Firm Dispersion and Business Cycles: Estimating Aggregate Shocks using Panel Data

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Introduction

- Recent focus in business cycle literature on learning from micro-data.
- In particular, firm distribution becomes more “spread out” in recessions.
  - Std. dev. of sales growth across firms is *countercyclical*.

*Aggregate Output* is Real GDP, taken from BEA National Income and Product Accounts, logged and detrended using a one-sided Hodrick-Prescott filter. *Std Dev of sales growth* is the cross-sectional standard deviation of the percentage change in quarterly sales across firms, using listed US firms in Compustat, with a cubic trend removed.
Motivation and approach

Question: is increased dispersion a cause or an effect of cycle?

Our approach:

- Model of heterogeneous firms with aggregate shocks
  - Standard macro shocks
    - productivity
    - discount factor
    - labor supply
  - Uncertainty shock
    - direct shock to firm dispersion
- Estimate the model using
  - aggregate time series
    - output, consumption, hours worked
  - time series of firm dispersion
    - std. dev. sales growth over time, constructed from panel data
- Obtain estimates for
  - contribution of uncertainty shock to aggregates (cause)
  - contribution of macro shocks to dispersion (effect)
Results preview

Q: Is firm dispersion a cause or effect of the business cycle?
A: Neither!

Estimation results show that
- almost all fluctuation in aggregates is due to macro shocks
- almost all fluctuation in dispersion is due to uncertainty shock

Uncertainty shocks can generate recessions but don’t.
DSGE overview

Model

\[ u(C_t) = X_t^\beta R_t u(C_{t+1}) \]
\[-X_t^\psi v(N_t) = u(C_t) \]
\[ Y_t = X_t^Z N_t^\nu K_t^{1-\nu} \]
\[ Y_t = C_t + (K_{t+1} - (1 - \delta)K_t) \]

\[ \theta = (u, \nu, \beta, \nu, \delta) \]

⇒ Impulse responses

⇒ Likelihood

\[ P(\text{data}|\theta) \]

⇒ Shock decomposition
Model: environment

- **Discrete time, infinite horizon.**
- **Household**
  - supplies labor $N_t^s$ at competitive wage $W_t$
  - consumes consumption good $C_t$
  - receives dividends $\Pi_t$ from firms.
- **Firms** Continuum of firms, each with state $(k_{it}, z_{it}, \xi_{it})$
  - hire labor $n_{it}$ at competitive wage $W_t$,
  - produce using labor $n_{it}$ and pre-installed capital stock $k_{it}$
    - decreasing returns to scale production function
      \[
      y_{it} = X_t^Z z_{it} \left( n_{it}^{\nu} k_{it}^{1-\nu} \right)^{\kappa}, \quad \kappa < 1
      \]
  - choose whether to adjust capital or not
    - pay $\xi_{it} \sim F(\xi)$ to adjust
    - if adjusting, choose next period’s capital stock $k_{i,t+1}$
  - pay out dividends $\pi_{it}$
Firm problem (adjust or not)

• The firm’s state is \((k, z, \xi; S)\).
• Let \(v(k, z, \xi; S)\) be the discounted present value of the firm.
• Each period, the firm chooses whether to adjust or not:

\[
v(k, z, \xi; S) = \max \left\{ -\xi W(S) + v^{\text{adj}}(k, z; S), v^{\text{stay}}(k, z; S) \right\}
\]

• Gives threshold rule \(\xi^*(k, z; S)\).
Firm problem (continued)

Value of adjusting:

\[
\nu^{\text{adj}}(k, z; S) = \max_{d, k' \geq 0, n} \left[ d + \mathbb{E}[M(S, S') \nu(k', z', \xi'; S')] \right]
\]

subject to

\[
d = X^\kappa z (n^\nu k^{1-\nu})^\kappa - W(S)n - (k' - (1 - \delta)k)
\]

\[
\log z' = \rho_z \log z + \varepsilon'_z, \quad \varepsilon'_z \sim \mathcal{N}
\left( -\frac{1}{2} \frac{(X^\sigma \sigma_z)^2}{1 + \rho_z}, (X^\sigma \sigma_z)^2 \right)
\]

\[
S' \sim \mathcal{H}(S, S')
\]

Value of not adjusting:

\[
\nu^{\text{stay}} \text{ same as } \nu^{\text{adj}} \text{ but } \frac{k' - (1 - \delta)k}{k} \in [-b, b]
\]
Household problem

Let \( W(S) \) be the household’s expected present discounted value:

\[
W(S) = \max_{C,N,A'} u(C - H) - X^\psi v(N) + \beta X^\beta E[W(S') | S] \\
C + Q(S) A' = W(S) N + (Q(S) + \Pi(S)) A
\]

Optimality conditions:

- stochastic discount factor

\[
M(S,S') = \beta X^\beta (S) \frac{u'(C(S') - H(S'))}{u'(C(S) - H(S))}
\]

- labor supply

\[
W(S) = X^\psi (S) \frac{v'(N(S))}{u'(C(S) - H(S))}
\]
Aggregate state

- The aggregate state is the distribution of firms and the aggregate shocks:
  \[ S = \left( \mu(k, z, \xi), X^Z, X^\beta, X^\psi, X^\sigma \right) \]

  where the aggregate shocks are:
  - \( X^Z \), a shock to aggregate TFP,
  - \( X^\beta \), a shock to the household’s discount factor,
  - \( X^\psi \), a shock to the household disutility of labor,
  - \( X^\sigma \), an uncertainty shock.

- Aggregate shocks follow AR(1) processes
  - in logs for \((Z, \beta, \psi)\):
    \[ \log X^j_{t+1} = \rho^j \log X^j_t + \sigma^j \epsilon_t, \quad \epsilon \sim \mathcal{N}(0, (\sigma^j)^2) \]
  - in levels for \(\sigma\):
    \[ X^\sigma_{t+1} = \rho^\sigma X^\sigma_t + \sigma^\sigma \epsilon_t, \quad \epsilon \sim \mathcal{N}(0, (\sigma^\sigma)^2) \]
An equilibrium of this model consists of

- firms’ value function \( v(k, z, \xi; S) \),
- firms’ policy functions \( k'(k, z, \xi; S) \), \( n(k, z, \xi; S) \) and \( \xi^*(k, z; S) \),
- households’ policies \( C(S) \), \( N(S) \) and associated discount factor \( M(S, S') \),
- a wage \( W(S) \),
- a law of motion for the distribution of firms \( \Gamma(\mu, \mu'; S) \),

such that

1. the firms’ policies are optimal,
2. the households’ policies are optimal,
3. the labor market clears,
   \[
   N(S) = \int n(k, z, \xi; S) \, d\mu(k, z, \xi; S)
   \]
4. the law of motion of the distribution is consistent with the firm’s policies.
Solution method

Solve numerically using a computer. 4 steps:

1. Solve for “steady state”, i.e. eqbm without aggregate shocks.
   - A fixed-point problem in the wage $W$.

2. Construct discrete approximations of eqbm objects and conditions.

3. Take first-order Taylor approximation of discretized eqbm conditions.
   - Expand around steady state from Step 1.

4. Solve resulting first-order difference equation using linear algebra.
Impulse response functions

TFP

Discount

Labor

SD sales growth
Uncertainty shock

**Output, Consumption, Labor**

**Investment, Capital, Fraction of firms adjusting**

**Firm capital choice**

Steady state, 1 period after uncertainty shock
Estimation

1. Fix some parameters externally:

\[ \theta_1 = (\beta, \sigma, \eta, \delta, \nu) \]

2. Estimate steady-state parameters using simulated method of moments:

\[ \theta_2 = (\bar{\xi}, \rho_z, \sigma_z, \kappa) \]

- to match moments of long-run firm distribution (Compustat)

3. Estimate shock process parameters using Bayesian methods:

\[ \theta_3 = \{ \rho^j, \sigma^j \} _{j \in \{Z, \psi, \beta, \sigma\}} \]

using as observable time series
- Aggregate output, consumption and hours worked (NIPA)
- Cross-sectional time series of SD sales growth (Compustat)
Data: cross-sectional

- **Compustat**: quarterly accounting data for publicly listed firms.
- **Sample selection**
  - US firms, 1989q1 to 2014q4
  - We drop from the sample
    - quarters in which a firm is involved in M&A
    - financial firms and utilities
    - each firm’s first and last quarter
  - Resulting sample: 360,591 firm-quarter observations
- **Variable definitions**

\[
\text{sales growth}_{it} = \frac{\text{sales}_{it} - \text{sales}_{i,t-1}}{\frac{1}{2} (\text{sales}_{it} + \text{sales}_{i,t-1})}, \quad \text{investment rate}_{it} = \frac{\text{ppe}_{it} - \text{ppe}_{i,t-1}}{\frac{1}{2} (\text{ppe}_{it} + \text{ppe}_{i,t-1})}
\]

\[
\text{inaction rate}_t = \frac{1}{N_t} \sum_{i=1}^{N_t} 1\{|\text{investment rate}_{it}| > 0.01\}
\]
Data: time series

- **Aggregate time series**
  - Source: National Income and Product Accounts (BEA)
  - Variables
    - Output $Y_t$: Real Gross Domestic Product
    - Consumption $C_t$: Real Personal Consumption Expenditures
    - Labor $N_t$: Index of Aggregate Weekly Hours
  - Logged and detrended using a one-sided HP-filter.

- **Cross-sectional time series:**
  - SD sales growth from Compustat.
  - Detrended using cubic time trend.
Estimation (1)

(1) Fix some parameters externally:

\[ u(C - H) = \frac{(C - H)^{1-\sigma}}{1-\sigma}, \quad v(N) = \frac{N^{1+\eta}}{1+\eta} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>( \beta )</td>
<td>0.99</td>
</tr>
<tr>
<td>Curvature of utility function</td>
<td>( \sigma )</td>
<td>1</td>
</tr>
<tr>
<td>Inverse Frisch elasticity</td>
<td>( \eta )</td>
<td>2</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>( \delta )</td>
<td>0.03</td>
</tr>
<tr>
<td>Labor elasticity of output</td>
<td>( \nu )</td>
<td>0.65</td>
</tr>
</tbody>
</table>

(2) Estimate steady-state parameters using SMM:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjustment costs</td>
<td>( \xi )</td>
<td>( 3.7 \times 10^{-4} )</td>
<td>Investment inaction rate</td>
<td>0.263</td>
</tr>
<tr>
<td>Decreasing RTS</td>
<td>( \kappa )</td>
<td>0.744</td>
<td>Dividends / output</td>
<td>0.070</td>
</tr>
<tr>
<td>Persistence idio. prod.</td>
<td>( \rho_z )</td>
<td>0.311</td>
<td>SD sales growth</td>
<td>0.285</td>
</tr>
<tr>
<td>Avg SD idio. prod.</td>
<td>( \bar{\sigma}_z )</td>
<td>0.320</td>
<td>SD investment rates</td>
<td>0.154</td>
</tr>
</tbody>
</table>
(3) Estimate shock process parameters using **Bayesian methods**:

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Type</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation, TFP shock</td>
<td>$\rho^Z$</td>
<td>Beta 0.600 0.260</td>
<td>0.983 0.015</td>
</tr>
<tr>
<td>Autocorrelation, discount shock</td>
<td>$\rho^\beta$</td>
<td>Beta 0.600 0.260</td>
<td>0.739 0.054</td>
</tr>
<tr>
<td>Autocorrelation, labor supply shock</td>
<td>$\rho^\psi$</td>
<td>Beta 0.600 0.260</td>
<td>0.986 0.006</td>
</tr>
<tr>
<td>Autocorrelation, uncertainty shock</td>
<td>$\rho^\sigma$</td>
<td>Beta 0.600 0.260</td>
<td>0.775 0.257</td>
</tr>
<tr>
<td>Standard deviation, TFP shock</td>
<td>$\sigma^Z$</td>
<td>Exp 0.100 0.100</td>
<td>0.012 0.001</td>
</tr>
<tr>
<td>Standard deviation, discount shock</td>
<td>$\sigma^\beta$</td>
<td>Exp 0.100 0.100</td>
<td>0.002 0.001</td>
</tr>
<tr>
<td>Standard deviation, labor supply shock</td>
<td>$\sigma^\psi$</td>
<td>Exp 0.100 0.100</td>
<td>0.012 0.001</td>
</tr>
<tr>
<td>Standard deviation, uncertainty shock</td>
<td>$\sigma^\sigma$</td>
<td>Exp 0.100 0.100</td>
<td>0.002 0.005</td>
</tr>
</tbody>
</table>

Posterior mode and SD are computed by drawing a sample of $10^6$ points from the posterior distribution using the Metropolis-Hastings algorithm. We verify that Gelman's $\sqrt{R}$ statistic is less than 1.05 for each parameter separately.
Results: FEVDs

A. Output

B. Consumption

C. Hours

A. SD sales growth
Results: shock decomposition

A. Output

B. Consumption

C. Labor

D. SD sales growth

-0.04
-0.02
0
0.02
0.04

-0.03
-0.02
-0.01
0
0.01
0.02
0.03

-0.1
-0.05
0
0.05

-0.03
-0.02
-0.01
0
0.01
0.02
0.03
Uncertainty-driven business cycles

- Compute uncertainty shock series that matches output fluctuations.
- Counterfactual implications for SD sales growth:

![SD sales growth graph]

- Model
- Data
**Conclusion**

- **Contribution**
  - We build a macro model consistent with micro evidence on dispersion.
  - We estimate the model on time series of aggregates and cross-sectional dispersion.

- **Findings**
  - Macro shocks alone not sufficient to explain fluctuations in dispersion.
  - Uncertainty shocks alone not sufficient to explain fluctuations in aggregates.

- **Future work**
  - Bayesian estimation of full parameter vector.
    - Computational speed is a barrier.
  - More comprehensive data.
  - A better model?
1. **Firm dispersion and uncertainty**

2. **Estimated macro models**

3. **Heterogeneous agent macro models**

4. **Linearizing heterogeneous agent models**
Sales growth interquartile range

IQR sales growth

Median sales growth
Sales growth dispersion by industry

IQR sales growth

- Mining & Construction
- Manufacturing (A)
- Manufacturing (B)
- Services

Year:
- 1990
- 1995
- 2000
- 2005
- 2010
- 2015