infection Risk**
\[
\begin{align*}
\Phi^{eq}_t(\pi_t) &\quad \Phi^*_t(\pi_t)
\end{align*}
\]
Dynamic Efficiency Conditions

- Focus on reduced form trade-off between $R$ and $\mathcal{V}$:

  \[ EQ : U_t(R_t, R_t) = \Phi_t^{eq} \]

  \[ CP : \mathcal{V}^*(R_t) = \Phi_t^* \]
Focus on reduced form trade-off between $R$ and $\mathcal{V}$:

\[ EQ : \mathcal{U}_t(R_t, R_t) = \Phi^e_t \]
\[ CP : \mathcal{V}^*(R_t) = \Phi^*_t \]

Generically

\[ \Phi_t \equiv \frac{\beta}{1-\beta} \pi_t(S)\pi_t(i) \left( \frac{\partial \nu(\pi_{t+1})}{\partial \pi(s)} - \frac{\partial \nu(\pi_{t+1})}{\partial \pi(i)} \right) \]
Dynamic Efficiency Conditions

- Focus on reduced form trade-off between $R$ and $\mathcal{V}$:

  $\text{EQ} : \mathcal{U}_t(R_t, R_t) = \Phi^\text{eq}_t$

  $\text{CP} : \mathcal{V}^*(R_t) = \Phi^*_t$

- Generically

  $$\Phi_t \equiv \frac{\beta}{1-\beta} \pi_t(s)\pi_t(i) \left( \frac{\partial \nu(\pi_{t+1})}{\partial \pi(s)} - \frac{\partial \nu(\pi_{t+1})}{\partial \pi(i)} \right)$$

  Proportional to propagation
DYNAMIC EFFICIENCY CONDITIONS

- Focus on reduced form trade-off between $R$ and $\mathcal{V}$:

$EQ : \mathcal{U}_t(R_t, R_t) = \Phi_t^{eq}$

$CP : \mathcal{V}^*(R_t) = \Phi_t^*$

- Generically

$$\Phi_t \equiv \frac{\beta}{1 - \beta} \pi_t(S) \pi_t(i) \left( \frac{\partial v(\pi_{t+1})}{\partial \pi(s)} - \frac{\partial v(\pi_{t+1})}{\partial \pi(i)} \right)$$

Rate of Interaction

Proportional to propagation
Dynamic Efficiency Conditions

- Focus on reduced form trade-off between $R$ and $\mathcal{V}$:

  $$EQ : \mathcal{U}_t(R_t, R_t) = \Phi_t^{eq}$$
  $$CP : \mathcal{V}_t^*(R_t) = \Phi_t^*$$

- Generically

  $$\Phi_t \equiv \frac{\beta}{1-\beta} \pi_t(S) \pi_t(i) \left( \frac{\partial \mathcal{V}(\pi_{t+1})}{\partial \pi(S)} - \frac{\partial \mathcal{V}(\pi_{t+1})}{\partial \pi(i)} \right)$$

  Speed of Propagation

  Proportional to propagation
Focus on reduced form trade-off between $R$ and $V$:

\[
EQ : \mathcal{U}_t(R_t, R_t) = \Phi^eq_t
\]

\[
CP : \mathcal{V}^*(R_t) = \Phi^*_t
\]

Generically

\[
\Phi_t \equiv \frac{\beta}{1-\beta} \pi_t(S)\pi_t(i) \left( \frac{\partial \nu(\pi_{t+1})}{\partial \pi(S)} - \frac{\partial \nu(\pi_{t+1})}{\partial \pi(i)} \right)
\]

Marginal Cost of Additional Infection
Focus on reduced form trade-off between $R$ and $V$:

\[ \text{EQ} : \mathcal{U}_t(R_t, R_t) = \Phi^e_t \]
\[ \text{CP} : V^*(R_t) = \Phi^*_t \]

Generically

\[ \Phi_t \equiv \frac{\beta}{1-\beta} \pi_t(S)\pi_t(i) \left( \frac{\partial v(\pi_{t+1})}{\partial \pi(s)} - \frac{\partial v(\pi_{t+1})}{\partial \pi(i)} \right) \]

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Dynamic Efficiency Conditions

- Focus on reduced form trade-off between $R$ and $\mathcal{V}$:

$$\text{EQ} : \mathcal{U}_t(R_t, R_t) = \Phi_{eq}^t$$

$$\text{CP} : \mathcal{V}^*(R_t) = \Phi^*_t$$

- Generically

$$\Phi_t \equiv \frac{\beta}{1 - \beta} \pi_t(S)\pi_t(i) \left( \frac{\partial v(\pi_{t+1})}{\partial \pi(s)} - \frac{\partial v(\pi_{t+1})}{\partial \pi(i)} \right)$$
**Marginal Values**

- **Social Marginal Value of Recovery** \((i\rightarrow r)\)

\[-\frac{\partial v(\pi)}{\partial \pi(i)} = v_r(\pi) - v_i(\pi) - \pi(s) \frac{\partial v_s(\pi)}{\partial \pi(i)} - \pi(i) \frac{\partial v_i(\pi)}{\partial \pi(i)} - (1 - \pi(s) - \pi(i)) \frac{\partial v_r(\pi)}{\partial \pi(i)}\]

- **Social Marginal Value of Immunization** \((s\rightarrow r)\)

\[-\frac{\partial v(\pi)}{\partial \pi(s)} = v_r(\pi) - v_s(\pi) - \pi(s) \frac{\partial v_s(\pi)}{\partial \pi(s)} - \pi(i) \frac{\partial v_i(\pi)}{\partial \pi(s)} - (1 - \pi(s) - \pi(i)) \frac{\partial v_r(\pi)}{\partial \pi(s)}\]

**Direct Benefit (Internalized)**

**Dynamic Infection Externalities:**
Indirect effects of a marginal decrease in infection rate on \(s, i, r\)

**Dynamic Immunization Externalities:**
Higher immunization lowers the need for economic restrictions
MARGINAL VALUES

• Social Marginal Value of Recovery \((i \rightarrow r)\)

\[
- \frac{\partial v \pi \left( \pi \right)}{\partial \pi \left( i \right)} = v_r \left( \pi \right) - v_i \left( \pi \right) - \pi \left( S \right) \frac{\partial v_s \pi \left( \pi \right)}{\partial \pi \left( i \right)} - \pi \left( i \right) \frac{\partial v_i \pi \left( \pi \right)}{\partial \pi \left( i \right)} - \left( 1 - \pi \left( S \right) - \pi \left( i \right) \right) \frac{\partial v_r \pi \left( \pi \right)}{\partial \pi \left( i \right)}
\]

\[
\text{direct benefit (Internalized)}
\]

• Social Marginal Value of Immunization \((s \rightarrow r)\)

\[
- \frac{\partial v \pi \left( \pi \right)}{\partial \pi \left( s \right)} = v_r \left( \pi \right) - v_s \left( \pi \right) - \pi \left( S \right) \frac{\partial v_s \pi \left( \pi \right)}{\partial \pi \left( s \right)} - \pi \left( i \right) \frac{\partial v_i \pi \left( \pi \right)}{\partial \pi \left( s \right)} - \left( 1 - \pi \left( S \right) - \pi \left( i \right) \right) \frac{\partial v_r \pi \left( \pi \right)}{\partial \pi \left( s \right)}
\]

\[
\text{direct benefit (Internalized)}
\]
MARGINAL VALUES

- Social Marginal Value of Recovery \((i \rightarrow r)\)

\[
- \frac{\partial v(\pi)}{\partial \pi(i)} = v_r(\pi) - v_i(\pi) - \pi(s) \frac{\partial v_s(\pi)}{\partial \pi(i)} - \pi(i) \frac{\partial v_i(\pi)}{\partial \pi(i)} - (1 - \pi(s) - \pi(i)) \frac{\partial v_r(\pi)}{\partial \pi(i)}
\]

Dynamic Infection Externalities:
Indirect effects of a marginal decrease in infection rate on \(s, i, r\)

- Social Marginal Value of Immunization \((s \rightarrow r)\)

\[
- \frac{\partial v(\pi)}{\partial \pi(s)} = v_r(\pi) - v_s(\pi) - \pi(s) \frac{\partial v_s(\pi)}{\partial \pi(s)} - \pi(i) \frac{\partial v_i(\pi)}{\partial \pi(s)} - (1 - \pi(s) - \pi(i)) \frac{\partial v_r(\pi)}{\partial \pi(s)}
\]

Dynamic Immunization Externalities:
Higher immunization lowers the need for economic restrictions
Dynamic Efficiency Conditions

- Focus on reduced form trade-off between $R$ and $V$:

\[
EQ : U_t(R_t, R_t) = \Phi^e_t
\]

\[
CP : V^*(R_t) = \Phi^*_t
\]

- Generically

\[
EQ : U_t(R_t, R_t) = \Phi^e_t = \frac{\beta}{1 - \beta} \pi_t(s) \pi_t(i) \left( \frac{V_s(\pi_{t+1}) - V_i(\pi_{t+1})}{\neq 0} \right)
\]

\[
CP : V^*(R_t) = \Phi^*_t = \frac{\beta}{1 - \beta} \pi_t(s) \pi_t(i) \left( \frac{V_s(\pi_{t+1}) - V_i(\pi_{t+1}) + \text{External Costs}}{\neq 0} \right)
\]

- Useful to focus on $\beta \rightarrow 1$ (Instantaneous propagation).
Focus on reduced form trade-off between $R$ and $V$:

$$EQ : U_r(R_t, R_t) = \Phi^e_t$$

$$CP : V^{*'}(R_t) = \Phi^*_t$$

Generically

$$EQ : U_r(R_t, R_t) = \Phi^e_t = \frac{\beta}{1 - \beta} \pi_t(s) \pi_t(i) \left( \frac{V_s(\pi_{t+1}) - V_i(\pi_{t+1})}{\neq 0} \right)$$

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Useful to focus on $\beta \to 1$ (Instantaneous propagation).
A USEFUL LIMIT CASE: INSTANTANEOUS PROPAGATION ($\beta \to 1$)

- Equilibrium:
  - Hammer:
  - Extreme confinement: $R_{eq} = R + \eta$, $\eta > 0$
  - Keep $\pi_t(i) \in (0, \eta)$ until herd immunity is reached

- Dance:
  - Keep $\pi_t(i)$ within $(0, \eta)$ until herd immunity is reached

- Central Planner:
  - Initially (in $t = 0$) let the infection propagate
  - Strict Hammer: bring infection and recovery to Long Run optimal level
  - Never ending Dance: keep infection under control to Long Run optimal level ($R = e^{R} < R_{eq}$)

- SR: Immunization externality: hold-out motive that inefficiently slows the propagation

- LR: infection externality: agents exit confinement too fast, relative to the CP: increases LR mortality.

$V(R)$

Pareto Frontier

Equilibrium Frontier

$V$, $\bar{V}$
A useful limit case: instantaneous propagation ($\beta \to 1$)

- **Equilibrium:**
  - **Hammer:** Extreme confinement: $R_0^{eq} = R + \eta$, $\eta > 0$
    until $\pi_t(i) \in (0, \eta)$, then

---

\[ R_0^{eq} \]
A USEFUL LIMIT CASE: INSTANTANEOUS PROPAGATION ($\beta \to 1$)

- **Equilibrium:**
  - **Hammer:** Extreme confinement: $R_0^{eq} = R + \eta$, $\eta > 0$
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  - **Dance:** keep $\pi_t(i)$ within $(0, \eta)$ until herd immunity is reached $\to$ very slow ($\pi_t(s)$)
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\[ R_0^{eq} = 1 - e^{R - \bar{R}} \]
A USEFUL LIMIT CASE: INSTANTANEOUS PROPAGATION ($\beta \rightarrow 1$)

- **Equilibrium:**
  - **Hammer:** Extreme confinement: $R_0^{eq} = R + \eta, \eta > 0$ until $\pi_t(i) \in (0, \eta)$, then
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- **Central Planner:**
  - Initially (in $t = 0$) let the infection propagate
  - **strict Hammer** $\rightarrow$ bring infection and recovery to Long Run optimal level
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- **SR: Immunization externality:** hold-out motive that inefficiently slows the propagation

- **LR: infection externality:** agents exit confinement too fast, relative to the CP $\rightarrow$ increases LR mortality.
Proposition

Start from an (arbitrarily) small, positive fraction $\pi_0 (i) > 0$ of infected agents in the population, the sequential planner’s solution and equilibrium $\{R_t^*, \pi_t^*\}$ and $\{R_t^{eq}, \pi_t^{eq}\}$ both satisfy:

1. Flatten the Curve (Short Run): Starting from $R_0^*$ and $R_0^{eq}$ arbitrarily close to $R$, both policy sequences are initially decreasing to “flatten the curve” and delay infections $\Rightarrow$ The Hammer phase

2. Herd Immunity (Long-run): In the long run, $R_t^*$ and $R_t^{eq}$ converge to $R$, and the economy returns to the pre-pandemic equilibrium in a state of herd immunity $\Rightarrow$ The Dance phase
THE HAMMER AND THE DANCE

Proposition

Start from an (arbitrarily) small, positive fraction \( \pi_0 \) of infected agents in the population, the sequential planner’s solution and equilibrium \( \{R^*_t, \pi^*_t\} \) and \( \{R^\text{eq}_t, \pi^\text{eq}_t\} \) both satisfy:

1. **Flatten the Curve (Short Run):** Starting from \( R^*_0 \) and \( R^\text{eq}_0 \) arbitrarily close to \( \bar{R} \), both policy sequences are initially decreasing to “flatten the curve” and delay infections → **The Hammer phase**
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Start from an (arbitrarily) small, positive fraction $\pi_0 (i) > 0$ of infected agents in the population, the sequential planner’s solution and equilibrium $\{R^*_t, \pi^*_t\}$ and $\{R^{eq}_t, \pi^{eq}_t\}$ both satisfy:

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EXTERNALITIES

Social Marginal Cost of Infection

Direct Cost \((v_2(\pi) - v_1(\pi))\)

External Cost

Equilibrium Optimal Allocation
Economic Externalities

- Infection Risk Externalities

**Protect/Subsidize**
- Essential, Non Critical:
  - Banking and finance
  - Pharmaceuticals
  - Basic research
  - Online services

**Leave Alone**
- Neither critical
- Nor essential:
  - Home activities,
  - Online video ...

**Manage Carefully**
- Critical but Essential:
  - Groceries
  - Healthcare
  - Education
  - Essential travel

**Restrict**
- Critical & Inessential:
  - Entertainment
  - Large Social Events
  - Restaurants
  - Non-essential travel

\[ \phi^* > \phi^{eq} \]
\[ \phi^* = \phi^{eq} \]
\[ \phi^* < \phi^{eq} \]
**Economic Externalities**

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Economic Externalities

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Pharmaceuticals
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EXTENSIONS

• Additional Externalities
  • Static Externalities
  • Congestion Externalities
  • Hope for vaccine and cure
  • Transitory immunization

• Shifts in Pareto and Equilibrium Frontiers
  • Face Masks
  • Testing and contact tracing
Concluding Remarks

- Dynamic model integrating economic interactions and infection risk
- Fast Propagation → **Hammer and Dance** Dynamics
- Highlight the role of static and dynamic externalities

Clear policy implications:

- **The hammer:**
  - Early, decisive action is warranted if saves lives in LR.
  - If only delays infections in SR → just lengthens the recovery and inflicts higher economic costs.

- **The dance:**
  - Optimal deconfinement keeps the pandemic’s transmission rate close to 1.
  - Policy must control the epidemic, not the other way around.
CONCLUDING REMARKS

- Dynamic model integrating economic interactions and infection risk
- Fast Propagation → **Hammer and Dance** Dynamics
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**Clear policy implications:**
- **The hammer:**
  - Early, decisive action is warranted if saves lives in LR.
  - If only delays infections in SR → just lengthens the recovery and inflicts higher economic costs
- **The dance:** Optimal deconfinement
  - Keeps the pandemic’s transmission rate close 1.

→ Policy must control the epidemic, not the other way around.
THANK YOU!
SIR Dynamics

- $\Lambda_t$ is the mass of agents
- Susceptible: $S_t = \pi_t(s)\Lambda_t$, Infected: $I_t = \pi_t(i)\Lambda_t$, Recovered: $R_t = \pi_t(r)\Lambda_t$
• $\Lambda_t$ is the mass of agents
• Susceptible: $S_t = \pi_t(s)\Lambda_t$, Infected: $I_t = \pi_t(i)\Lambda_t$, Recovered: $R_t = \pi_t(r)\Lambda_t$

\[
S_{t+1} = S_t - R_t S_t \\
I_{t+1} = R_t S_t + (1 - \gamma - \delta)I_t \\
R_{t+1} = R_t + \gamma I_t
\]

where $R_t = R(x_t, X_t)\pi_t(i)$. 
**SIR Dynamics**

- $\Lambda_t$ is the mass of agents
- Susceptible: $S_t = \pi_t(s)\Lambda_t$, Infected: $l_t = \pi_t(i)\Lambda_t$, Recovered: $R_t = \pi_t(r)\Lambda_t$

\[
S_{t+1} = S_t - R_t S_t \\
l_{t+1} = R_t S_t + (1 - \gamma - \delta)l_t \\
R_{t+1} = R_t + \gamma l_t
\]

where $R_t = R(x_t, X_t)\pi_t(i)$.

- Dynamics of population: $\Lambda_{t+1} = (1 - \delta \pi_t(i))\Lambda_t$. Dividing the previous system by $\Lambda_t$

\[
\pi_{t+1}^k(s) = \frac{\pi_t^k(s) - r_t\pi_t^k(s)\pi_t(i)}{1 - \delta \pi_t^k(i)} \\
\pi_{t+1}^k(i) = \frac{\pi_t^k(i) + r_t\pi_t^k(s)\pi_t(i) - \gamma \pi_t^k(i) - \delta \pi_t^k(i)}{1 - \delta \pi_t^k(i)} \\
\pi_{t+1}^k(r) = \frac{\pi_t^k(r) + \gamma \pi_t^k(i)}{1 - \delta \pi_t^k(i)}
\]
EXTENSIONS: STATIC EXTERNALITIES

- **Susceptible**
- **Infected**
- **Recovered**
- **Share of Deads**
- \( R_0 \)
- **Shadow Cost (\( \Phi \))**

Graphs showing the dynamics over weeks for different scenarios:
- **Eq. (Static Infection Ext.)**
- **Eq. (Static Economic Ext.)**
- **Eq. (benchmark)**
- **Opt. (benchmark)**
EXTENSIONS: CONGESTION EXTERNALITIES

- Congestion of the medical sector: death probability is an increasing and convex function of share of infected

- **Equilibrium:**
  - Stronger hammer to avoid infection and face a higher risk of death
  - But do not understand the congestion → not strong enough, larger death toll

- **Planner:**
  - Much higher shadow cost of infection due to infection externality
  → Stronger Hammer to delay infection and save lives by avoiding congestion
EXTENSIONS: HOPES FOR VACCINES AND CURES

• Probability that all susceptible are moved to recovery state in each period
• Comes too late for those who are infected

• **Equilibrium:**
  • The private benefits of a vaccine are remote
    → No significant change in behavior.

• **Planner:**
  • Very high initial shadow cost of infection
    → Strong Hammer to delay infection and save lives while waiting for vaccine

• Depends very much on expected time to a vaccine/cure.
EXTENSIONS: HOPES FOR VACCINES AND CURES (1 YEAR)

Graphs showing the dynamics of susceptible, infected, recovered, share of deads, $R_0$, and shadow cost over weeks. The graphs compare equilibrium and optimal allocation scenarios with benchmark models.
EXTENSIONS: TRANSITORY IMMUNITY (1 YEAR)

- Susceptible
- Infected
- Recovered
- Share of Deads
- $R_0$
- Shadow Cost ($\phi$)

Graphs showing the dynamics over weeks with different scenarios and labels for equilibrium, optimal allocation, and benchmark.
EXTENSIONS: FACE MASKS

- Wearing face mask does not affect utility but lowers infection risk.
- Costly to produce masks
EXTENSIONS: FACE MASKS

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  → Gives the option to push infection risk below $R$ → Faster control of epidemics
• Wearing face mask does not affect utility but lowers infection risk.
• Costly to produce masks
  → flattens and expands the set of payoffs/actions
  → Gives the option to push infection risk below $R$ → Faster control of epidemics
  → Relaxes the Pareto frontier to achieve higher utility during the dance phase.
• Lowers consumption losses.
EXTENSIONS: FACE MASKS

- Susceptible
- Infected
- Recovered
- Share of Deads
- $R_0$
- Shadow Cost ($\Phi$)

Graphs showing the dynamics of susceptible, infected, and recovered populations, as well as the share of deaths and $R_0$ over time. The graphs compare equilibrium and optimal allocation scenarios with benchmark values.

Equilibrium: 
Optimal Allocation: 
Eq. (benchmark): 
Opt. (benchmark):
- test $\rightarrow$ quarantine (temp. exit from game)
- Reduce undetected infections to $\pi(i)(1 - P(\text{test}|i))$
Extensions: Testing and Contact-Tracing

- Test → quarantine (temp. exit from game)
- Reduce undetected infections to \( \pi(i)(1 - \mathbb{P}(\text{test}|i)) \)

- Improves static efficiency frontier (better control of infections)
- But \( \neq \) face masks as it lowers threshold level of recoveries for
  - herd immunity,
  - virus eradication,
  - elimination of economic restrictions.
- Shift \( \bar{V} \) to the left.
DYNAMICS ALONG THE PARETO FRONTIER

Shadow Cost ($\Phi$)

Pareto Frontier

weeks

$R_t$

$\forall(R_t)$

back
DYNAMICS ALONG THE PARETO FRONTIER

Shadow Cost ($\Phi$)

- $\Phi$ vs. weeks

Pareto Frontier

- $\gamma(R_t)$ vs. $R_t$

Hammer Phase

- Transition from low to high shadow cost
DYNAMICS ALONG THE PARETO FRONTIER

Shadow Cost (\(\Phi\))

Pareto Frontier

\(V(R_t)\)

weeks

\(R_t\)

Hammer Phase

Dance Phase

back
• Individual utility function

\[
\left( \frac{U^{eq}(r, R) - V}{\bar{V} - \bar{V}} \right)^2 + \alpha \left( \frac{r - \bar{R}}{\bar{R} - R} \right)^2 + (1 - \alpha) \left( \frac{R - \bar{R}}{\bar{R} - R} \right)^2 = 1.
\]

→ Leads to

\[
\left( \frac{\nu(R) - V}{\bar{V} - \bar{V}} \right)^2 + \left( \frac{R - \bar{R}}{\bar{R} - R} \right)^2 = 1.
\]