Labor Markets during Pandemics

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Abstract

We study how a COVID-19 like pandemic spreads through the labor market, and what it implies for the dynamics of wages and unemployment rates. The model predicts a segmentation of the labor market between those that have recovered and those that are not yet infected. Wages fall during the early phases of the pandemic, and then rise as the pandemic progresses. The unemployment rate increases among those not yet infected, decreases among those recovered, and increases overall.

We also characterize the efficient allocations and optimal policies. It is optimal to shut down businesses and impose a quarantine several weeks before the pandemic peaks and, in addition, to tax business creation. It is optimal to move approximately one quarter of workers out of employment. The optimal policies can reduce the fraction of people infected by about 10 percentage points.

1 Introduction

We study how the recent outbreak of the COVID-19 pandemic affects the behavior of the labor market, and what it implies for government policies. Labor markets are deeply affected by the ongoing pandemic in several ways. First, infected and deceased people directly reduce labor market participation. Second, labor market activities involve social interactions and labor market participation thus contributes to the spread of the infection, but may also reduce the willingness to participate in the labor market itself if people are worried about getting infected. Finally, a transmission of the virus is an externality and the labor market will not, by itself, function efficiently during a pandemic. This not

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only has potentially important implications for the wage dynamics, but also calls into question what the optimal policy responses are.

We use a standard Mortensen-Pissarides (Pissarides (1985), Mortensen and Pissarides (1994)) model of a frictional labor market, and incorporate aspects of a canonical epidemiological SIR model (Kermack and McKendrick (1927)) to address those questions. Specifically, we use this framework to study how labor market responds in terms of wages and unemployment rates, whether the labor markets helps to spread the epidemics or inhibits it, whether (and when) is it optimal to limit labor market participation, and spread the epidemics over longer periods, and whether a tax on business creation is optimal during the pandemic.

There are two key ingredients in the model. First, as is standard in the SIR models, the probability of getting infected depends on the number of infected people in the economy. Second, the probability of getting infected is higher if one participates in the labor market. The individuals participating in the labor market take into account that they have a higher chance of getting infected, but they do not take into account that there is a higher chance that they will then infect others. This generates a negative externality and a spread of the virus that is too high.

The labor market during the pandemic temporarily separates into two: a labor market for the people that have already recovered, and a labor market for those that have not yet been infected, and are susceptible to the infection. The effect on wages for susceptible workers is non-monotonic, with wages first falling and then rising as the pandemic progresses. Workers are wary of becoming infected while employed, leading to increased reservation wages. Moreover, as more people become infected this effect gets stronger. However, the value of a match declines and reaches a minimum when the probability of becoming infected is highest. At the beginning of the pandemic wages fall as the job value effect dominates. As the pandemic progresses, the reservation wage effect dominates and wages rise.

The overall unemployment rate rises as the pandemic takes hold. Depending on the scenario, we find that the unemployment rate at the peak of the pandemic ranges between 10%-30%. This result is driven by the labor market for the susceptible workers who are less willing to participate in the labor market, and are also more costly for the potential employers because of a shorter expected tenure and higher cost. The unemployment rate for recovered workers, on the other hand, falls. This is because of a selection effect: employed people are more likely to get infected and so enter the pool of recovered people at a higher rate than the unemployed. In the long run, the unemployment rate converges back to its steady state value.

We also characterize the labor market with wages that are rigid and stay unchanged during the pandemic. Rigid wages increase unemployment during the pandemic, but that reduces the externality in the infection propagation. Interestingly, rigid wages then dominate flexible wages in our scenario, even when the Hosios condition holds, and labor markets with flexible wages would be efficient in the absence of the pandemic. Flexible wages, and especially their decline at the beginning, thus contribute to the spread of the infection.

Turning to the efficient allocations and optimal policies, we study two versions of the planning problem. In one, the social planner is able to move people out of employment, essentially shutting down businesses and quarantining workers. We call this a planning problem with quarantine. In the other one, the planning problem without quarantine, the planner is not able to do so and must respect the labor market separation rate given exogenously. We find that the social value of the match becomes negative before the pandemic peaks, even though its private value is still positive and firms, left by themselves, would like to create more vacancies. Labor market tightness (vacancies to unemployment ratio) then goes to zero and it is optimal for the planner to impose a quarantine. It does so only once, at the onset of the pandemic, and reallocates about a quarter of the employed people out of employment. This saves a substantial number of lives: the fraction of people infected or dead is reduced by more then 10 percent, saving approximately 339,000 lives. Regardless of whether the planner can impose quarantine or not, it is also optimal to tax the creation of vacancies, especially during the peak of the pandemic. Quarantine itself is thus not enough to implement the efficient allocation.

2 Related Literature on Pandemics

The progression of an epidemics has been typically analyzed by means of epidemiology models, typically a variant of a SIR model (Kermack and McKendrick (1927), that characterizes the dynamics of people that are susceptible to an infection, infected, and recovered. The epidemiology models are useful for tracking out pandemics for a given set of transition probabilities, but are, by themselves, not useful for studying the mutual interaction between epidemics and the economy. They are also not a useful framework for studying optimal policy responses.

This paper joins a rapidly growing literature that tries to understand and quantify the forces present in the SIR model (or the SEIR model, which adds an exposed stage), and to embed them within an economic framework. Atkeson (2020) solves for various scenarios of the COVID-19 pandemic within a canonical SIR model. Alvarez et al. (2020) and Piguillem and Shi (2020) study optimal lockdowns, while Berger et al. (2020) study optimal testing and quarantines, within essentially the same framework of a standard SIR model. None of those papers explicitly models labor markets during the pandemic.

Eichenbaum et al. (2020) merge the SIR model with a dynamic representative agent framework study optimal policy responses to a pandemic. Like in our paper, one of the central features of their model is an externality generated by individual behavior. In their case it is both higher labor supply and higher consumption that increase the probability of spreading the infection. In contrast to them, we focus on the behavior of labor markets and its role in spreading (or slowing down) the pandemic, and on labor market policies. Also, our model is a heterogeneous agent version of a SIR model, where the key factor is that employed people have a different probability of getting infected than nonemployed.

One of the key aspects of the pandemic is that its impact on the population is heterogeneous, both in terms of its direct losses, and in terms of its implications for individual behavior. Glover et al. (2020) show how the epidemics can bring about intergenerational conflicts and new trade-offs for the government. Guerrieri et al. (2020) show how a negative supply shock can create negative demand shocks of even larger magnitudes. A key ingredient in their model is a heterogeneity across sectors of the economy, just like a heterogeneity in the labor market status is a key aspect of our model. However, their mechanisms are quite different from ours, and they do not explicitly model the propagation of epidemics itself. Naturally, some of the policy implications are different as well: while in our model it is efficient to move people out of employment to reduce the externality, in their model it is optimal to subsidize businesses to reduce exits and stimulate demand. It is, of course, natural, that both mechanisms are in place at some point during the pandemic, perhaps with different intensities at different stages.[1]

[1] Demand shocks arising from a pandemic are also studied, within a DSGE framework, by Faria e Castro (2020), who characterizes the optimal fiscal policy during pandemics.

3 An Accounting Framework

We start by characterizing the flows in the economy, both across the health status and across the employment status. The initial population is normalized to one. The population of living people at time t is denoted by N_t and the population that has died by the end of period t is denoted by D_t. Since people can be either dead or alive, the sum of both is equal to one:

$$N_t + D_t = 1.$$

We categorize the living population in two dimensions. Along the labor market dimension, people can be either employed (E) or unemployed (U). The other one is health status: people can be susceptible to infection but not yet infected (S), infected (I), or recovered (R). Recovered people are no longer susceptible to repeated infection. The total number of unemployed in period t, U_t, is the sum of unemployment across the health categories, and E_t is the sum of the employment across the health categories:

$$U_t = US_t + UI_t + UR_t$$
$$E_t = ES_t + EI_t + ER_t,$$

where US_t denotes the stock of susceptible unemployed in the beginning of period t, UI_t denotes the stock of infected unemployed in the beginning of period t, and UR_t denotes the stock of recovered unemployed in the beginning of period t. Similarly, ES_t, EI_t and ER_t are employed susceptible, employed infected, and employed recovered. Aggregating across health status, we get the total number of susceptible, infected and recovered:

$$S_t = US_t + ES_t$$
$$I_t = UI_t + EI_t$$
$$R_t = UR_t + ER_t.$$

The total living population consists of only unemployed and employed or, alternatively, of only the susceptible, infected and recovered, and so the following equalities hold:

$$N_t = U_t + E_t = S_t + U_t + R_t.$$

Health transition probabilities. We now describe the probabilities of transiting from one state to another. In the accounting framework, we take all the transition probabilities as given. We will later provide a theory that endogenizes some of them.

We assume that the rate at which susceptible people get infected depends on their labor market status. Employed individuals have probability π_t^{EI} of getting infected in the course of their employment, while unemployed individuals have a probability π_t^{UI} of getting infected by interacting with other people. We think of those probabilities as reflecting the number of social interactions that people have. The underlying assumption is that the unemployed interact less with other people because they do not go to work, and their probability of getting infected is lower. Hence we expect $\pi_t^{UI} < \pi_t^{EI}$.

Once people get infected,they recover from the infection with probability π_R. With probability π_D, they die from the infection. With the remaining probability $1 - \pi_R - \pi_D$ they continue being infected next period. Both probabilities are assumed to be independent of the employment status.

Employment transition probabilities. The dynamics of the labor market is driven by the job finding and job separation probabilities. The probability of finding a job depends on the health status. Susceptible people can find a job with probability p_t^S while recovered people can find a job with probability p_t^R. We assume that the infected people cannot look for a job at all. If they get infected while being unemployed, they will stay unemployed until they recover or die. If a susceptible unemployed finds a job and gets infected at the same time, he cannot take the job and remains unemployed.

On the other hand, we assume that the probability with which the employer and employee separate, λ, is independent of the health status of the employee. That is, employees cannot get fired just because they get infected. If a susceptible employed gets infected, he cannot be fired and stays employed, although he is temporarily on a sick leave and is unproductive.

There are six types of living people in the economy, given by the combination of three health states of living people and two employment states. The transitional probabilities above determine the laws of motion for each category of people. For employed people, the inflows and outflows are as follows:

$$ES_{t+1} = (1-\lambda)(1-\pi_t^{EI})ES_t + p_t^S(1-\pi^{UI})US_t \tag{1a}$$

$$EI_{t+1} = (1-\lambda)(1-\pi_R-\pi_D)EI_t + (1-\lambda)\pi_t^{EI}ES_t \tag{1b}$$

$$ER_{t+1} = (1-\lambda)ER_t + (1-\lambda)\pi_R EI_t + p_t^R UR_t. \tag{1c}$$

The number of employed susceptible next period is reduced because a fraction $(1-\lambda)\pi_t^{EI}$ is not separated from the employer but gets infected, a fraction $\lambda(1-\pi_t^{EI})$ does not get infected but gets separated, and a fraction $\lambda\pi_t^{EI}$ gets both separated and infected. Thus, only a fraction $(1-\lambda)(1-\pi_t^{EI})$ remains. On the other hand, a fraction $p^S(1-\pi_t^{UI})$ of unemployed susceptible finds a job and does not get infected, and so becomes employed susceptible. The number of employed infected is reduced because a fraction $(1-\lambda)\pi_R$ recovers while keeping the job, a fraction $\lambda(1-\pi_R-\pi_D)$ loses their job and continues being infected (i.e. does not die or recovers), $\lambda\pi_R$ gets separated and recovers, and a fraction π_D dies. On the other hand, the stock is increased by a fraction $(1-\lambda)\pi_t^{EI}$ of susceptible employees who become infected. Finally, the number of employed recovered decreases because a fraction λ loses their job, but a fraction $(1-\lambda)\pi_R$ of infected employees recover and stay on the job, and a fraction p^R if unemployed who are already recovered finds a job.

The dynamics of the unemployed categories is determined by the following flows:

$$US_{t+1} = (1-p^S)(1-\pi_t^{UI})US_t + \lambda(1-\pi_t^{EI})ES_t \tag{2a}$$

$$UI_{t+1} = (1-\pi_R-\pi_D)UI_t + \pi_t^{UI}US_t + \lambda\pi_t^{EI}ES_t + \lambda(1-\pi_R-\pi_D)EI_t \tag{2b}$$

$$UR_{t+1} = (1-p_t^R)UR_t + \pi_R UI_t + \lambda\pi_R EI_t + \lambda ER_t \tag{2c}$$

The pool of unemployed susceptible decreases because a fraction $(1-p_t^S)\pi_t^{UI}$ does not find a job but becomes infected, a fraction $p_t^S\pi_t^{UI}$ finds a job and become infected, and a fraction $p_t^S(1-\pi_t^{UI})$ finds a job but does not get infected. A fraction $\lambda(1-\pi_t^{EI})$ of employed susceptible workers gets separated without catching the virus and adds to pool. The unemployed infected are not looking for a job, and so the pool gets reduced only by recovering (fraction π_R) or by dying (fraction π_D). Increasing the pool of unemployed infected are susceptible unemployed who get infected (regardless of their job market outcome) with probability π^{UI}, and susceptible employed who get infected and also happen to get fired with probability $\lambda\pi_t^{EI}$. Infected employed can also become infected unemployed by losing their job while avoiding dying or recovering with probability $\lambda(1-\pi_R-\pi_D)$. Finally, the stock of unemployed recovered is reduced because a fraction p_t^R finds a job, and increased because a fraction π_R of unemployed infected recovers, a fraction $\lambda\pi_R$ of employed infected is fired but recovers at the same time, and a fraction λ of employed recovered loses their job.

It follows that the number of dead people keeps growing, with the increments being a fraction π_D of the infected people:

$$D_{t+1} = D_t + \pi_D(EI_t + UI_t).$$

We can aggregate across health status to express the law of motion for the total number of employed and unemployed:

$$\begin{aligned} E_{t+1} &= (1-\lambda)\left(E_t - \pi_D EI_t\right) + p_t^S US_t + p_t^R UR_t \\ U_{t+1} &= (1-p_t^S)US_t + (1-p_t^R)UR_t + (1-\pi_D)UI_t + \lambda(E_t - \pi_D EI_t) \end{aligned}$$

The evolution of aggregate employment and unemployment cannot be expressed in terms of the labor market aggregates themselves for three reasons. First, the number of people who leave the labor market by kicking the bucket depends only on the number of infected people. Second, infected people cannot look for a job, and so the rate at which people transition to the employment status depends on the number of infected unemployed. Third, job finding probabilities also depend on whether people are susceptible or recovered.

Aggregating across the employment status yields

$$\begin{aligned} S_{t+1} &= (1-\pi^{UI})S_t - (\pi^{EI} - \pi^{UI})ES_t \\ I_{t+1} &= (1-\pi_R-\pi_D)I_t + \pi^{UI}S_t + (\pi^{EI} - \pi^{UI})ES_t \\ R_{t+1} &= R_t + \pi_R I_t. \end{aligned}$$

The aggregation is again only partial, this time because unemployed and employed have a different probability of being infected. Only if $\pi_t^{UI} = \pi_t^{EI}$ then the law of motion can be expressed entirely in terms of health status aggregates S, I and R.[2]

[2] In particular, $S_{t+1} = (1-\pi_t^I)S_t$, $I_{t+1} = (1-\pi_R-\pi_D)I_t + \pi_t^I S_t$, and $R_{t+1} = R_t + \pi_R I_t$.

Dynamics. We think of the pandemic as follows. At the initial steady state, no one is infected or dead, and everyone is susceptible, either employed or unemployed. At time zero, a small fraction of individuals gets exogenously infected. The pandemic spreads throughout the population. At the end, the economy settles into new steady state, where everyone who is alive is recovered, and there are no susceptible or infected people.

4 The Model

Our goal now is to provide a theory of how some of the transition probabilities in the above accounting framework are determined in equilibrium. In particular, we want to characterize the job hiring probabilities p_t^S and p_t^R, and their dynamics during the transition. We will also endogenize the infection probabilities π_t^{EI} and π_t^{UI}. We also want to use the theory to characterize the dynamics of wages over the transition, welfare consequences of the pandemic, and optimal policies.

4.1 Labor Market

The labor market is frictional. We assume that the labor market for the susceptible people is separate from the labor market for the recovered people. It is beneficial for the recovered people to signal that they have already recovered, because they no longer face any potential disruptions to their productivity. The key assumption is that susceptible people cannot pretend to have recovered. It is reasonable to assume that it would be in the overall public interest to separate the two groups.[3] The assumption also simplifies the model substantially, because potential employers do not have to solve a screening problem that would otherwise arise.

Firms create vacancies in each of the two labor markets, VS_t and VR_t. The number of new jobs created among the susceptible and recovered, HS_t and HR_t, depends on the total number of unemployed and the number of vacancies in each labor market, and is given by a matching function m:

$$HS_t = m(US_t, VS_t)$$
$$HR_t = m(UR_t, VR_t).$$

[3] In the movie Epidemics (2011), people who have already been vaccinated wear special armbands. Presumably, counterfeiting the armbands would be criminalize.

The function m is common in both labor markets. It has standard properties, it is increasing and concave in both arguments, differentiable, and is constant returns to scale.

The labor market tightness will differ across both labor markets. We denote the labor market tightness in the susceptible labor market by $\theta_t^S = VS_t/US_t$, and in the recovered labor market by $\theta_t^R = VR_t/UR_t$. The job finding probabilities in both markets depend only on their respective labor market tightness and are

$$p_t^S = \frac{HS_t}{US_t} = p(\theta_t^S) \tag{3a}$$

$$p_t^R = \frac{HR_t}{UR_t} = p(\theta_t^R). \tag{3b}$$

where $p(\theta) = m(1, \theta)$. The probabilities of filling a vacancy in each market are $q_t^S = p(\theta_t^S)/\theta_t^S$ and $q_t^R = p(\theta_t^R)/\theta_t^R$. We note that the labor market tightness, and the associated job hiring probabilities, will only differ during the outbreak of the epidemics. Before and after the epidemics, they will be the same, because the matching function is the same.

4.2 Infection Propagation

The probability that a worker gets infected is endogenous and depends on the interactions with other people. We assume that if there is more infected workers around, the probability of getting infected increases.[4] We model it as follows: the probability that employed and unemployed workers get infected is proportional to the number of currently infected people:

$$\pi_t^{EI} = \frac{EI_t}{ES_t} = s^E I_t \tag{4a}$$

$$\pi_t^{UI} = \frac{UI_t}{US_t} = s^U I_t. \tag{4b}$$

The underlying assumption is that both employed and unemployed interact with all infected people, although at different rates, given by constants s^E and s^U.[5] The number

[4] It is reasonable to conjecture that, once a worker is revealed to be infected, he is quarantined and no longer interacts with other workers. However, he interacts with other workers during the incubation period, which we do not model explicitly for simplicity.

[5] We also considered a version where the employed people interact more with other employed people, and their infection probability depends on EI instead of I. The results were similar.

of newly infected employed and unemployed, EI_t and UT_t, is

$$EI_t = s^E I_t ES_t$$
$$UI_t = s^U I_t US_t.$$

4.3 Unemployed Workers

An unemployed worker enjoys non-monetary benefits b per period, regardless of its health status. Infected workers, however, incur additional utility loss c per period. The utility loss reflects pain and additional non-monetary costs associated with the infection.

Let K_t^{US}, K_t^{UI} and K_t^{UR} be the lifetime utility of an unemployed worker who is, respectively, susceptible, infected and recovered. The timing for the unemployed in the susceptible category is as follows. They search for a new job during the period. At the end of the period, it is revealed whether the worker has found a job and whether he got infected during the period. We assume that a worker who has found a match but is infected at the same time is prohibited from signing a contract, and the match is not formed. The wage, which is determined in a way that we describe below, is then formed only for workers that are susceptible, but will be able to work during the first period. The value function of a susceptible unemployed is

$$K_t^{US} = b + \beta \left[p_{t+1}^S (1 - \pi_{t+1}^{UI}) K_{t+1}^{ES} + \pi^{UI} K_{t+1}^{UI} + (1 - p_{t+1}^S)(1 - \pi_{t+1}^{UI}) K_{t+1}^{US} \right], \tag{5}$$

where K^{ES} is the value function of a susceptible employed worker. The infected unemployed incur the disease cost c but also face different transition probabilities, in particular they can recover or die, but cannot become employed. Their value function is:

$$K_t^{UI} = b - c + \beta \left[\pi_R K_{t+1}^{UR} + \pi_D D + (1 - \pi_R - \pi_D) K_{t+1}^{UI} \right], \tag{6}$$

where D is the value of death, which we normalize to zero. Finally, the recovered unemployed face a relatively standard labor search problem

$$K_t^{UR} = b + \beta \left[p_{t+1}^R K_{t+1}^{ER} + (1 - p_{t+1}^R) K_{t+1}^{UR} \right], \tag{7}$$

where K^{ER} is the value function of employed recovered.

4.4 Employed Workers

A job is formed by matching a firm with a worker. A match between susceptible or recovered workers and a firm produces y units of output per period. Infected people are unproductive until they recover and produce zero. Once a match is formed, the worker and the firm bargain about their wage. We describe the wage formation in detail below. The main assumption is that there are no long-term contracts and the wage is renegotiated continuously every period, including periods when the worker is infected and is on a sick leave.

The value functions of employed workers are K_t^{ES}, K_t^{EI} and K_t^{ER}. The value function of susceptible employees solves

$$K_t^{ES}(w) = w + \beta \left\{ \lambda \left[(1 - \pi_{t+1}^{EI}) K_{t+1}^{US} + \pi_{t+1}^{EI} K_{t+1}^{UI} \right] + (1 - \lambda) \left[(1 - \pi_{t+1}^{EI}) K_{t+1}^{ES} + \pi_{t+1}^{EI} K_{t+1}^{EI} \right] \right\} \tag{8}$$

When an employed worker gets infected, he is not productive. As mentioned previously, the worker cannot be fired just because he is infected, and so separates either because he is fired for other reasons (with probability λ), or because he dies (with probability π_D). During sickness, he collects a fraction of his wage, which is negotiated with the employer. If the worker recovers, he regains his previous productivity. The infected employees also incur a utility cost c. Their value function satisfies

$$\begin{aligned} K_t^{EI}(w) = w - c + \beta\lambda \left[\pi_R K_{t+1}^{UR} + (1 - \pi_R - \pi_D) K_{t+1}^{UI} \right] \\ + \beta(1 - \lambda) \left[\pi_R K_{t+1}^{ER} + (1 - \pi_R - \pi_D) K_{t+1}^{EI} \right] + \pi_D D. \end{aligned} \tag{9}$$

Finally, recovered employees have a value function

$$K_t^{ER}(w) = w + \beta \left[\lambda K_{t+1}^{UR} + (1 - \lambda) K_{t+1}^{ER} \right]. \tag{10}$$

4.5 Firms

Firms can post vacancies in the labor market for susceptible workers, or in the labor market for recovered workers. Firms in the first labor market take into account that a susceptible worker may find a job and get infected, either immediately, in which case a match is not formed, or later. Firms in the second labor market are, on the other hand, sure that the worker will not get infected. To characterize the firms problem, we first consider the value of a filled job conditional on a health status of its worker, J_t^S, J_t^I and J_t^R. The value of having a susceptible worker is

$$J_t^S(w) = y - w + \beta \left\{ \lambda V_{t+1} + (1-\lambda) \left[\pi_{t+1}^{EI} J_{t+1}^I + (1-\pi_{t+1}^{EI}) J_{t+1}^S \right] \right\}, \tag{11}$$

where V_{t+1} is the value of a vacancy in period $t+1$. The value of having an infected worker is

$$J_t^I(w) = -w + \beta \left\{ [\pi_D + \lambda(1-\pi_D)] V_{t+1} + (1-\lambda)\pi_R J_{t+1}^R + (1-\lambda)(1-\pi_R-\pi_D) J_{t+1}^I \right\}. \tag{12}$$

Finally, the value of having a recovered worker is

$$J_t^R(w) = y - w + \beta \left[\lambda V_{t+1} + (1-\lambda) J_{t+1}^R \right]. \tag{13}$$

Posting a vacancy has a cost k irrespective of the market. The value of vacancy in the market for susceptible workers is

$$V_t^S = -k + \beta \left[[1 - q_{t+1}^S (1-\pi_{t+1}^{UI})] V_{t+1} + q_{t+1}^S (1-\pi_{t+1}^{UI}) J_{t+1}^S \right]$$

Similarly, the value of vacancy in the market for recovered workers is

$$V_t^R = -k + \beta \left[q_{t+1}^R J_{t+1}^R + (1-q_{t+1}^R) V_{t+1} \right]$$

Free entry. Firms are free to enter any of the two markets anytime. This means that the value of a vacancy, V_t, equals zero. That is, in equilibrium, the cost of creating a vacancy equals the expected benefits from doing so,

$$\beta(1-\pi_{t+1}^{UI}) J_{t+1}^S = \frac{k}{q_{t+1}^S} \tag{14}$$

$$\beta J_{t+1}^R = \frac{k}{q_{t+1}^R} \tag{15}$$

4.6 Wage determination

The wages in both markets are determined as a result of Nash bargaining between both parties. We assume that the wage can be renegotiated continuously at the beginning of the period. Both parties know the health status of the worker and so have a symmetric information. The wage in the market for susceptible, infected and recovered workers is a solution to

$$w_t^S = \arg\max \left(J_t^S\right)^{1-\phi} \left(K_t^{ES} - K_t^{US}\right)^{\phi} \tag{16}$$

$$w_t^I = \arg\max \left(J_t^I\right)^{1-\phi} \left(K_t^{EI} - K_t^{UI}\right)^{\phi} \tag{17}$$

$$w_t^R = \arg\max \left(J_t^R\right)^{1-\phi} \left(K_t^{ER} - K_t^{UR}\right)^{\phi}. \tag{18}$$

where $\phi \in [0, 1]$ is the bargaining weight of the worker. The bargaining weight is identical in all markets, and the bargaining problem already incorporates the result that the value of vacancies, the outside option for the firm, is equal to zero.

Since the value functions K^{UI}, K^{US} and K^{UR} are all independent of the bargained wage and there is a fixed surplus to be divided, the first-order conditions are that

$$K_t^{ES} - K_t^{US} = \frac{\phi}{1-\phi} J_t^S$$

$$K_t^{EI} - K_t^{UI} = \frac{\phi}{1-\phi} J_t^I$$

$$K_t^{ER} - K_t^{UR} = \frac{\phi}{1-\phi} J_t^R.$$

4.7 Equilibrium

The initial conditions are given by $(US_0, ES_0, UI_0, EI_0$, where the values of UI_0 and EI_0 can be thought of as being small, as they would be at the onset of a pandemic, and the number of recovered people is zero, $UR_0 = ER_0$. The equilibrium is given by type allocations $\{US_t, ES_t, UI_t, EI_t, UR_t, ER_t, D_t\}$, value functions $\{K_t^{US}, K_t^{UI}, K_t^{UR}, K_t^{ES}, K_t^{EI}, K_t^{ER}\}$ for the worker, value functions $\{J_t^S, J_t^I, J_t^R\}$ for the firm, wages $\{w_t^S, w_t^R\}$, labor market tightness $\{\theta_t^S, \theta_t^R\}$ such that i) worker value functions satisfy (5)-(10), ii) firm value functions satisfy (11)-(13),, iii) free entry conditions (14) and (15) hold, iv) wages solve (25)-(18), v) aggregates evolve according to (1) and (2), and vi) job finding and infection probabilities are given by (3) and (4).

5 Solving the Model

Since the workers always proceed from being susceptible to being infected to being recovered, the model can be solved recursively, starting from the recovered stage, then proceeding to the infected stage, and finally to the susceptible stage.

Recovered stage. The recovered stage has a time invariant solution. Market tightness, wage, and all the value functions are independent of time, and identical to their value before the pandemic started. The dynamics of the recovered stage is only reflected in the unemployment rate. The solution is a textbook one, and we provide it mainly so as to compare it to the solution in other stages. Define the total surplus of the match in the recovered stage as $F^R = J^R + K^{ER} - K^{UR}$. The value of the match is independent of the wage. Then $K^{ER} - K^{UR} = \phi F^R$ and $J^R = (1-\phi)F^R$. Adding together the value functions in the recovered stage, we obtain the value of the surplus

$$F^R = \frac{y-b}{1-\beta[1-\lambda-\phi p(\theta^R)]},$$

where we are now explicit about the relationship between the probability of finding a job and the market tightness in the recovered stage. Combining with the free entry condition (15) yields the textbook equation in the market tightness θ^R:

$$\frac{y-b}{k} = \frac{1/\beta - 1 + \lambda + \phi p(\theta^R)}{(1-\phi)q(\theta^R)}.$$

To characterize the equilibrium wage, we define the reservation wage as a wage that, if paid in the current period (not permanently), makes the worker indifferent between accepting and not accepting the job offer. It is defined by $K^{ER}(\overline{w}^R) = K^{UR}$, which can be solved for

$$\overline{w}^R = b + \beta p^R \phi F^R - \beta(1-\lambda)\phi F^R. \tag{19}$$

The reservation wage increases above the unemployment benefits because there is an option, with probability p^R, of waiting until next period, negotiating, and getting a fraction of the surplus ϕF^R tomorrow. On the other hand, the reservation wage decreases because there is an option of continuing on the job tomorrow, with probability $1-\lambda$.[6]

[6]While the expression can be further simplified, we prefer to keep it in the current form to facilitate a

The equilibrium wage w^R is then a sum of the reservation wage plus a fraction ϕ of the surplus of the match:

$$w^R = \overline{w}^R + \phi F^R. \tag{20}$$

The worker thus gets his reservation wage and his share of the match, given by his bargaining power. Replacing the reservation wage and the value of the surplus yields the equilibrium wage as a direct function of the market tightness ratio,

$$w^R = (1-\phi)b + \phi(y + k\theta^R)$$

As usual, higher market tightness means that workers are more likely to find a job, and can command a higher wage. We also solve explicitly for value functions of the worker K^{ER} and K^{UR}:

$$K^{UR} = \frac{b}{1-\beta} + \frac{\beta\phi p(\theta^R)}{1-\beta}F^R, \quad K^{ER} = \frac{w^R}{1-\beta} + \frac{\beta\phi\lambda}{1-\beta}F^R.$$

Infected stage Once the solution to the problem in the recovered stage is known, we can solve the problem in the infected stage. Since the probabilities of recovery and death are constant over time by assumption, the infected stage has a time invariant solution as well. The value of a match in the infected stage is $F^I = J^I + K^{EI} - K^{UI}$. The bargaining protocol again implies that it is divided according to the bargaining power. Since the problem is time invariant, F_t^I is also constant over time and is a solution to

$$F^I = \frac{-b + \beta(1-\lambda)\pi_R F^R}{1 - \beta(1-\lambda)(1-\pi_R-\pi_D)}.$$

The expression is different from F^R in one important aspect. The current surplus is $-b$ rather than $y - b$ because nothing is produced in the infected stage. Instead, the value of the match is given purely by its future surplus, once the worker recovers and becomes productive again. This is represented by the second term $\beta(1-\lambda)\pi_R F^R$, where the probability $(1-\lambda)\pi_R$ denotes the probability that the worker recovers and the match continues.

The reservation wage in the infected stage is determined differently. It is given by

comparison to the reservation wages in other stages.

the following expression:

$$\overline{w}^I = b - \beta\pi_R(1-\lambda)\left(K^{ER} - K^{UR}\right).$$

The reservation wage is lower than the unemployment benefits because infected people are not able to look for a job directly and so must enter the recovered stage as unemployed. If they could, they would then be willing to take a wage cut, relative to benefits, in order to obtain a job now and enter the recovered stage as employed. The size of the cut, $\beta\pi_R(1-\lambda)\left(K^{ER} - K^{UR}\right)$, depends on the gain from being employed in the recovered stage, $K^{ER} - K^{UR}$, and the probability that the worker transits employed to the recovered stage, $\pi_R(1-\lambda)$.

Analogously to (20), the equilibrium wage w^I is a weighted average of the reservation wage $\overline{w}^I$, and the surplus of the match. The surplus of the match is now different, however. It is given by $\phi\beta(1-\lambda)\pi_R J^R$ and so the wage is

$$w^I = \phi\beta(1-\lambda)\pi_R J^R + (1-\phi)\overline{w}^I. \tag{21}$$

Combining both expressions, we obtain that the equilibrium wage in the infected stage is simply given by

$$w^I = (1-\phi)b.$$

On one hand, the future surplus $\beta(1-\lambda)\pi_R J^R$ in (21) drives the wage of the infected above the reservation wage. On the other hand, the reservation wage is lower than the unemployment benefits. In equilibrium, the second effect dominates, and the wage is a fraction of the unemployment benefits. The equilibrium wage in turn yields the equilibrium value of the firm J^I,

$$J^I = \frac{-(1-\phi)b + \beta(1-\lambda)\pi_R J^R}{1-\beta(1-\lambda)(1-\pi_R-\pi_D)},$$

the value of an unemployed infected,

$$K^{UI} = \frac{b - c + \beta\pi_R K^{UR}}{1-\beta(1-\pi_R-\pi_D)},$$

and, finally, the value of infected employees:

$$K^{EI} = \frac{(1-\phi)b - c + \beta\pi_R\left[\lambda K^{UR} + (1-\lambda)K^{ER}\right] + \beta\lambda(1-\pi_R-\pi_D)K^{UI}}{1-\beta(1-\lambda)(1-\pi_R-\pi_D)}.$$

Susceptible stage. Unlike the last two stages, the problem in the susceptible stage does not have a time invariant solution because the probabilities of being infected are not constant over the pandemic. We again define the value of a match in the susceptible stage by $F_t^S = J_t^S + K_t^{ES} - K_t^{US}$ and divide it between both parties according to their bargaining power, $K_t^{ES} - K_t^{US} = \phi F_t^S$ and $J_t^S = (1-\phi)F_t^S$.

We first characterize the law of motion of the output match F_t^S. The current value F_t^S depends not only on future match values F_{t+1}^S and F^I, but also on the utility loss that a susceptible person gets from getting infected $\Delta_{t+1}^S = K^{UI} - K_{t+1}^{US}$. By rearranging the value functions for the firm and the workers, we find that the law of motion is given by

$$\begin{aligned} F_t^S = {} & y - b + \beta(1-\lambda)\left[\pi_{t+1}^{EI}F^I + (1-\pi_{t+1}^{EI})F_{t+1}^S\right] - \beta\phi p_{t+1}^S(1-\pi_{t+1}^{UI})F_{t+1}^S \\ & + \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\Delta_{t+1}^S \end{aligned} \tag{22}$$

The law of motion for the loss from getting infected, Δ_t^S, is

$$\Delta_t^S = (1-\beta)K^{UI} - b - \beta(1-\pi_{t+1}^{UI})\left(\phi p_{t+1}^S F_{t+1}^S - \Delta_{t+1}^S\right). \tag{23}$$

The loss from getting infected consists of several parts. First, there is the difference between the period utilities $(1-\beta)K^{UI} - b$, which is constant over time. Second, the loss in utility comes from the fact that an individual cannot get a job while being infected, while a susceptible worker gets a job with a probability $(1-\pi_{t+1}^{UI})p_{t+1}^S$, in which case the worker gets a share ϕ of the surplus. Finally, we correct for the fact that the worker might get infected tomorrow, in which case he has an additional loss Δ_{t+1}^S.

In addition to (22) and (23), the free entry condition (14) must hold as well. This gives us a third set of equations. Those equations are to be solved for equilibrium sequences $\{F_t^S, \Delta_t^S, \theta_t^S\}$. The system can be solved as follows. Using the above system of equations, given F_{t+1}^S, Δ_{t+1}^S and θ_{t+1}^S, solve for F_t^S and Δ_t^S. Then use the free entry condition to obtain θ_t^S. Iterate until the beginning. We start by assuming that the system is in steady state after some sufficiently distant date T.

We now characterize the sequence of wages $\{w_t^S\}$. The reservation wages $\{\overline{w}_t^S\}$ are such that a currently susceptible worker is indifferent between working and not working:

$$K_t^{US} = K_t^{ES}(\overline{w}_t^S).$$

Rearranging, we obtain

$$\overline{w}_t^S = b + \beta\phi p_{t+1}^S(1-\pi_{t+1}^{UI})F_{t+1}^S - \beta(1-\lambda)\phi\left[\pi_{t+1}^{EI}F^I + (1-\pi_{t+1}^{EI})F_{t+1}^S\right] \\ - \beta\left(\pi_{t+1}^{EI}-\pi_{t+1}^{UI}\right)\Delta_{t+1}^S. \tag{24}$$

The reservation wage is a product of several forces. The second and third term on the right-hand side are analogous to the second term on the right-hand side of the reservation wage in the recovered stage (20). An option of waiting, not getting infected and negotiating tomorrow increases the reservation wage, while an option of continuing on the job tomorrow decreases the reservation wage. A reasonable guess is that a person is more likely to be an employed susceptible tomorrow if it takes the job today, $(1-\lambda)(1-\pi_{t+1}^{EI}) > p_{t+1}^S(1-\pi_{t+1}^{UI})$: while the infection probability is somewhat higher on the job, the probability of losing the job λ is likely to be much smaller than the probability of finding a job p_{t+1}^S. Under those conditions, a higher value of a susceptible match F_{t+1}^S makes the workers more willing to accept job and reduces the reservation wage. The last term in the reservation wage equation (24) is novel. It decreases the reservation wage, because accepting a job increases the probability of getting infected.[7] For example, if both infection probabilities get proportionally higher, the reservation wage will decrease.

The equilibrium wage is then again equal to the reservation wage plus a fraction ϕ if the match:

$$w_t^S = \overline{w}_t^S + \phi F_t^S. \tag{25}$$

The equation (25) shows that the equilibrium wage is a product of two forces that will be under detailed scrutiny later: the reservation wage and the value of the match.

[7]The reservation wage increases because $\pi_{t+1}^{EI} - \pi_{t+1}^{UI} > 0$ and the Δ_{t+1}^S denotes the loss from getting infected, and so is negative.

6 Efficient Allocations and Optimal Quarantine

The equilibrium of the model is not efficient, even when the Hosios condition holds, because of the externality coming from the propagation of the infection. Individuals obviously do take into account that, if they choose to work, they increase the probability that they would themselves get infected. They do not, however, take into account that, if they get infected, the probability that others will get infected increases. We have already seen that rigid wages can dominate a flexible wage scenario, because they decrease the employment rate and save lives. But, of course, there is no reason to believe that rigid wages would be the best possible scenario. We now consider the planning problem directly.

We consider two versions of the planning problem. The first version, less authoritative, takes the separation rate λ as given. That is, the planner cannot forcibly move the individuals from employment to unemployment and vice versa. This is the usual notion of the planning problem in this context. The second version of the planning problem allows the planner to terminate job matches, and move people from employment to unemployment in excess of what is dictated by the separation rate. We think of the extra option of forcibly moving people out of employment and destroying labor market matches as a version of a quarantine. Correspondingly, we will call the outcomes of the first problem as *efficient allocations without quarantine*, and the outcomes of the second problem as *efficient allocations with quarantine*. Note that, absent a pandemic, both problems would coincide, because the equilibrium would be efficient planner would not want to voluntarily destroy any matches. As we will see, this result no longer holds during pandemics.

We now formulate both versions of the planning problem. In the planning problem without quarantine, the social planner chooses the flows $\{US_t, ES_t, UI_t, EI_t, UR_t, ER_t, D_t\}$, as well as the number of vacancies $\{VS_t, VR_t\}$, to maximize the present value of resources,

$$\sum_{t=0}^{\infty} \beta^t \left[(ES_t + ER_t)y + (US_t + UI_t + UR_t)b - (UI_t + EI_t)c - (VS_t + VR_t)k\right],$$

subject to laws of motion (1) and (2), where the probabilities of being hired are given by (3) and the infection probabilities are given by (4a) and (4b). In the planning problem with quarantine, we define Q_t to be the number of quarantined people in the susceptible stage in period t. Quarantined people change the labor market flows in the susceptible stage from (1a) and (2a) to[8]

$$ES_{t+1} = (1-\lambda)(1-\pi^{EI})ES_t + p^S(1-\pi^{UI})US_t - Q_t \tag{26}$$

$$US_{t+1} = (1-p^S)(1-\pi^{UI})US_t + \lambda(1-\pi^{EI})ES_t + Q_t. \tag{27}$$

The social planner's problem with quarantine modifies the problem without quarantine by replacing (1a) and (2a) with (26) and (27), and by allowing the planner to choose the number of quarantined people $\{Q_t^S\}$ subject to the nonnegativity constraint $Q_t^S \geq 0$. Additional details of the solution to both planning problems are in the Appendix A.

We note that in both planning problems, the objective function can be considered somewhat restrictive: the loss of one's life is measured purely in terms of the loss of output. It is, of course, reasonable to argue that a loss of life has higher cost, not internalized by individuals, as in, for example, Alvarez et al. (2020). It is in fact easy to extend the planning problem in this way. However, we wish to stay away from assigning arbitrary values to one's life. In this way, our planning problems, and the policies that they imply, can be seen as a sort of a lower bound on such policies when a life has larger cost.[9]

The efficient allocations can be characterized by the employment and unemployment flows, and by the labor market tightness ratios. The recovered stage is efficient as long as the Hosios condition holds, and has the same labor market tightness. The infected and susceptible stage are not efficient. The solution to the social planner's problem yields the following first-order conditions in the infected stage:

$$\mu_t^{UI} = b - c + \beta\left[(1-\pi_R-\pi_D)\mu_{t+1}^{UI} + \pi_R\mu^{UR}\right] - \psi_t \tag{28}$$

$$\mu_t^{EI} = -c + \beta\lambda\left[(1-\pi_R-\pi_D)\mu_{t+1}^{UI} + \pi_R\mu_{t+1}^{UR}\right] + \beta(1-\lambda)\left[(1-\pi_R-\pi_D)\mu_{t+1}^{EI} + \pi_R\mu_{t+1}^{ER}\right] - \psi_t, \tag{29}$$

where μ_t^{UI} is the social value of unemployment, and μ_t^{EI} is the social value of employment. They have their equilibrium counterparts in the private value of unemployment

[8] We do not allow the social planner to move people in the infected or recovered stage. The social planner has nothing to gain by doing so.

[9] See Hall et al. (2020) for calculations regarding the trade-off between lost consumption and lives,

K_t^{UI} and private value of employment $K_t^{EI} + J_t^I$. The last term,

$$\psi_t = \beta(\mu_t^{ES} - \mu_t^{US}) s^U p_t^S US_t + \beta(\mu_t^{ES} - \mu_t^{EI})(1-\lambda) s^E ES_t + \beta(\mu_t^{US} - \mu_t^{UI})(s^U US_t + \lambda s^E ES_t) \tag{30}$$

is the size of the externality, where μ_t^{EI} and μ_t^{EI} are the social values of employment and unemployment. Comparing conditions (28) and (29) to (6) and the sum of (9) and (12) shows that the difference is exactly the externality ψ_t. However, while the value of employment and unemployment is, in the optimum, reduced by ψ_t, their difference is not: after cancelling terms, we find that the difference is not only independent of time, but satisfies

$$\mu_t^{EI} - \mu_t^{UI} = -b + \beta(1-\lambda)\left[(1-\pi_R-\pi_D)(\mu_t^{EI} - \mu_t^{UI}) + \pi_R(\mu^{ER} - \mu^{UR})\right]$$

which has the same time invariant solution as F^I. That is, the value of the match in the infected stage has the socially optimal value in equilibrium. But the components do not, and each of them is reduced by ψ_t.

The externality ψ_t in (30) consists of three terms. They correspond to three types of external effects that an infected individual imposes on others: reduction in the effective job finding rate, loss of value from employment, and loss of value from unemployment. The effective job finding rate in the model is $p_t^S(1-\pi_t^{UI})$ because an unemployed susceptible who becomes infected cannot accept a job, even if a match is found. If the number of infected individuals increases by one person, the probability that an unemployed individual accepts a job decreases $s^U p_t^S$ and, since there is US_t unemployed people, $s^U p_t^S US_t = s^U m(1, \theta_t^S)$ is the number of people that will lose the value of employment $\mu_t^{ES} - \mu_t^{US}$. Second, an increase of infected people by one person increases the probability that employed people do not lose job but get infected by $(1-\lambda)s^E$. Those people, ES_t of them, become employed infected, and lose value $\mu_t^{ES} - \mu_t^{EI}$. Third, an increase of infected people by one person increases the number of unemployed people who are infected by $s^U US_t + \lambda s^E ES_t$, where the first term represent those that were unemployed, and the second term represents those that were employed and lost their job. Each of those individuals loses value $\mu_t^{US} - \mu_t^{UI}$. The costs accrue only tomorrow, and so are discounted.

7 Calibration

A period is one week. an annual interest rate is 2 percent, and so the discount factor is set to $\beta = 0.98^{1/52}$. The calibration of the labor sector is standard. Output is set to one, and the value of being unemployed is set to 0.4.[10] The separation rate λ is set equal to 1/130, producing an average job duration of 130 weeks or 2.5 years. We target a quarterly job finding probability of 0.45 which translate to a weekly job finding probability of $0.45 \times 12/52 = 0.104$. The bargaining power of the workers is 0.72. We assume that the matching function is Cobb-Douglas:

$$m(U, V) = aU^{\alpha}V^{1-\alpha}. \tag{31}$$

The elasticity of the matching function α is also set to 0.72, ensuring that the Hosios condition holds, and the only source of inefficiency comes from the transmission of the epidemic. The steady state labor market tightness is normalized to one. This produces the value of a equal to the job finding probability and the value of posting a vacancy $k = 0.21$.

As for the calibration of the epidemic, we follow Atkeson (2020) and Eichenbaum et al. (2020), and set the probability of death $\pi_D = 7 \times 0.01/18$, and the probability of recovery to $\pi_R = 7 \times 0.99/18$. Those values ensure that, in the long run, one percent of the infected dies. The ratio s^U/s^E determines the probability that the unemployed get infected relative to the employed. We calibrate it as follows. Edmunds et al. (1997) report the results of a survey of mixing patterns among a sample of adults. Their figure 4 implies that participants in the sample has approximately 125 contacts per week. Out of those, 66.5 contacts were work related; the rest of the contacts were social, at home, shopping and travel related. That is, non-work related contacts were 46.8 percent of total contacts. Accordingly, we set $s^U/s^E = 0.468$. Finally, we target a steady state value of infected and recovered people to be two thirds. This yields $s^E = 0.6598$. Table 1 shows the calibrated parameters. Finally, we set the cost of being infected $c = 0.2$, thus reducing the value of being unemployed to a half.

We initialized the pandemic by assuming that 0.1 percent of the population gets infected for exogenous reasons. We assume that the proportion of infected workers is the same among employed and unemployed workers.

[10] The level of output and benefits is actually not a normalization, because the value of death is already set to zero. We find that values higher than one do not have a significant effect on the results.

Table 1: Benchmark Parameters

parameter	value	target
β	0.9996	2% annual interest rate
y	1	normalization
b	0.4	Shimer (2005)
λ	0.0077	average job duration of 2.5 years
ϕ	0.72	Shimer (2005)
a	0.1038	normalization
α	0.72	Hosios condition
π_D	0.0039	Eichenbaum et al. (2020)
π_R	0.3850	Eichenbaum et al. (2020)
s^U	0.3088	relative probability of infection
s^E	0.6597	2/3 of the population eventually infected

8 The Results

Figure 1 shows the equilibrium probability of getting infected over the course of the first one hundred weeks. The infection peaks after 28 weeks, when about 6 percent of employed workers and 3 percent of unemployed workers gets infected. After approximately 60 weeks the probabilities of getting infected drop close to zero and the fraction of people who are either dead or infected stabilizes at two thirds of the population. The pandemic costs lives of 0.67 percent of the population.

To understand the dynamics of the labor market, two factors are critical: the value of the match, and labor market tightness. Figure 2a shows the match value for all three categories of workers. Once the workers recover, the value of the match is obviously the highest, as there are no additional disruptions to productivity. The value of the match in the infected stage is lower, because the workers are temporarily unproductive. In both cases, the values are constant over time. The value of the match in the susceptible stage on the other hand exhibits substantial dynamics. It clearly co-moves negatively with the probability of getting infected, which is its main determining factor. It has a U-shaped profile and reaches its lowest value after 25 weeks, and so is ahead of the infection peak by about three weeks. Interestingly, the match in the susceptible stage is lower than the match in the infected stage between weeks 10 and 40. The reason is that the value of the match in the susceptible stage takes into account that workers have a higher probability

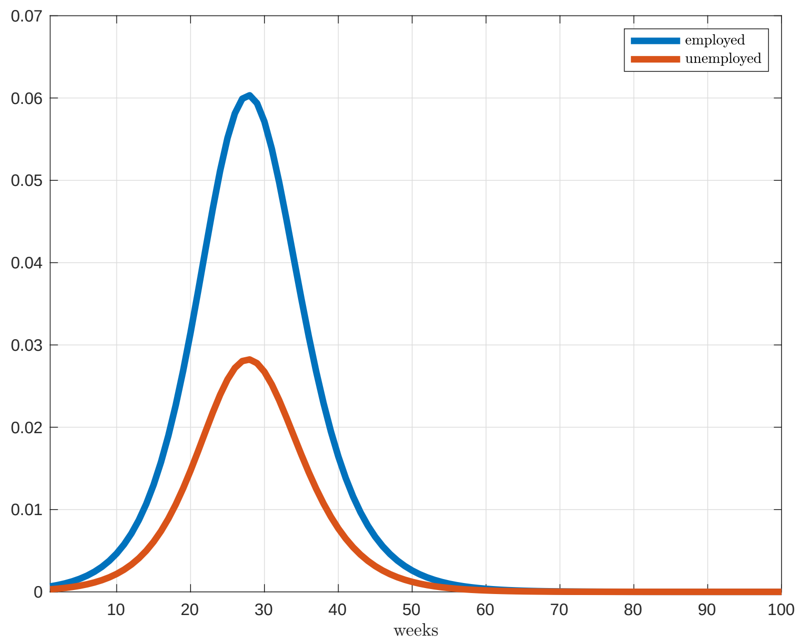


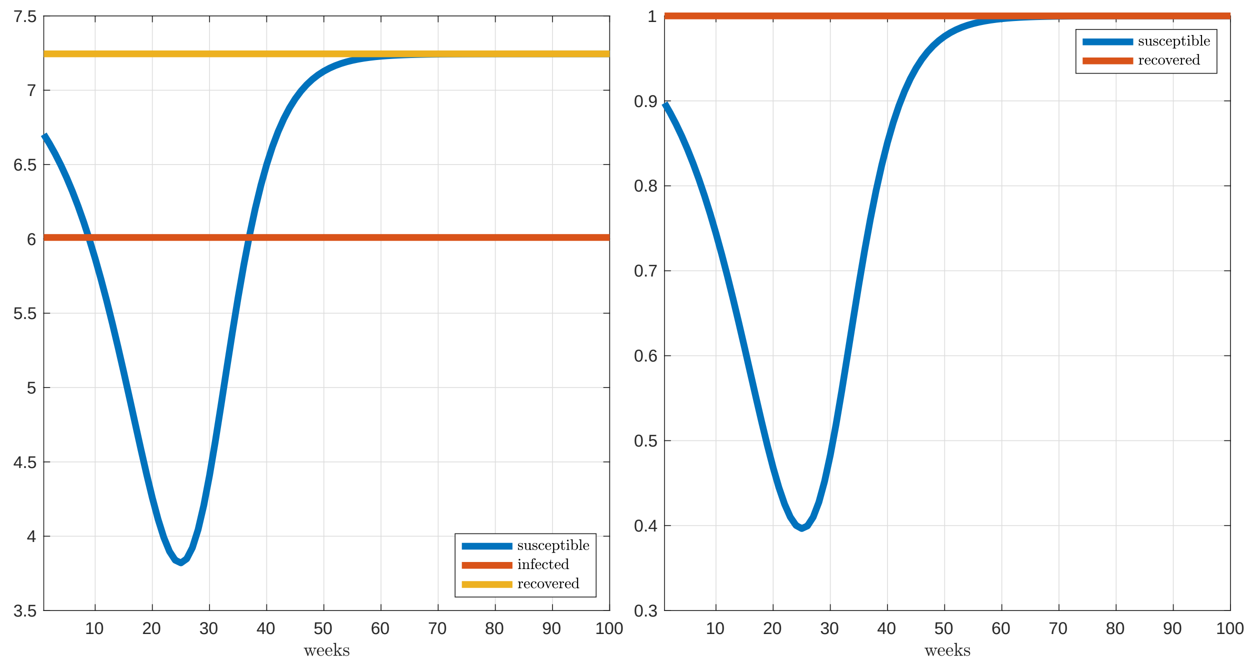
Figure 1: Probability of getting infected

of getting infected and lose utility, while this factor does not appear in the infected stage. When the probability of that happening is large, as it is in the peak weeks, the value of the match decreases substantially.

Labor market tightness is shown in Figure 2b. Recall that labor market tightness is normalized to one in the recovered market. We find that labor market tightness drops substantially in the susceptible market: at its minimum, in week 25, it drops to less than 40 percent of its steady state value. To see why, consider the free entry condition:

$$\frac{k}{q(\theta_t^S)} = \beta(1-\phi)(1-\pi_t^{UI})F_t^S.$$

The left-hand side is increasing in θ_t^S. Market tightness will then be lower if the value of the match is lower, because it does not pay off to post vacancies. The market tightness will also be lower if the probability of getting infected is higher, because this increases the chance that the match will be unproductive. We have seen in Figures 1 and 2a that both factors reinforce each other, because the value of a match is the lowest when the probability of getting infected is the highest. As a result, the labor market tightness co-moves strongly negatively with the probabilities of getting infected. In the long run, as the probability of getting infected converges to zero, the labor market tightness con-



(a) Match value in the susceptible, infected and recovered stage, F_t^S, F^I and F^R.

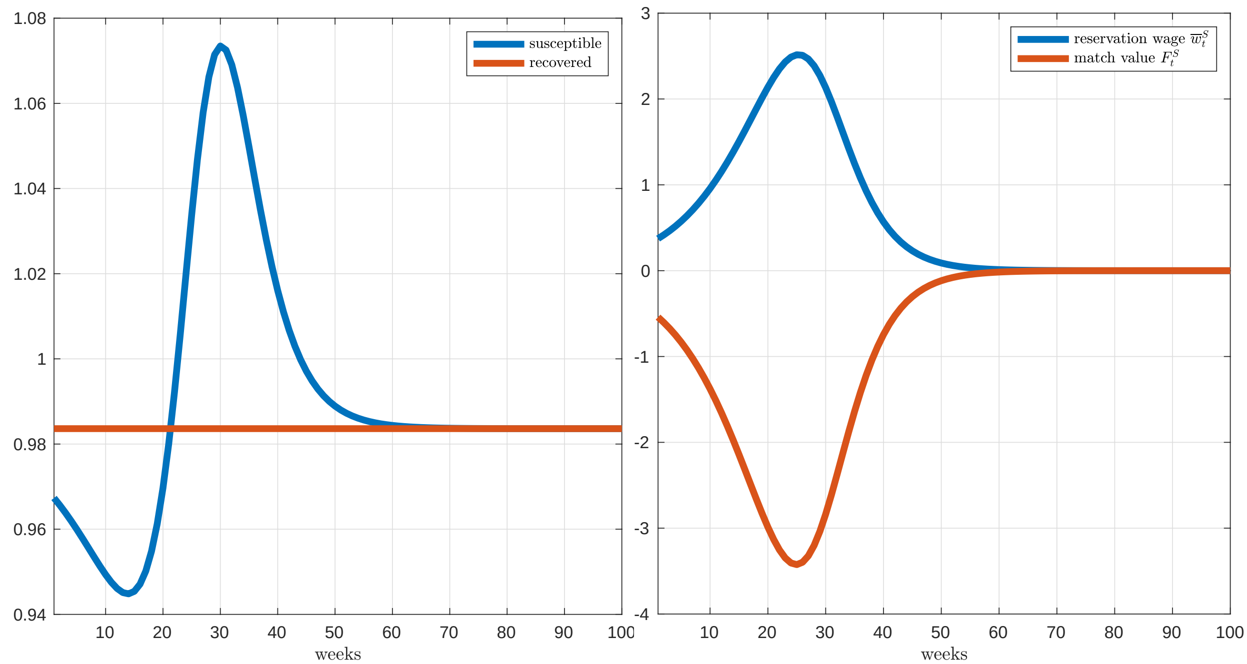
(b) Labor market tightness in the susceptible and recovered market θ_t^S and θ^R.

Figure 2: Labor market tightness and match value.

verges to its steady state level of one.

What do these forces imply for the dynamics of wages and unemployment? Figure 3a shows the wage rate. Immediately after the pandemic starts, the wage in the market for susceptible workers drops below its steady state value by about three percent. It continues decreasing, but then it reverses trend when the probability of getting infected takes off, and increases above its long-run value. After the epidemics peaks, the wage rate in the susceptible market start declining again. At its peak, the wage rate is almost 10 percent above the recovered wage, while at its minimum it is about 5 percent below. The model thus predicts, first, a relatively strong segmentation in the labor market and, second, substantial volatility of wages in the economy.

What explains the reversals in the wage rate? Consider again the expression for the equilibrium wage (25). The equilibrium wage is a product of two factors: the reservation wage, and the surplus match. We have already seen in Figure 2a that the match surplus is U-shaped. That tends to decrease the equilibrium wage rate. Figure 3b reproduces the match value and also shows the reservation wage. The reservation wage moves in the opposite direction than the value of the match. The expression for the reservation



(a) Wage rate in the susceptible and recovered market

(b) Reservation wage and match surplus in the susceptible stage.

Figure 3: Wage rate, match surplus and reservation wage. Match surplus and reservation wages represent deviations from their long-run values.

wage (24) makes it clear why. It is both because the value of the future match is low, and so the expected future gains are relatively low, and because there is a higher chance of getting infected when employed, and that chance is particularly high in those periods. So both forces move the equilibrium wage in the opposite direction. But the reservation wage is more forward-looking than the match value and so starts increasing earlier than the match value starts decreasing and then starts decreasing earlier, which produces the pattern in figure 3a.

Finally, consider the unemployment rates in Figure 4. As the wage among susceptible workers starts rising, fewer vacancies are created and market tightness starts dropping and the unemployment rate rises. It peaks in week 33, 5 weeks after the pandemic peaks, when it reaches more than 10 percentage points, which is almost 50 percent above its steady state level. On the other hand, the unemployment rate among the recovered starts decreasing right after the pandemic begins and stays lower for almost all periods. This is due to a selection effect since employed people are more likely to get infected, they enter the pool of recovered people at a higher rate than the unemployed and drive down

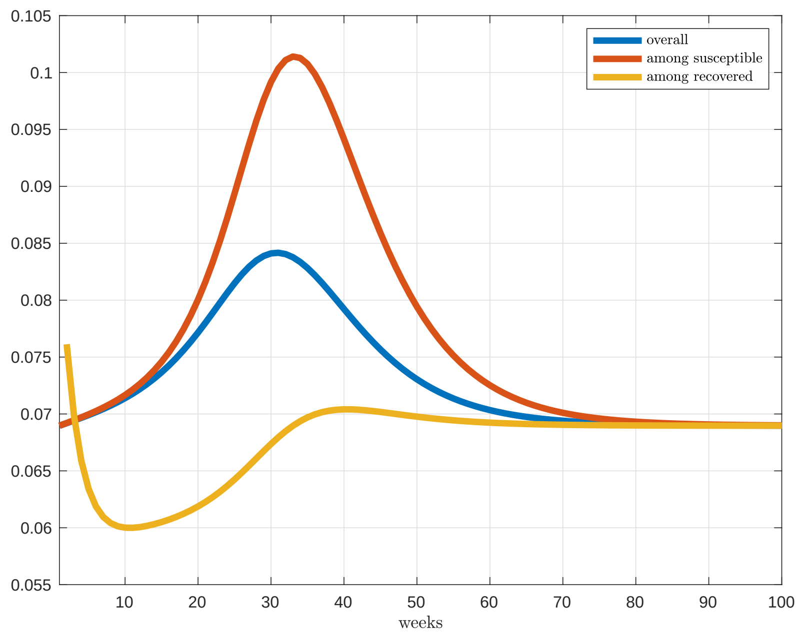


Figure 4: Unemployment rate in the susceptible and recovered stage, and overall unemployment rate.

the unemployment rate.

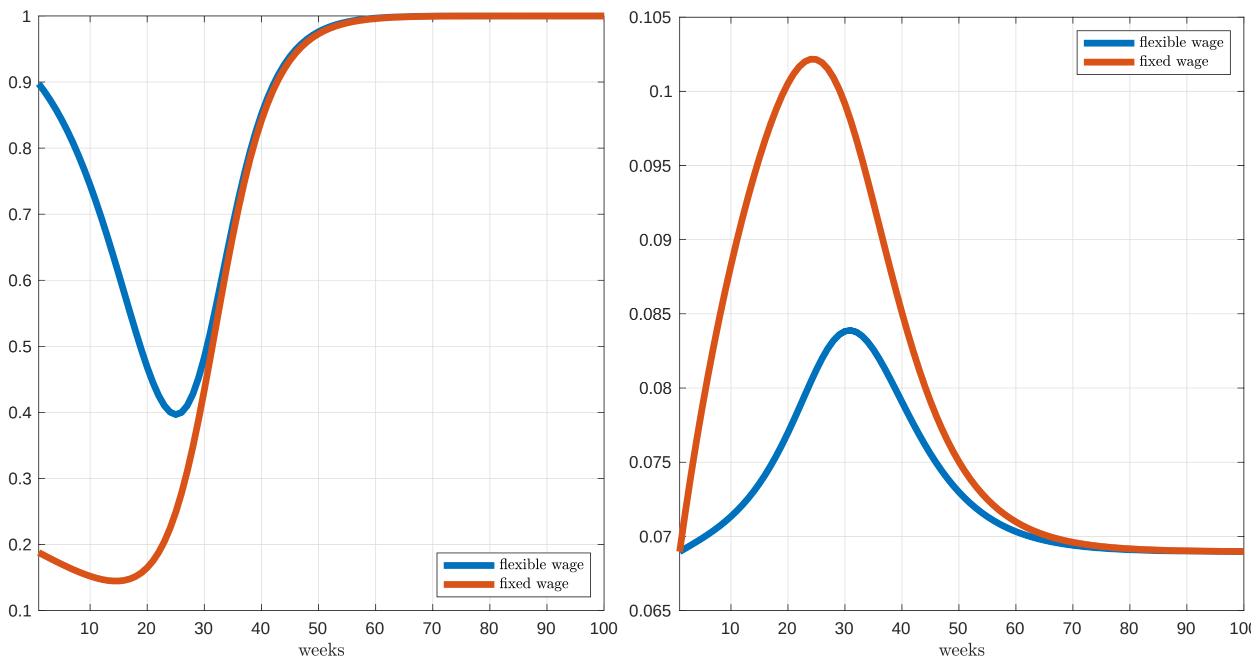
8.1 Rigid Wages

One may ask if the behavior of labor markets, and especially changes in wages, contribute or inhibit the spread of the infection. To that end, we now consider alternative labor market arrangements, where wages, instead of being renegotiated continuously every period, are rigid. They stay the same throughout the epidemics, at their initial steady state value. We denote the initial value by w_0. For brevity, we only characterize the dynamics of the firm's problem in the infected and susceptible stage, and the determination of the labor market tightness in the susceptible stage. The value function of the firm is

$$J_t^S(w_0) = y - w_0 + \beta(1-\lambda)\left[\pi_{t+1}^{EI} J^I(w_0) + (1-\pi_{t+1}^{EI}) J_{t+1}^S(w_0)\right]$$

$$J^I(w_0) = \frac{-w_0 + \beta(1-\lambda)\pi_R J^R(w_0)}{1-\beta(1-\lambda)(1-\pi_R-\pi_D)}.$$

Since the economy returns to the pre-epidemics steady state, the value of the firm in the recovered stage happens to be the same as in the model with flexible wages, and we



(a) Labor market tightness in the susceptible stage

(b) Overall unemployment rate.

Figure 5: Labor market: flexible vs rigid wages.

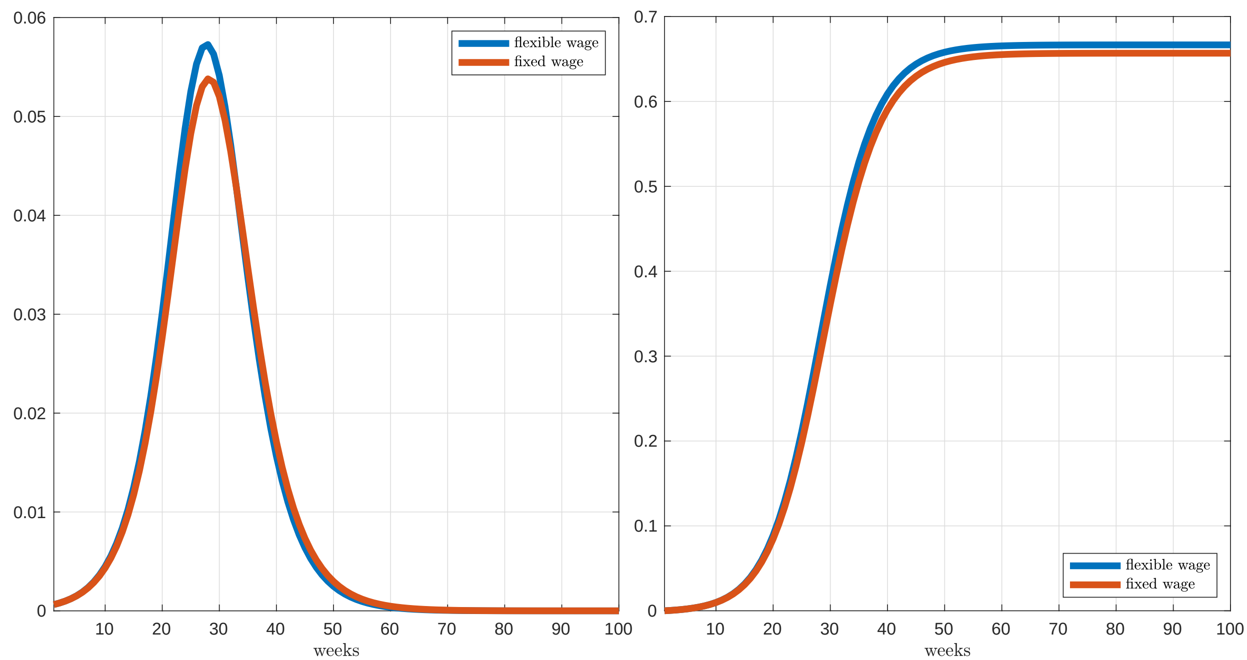
omit it. Labor market tightness is obtained so as to equate the value of a vacancy to the cost of a vacancy k,

$$J_t^S(w_0) = \frac{k}{\beta(1-\pi_{t+1}^{UI})q(\theta^S)}.$$

If the wage w_0 is higher than the flexible wage, the labor market tightness decreases and vice versa. Figure 3a has shown that flexible wages first drop below the steady state value, and then, before the epidemics peaks, rise above. The labor market tightness under rigid wages must therefore be initially lower. This is exactly what happens, as figure 5a shows. As a result, the unemployment rate increases more, and peaks at more than 10 percent, instead of 8.5 percent under flexible wages (figure 5b).[11]

Higher unemployment under rigid wages mean that the infection spreads at a lower rate. Figure 6a shows that, indeed, the probability of getting infected decreases by about half of a percent. The fraction of recovered and dead also decreases, by one percentage point, to 65.6 percent. Interestingly, although the benefits seems small, they outweigh

[11]Interestingly, labor market tightness is lower under rigid wages for the whole episode, even when the flexible wage rises. This is because there is now more unemployed people, and so the denominator of V/U is higher.



(a) Probability of getting infected

(b) Fraction of recovered and dead.

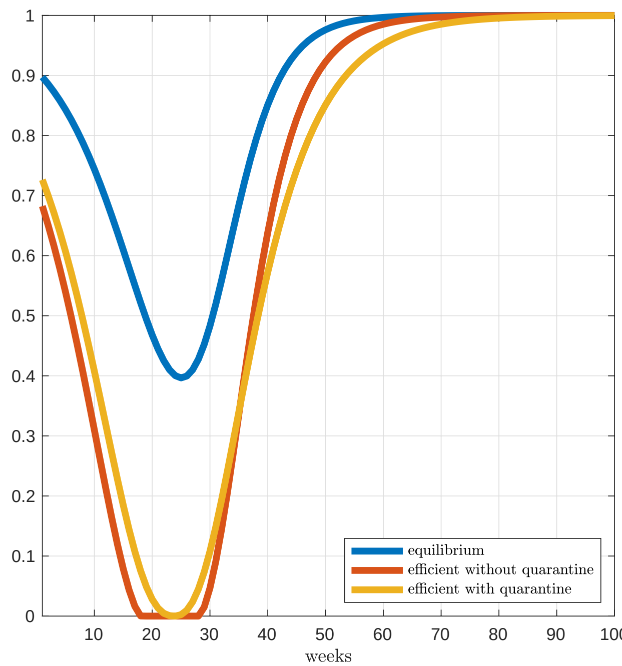
Figure 6: Pandemics: flexible vs rigid wages..

the cost of an inferior labor market allocation, and overall welfare slightly increases. This is purely due to the epidemics and the associated externality, and would not be possible in its absence.

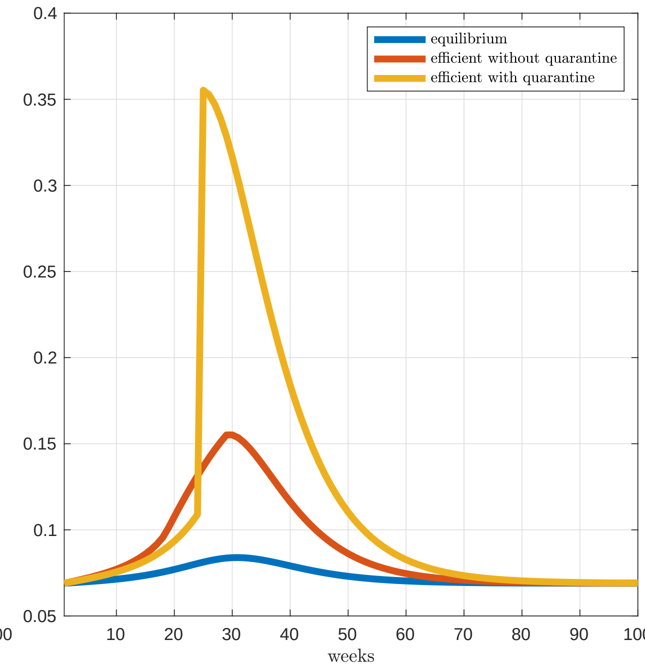
8.2 Efficiency

The red line in Figure 7a shows that, if the planner is not allowed to use quarantines, the market tightness drops to zero in week 18, when the pandemic is on the rise. This is because the social value of the match drops below zero: when the pandemic is on the rise, the negative externality from jobs is the strongest. In contrast, as evidenced from a strictly positive labor market tightness in the equilibrium allocation (as well as from Figure 2a directly), the private value of the match is still substantially positive and new jobs are still being created.

When the social value of the match becomes negative, the planner wants to destroy matches. This is exactly what a quarantine does here. The quarantine happens in week 24. The number of quarantined people is such that labor market tightness becomes exactly zero, as the yellow line in Figure 7a shows. It is worth noting that is is optimal



(a) Labor market tightness, susceptible stage.



(b) Overall unemployment rate.

Figure 7: Labor market: equilibrium vs. efficient allocation

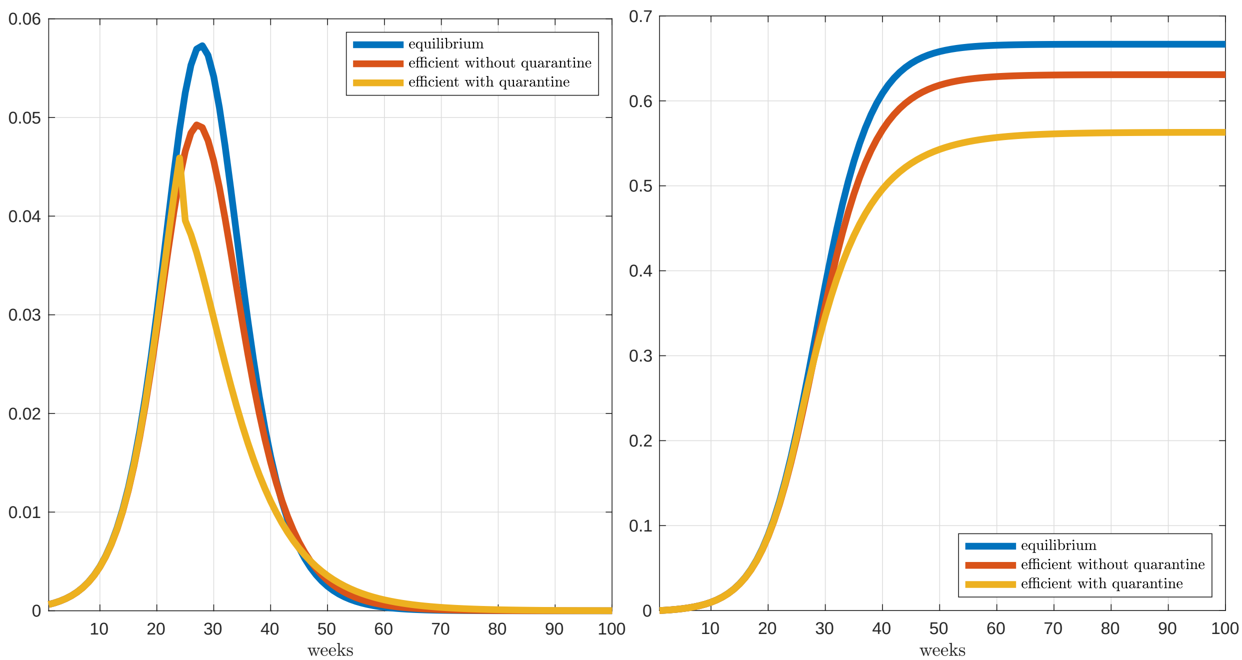
to move people from employment to unemployment only once, before the pandemic peaks, even though the planner has the option to do it repeatedly. This is because the rate at which people get back to employment is low when the pandemic peaks, and so there is no need to continue removing people from employment later. The job finding rate only accelerates after the pandemic peaks, but then the externalities are too small to warrant another quarantine. Note also that the labor market tightness is higher before the quarantine, relative to the efficient allocation without quarantine. It is less costly to create matches, because they can be destroyed during quarantine. In contrast, if quarantine is not feasible, the matches will persist and will contribute to the epidemics.

Figure 7b shows the efficient unemployment rate. In both planning problems, the overall unemployment rate is significantly above the equilibrium one. While the equilibrium unemployment rate is about 8.5 percent, in the optimum it almost doubles to more than 15 percent even without the use of a quarantine. If a quarantine is used, then it naturally rises even more and, as the results show, substantially more: 24 percent of all employees are optimally quarantined in period 18, and the unemployment rate rises to more than 35 percent. After that, the quarantined people, now mixed with the people who lost their jobs in a regular way, slowly get back to the pool of employed people. Since the job finding probability depends positively on the labor market tightness, Figure 7a shows that the job finding probability after week 35 exceeds the job finding probability in the efficient allocation without quarantine: at that point the peak of the pandemic has passed, and is is optimal to reduce the pool of unemployed fast.

What are the extra benefits gained in both efficient allocations? We look at the progression of the pandemic throughout the population in Figure 8. Figure 8a shows that the probability of getting infected is reduced by about one one percentage point in case of a quarantine, and by about half a percentage point without a quarantine. In case of a quarantine, the probability declines afterward very sharply and stays substantially lower during the peak of the pandemic. After the pandemic peaks, after week 46, the probability of getting infected is slightly higher in both efficient allocations, because some of the infections are effectively pushed forward towards later dates. If the quarantine is not used, then the fraction of the recovered and dead decreases by about 3.5 percentage points, to 63.1 percent. Since the death rate is one percent, it saves 0.035 percent of the population, which is about 117,000 lives.[12] If quarantine is used, the number of lives saved is substantially higher. The fraction of recovered and dead drops by more than 10 percentage points to 56.3 percent. This translates into savings of approximately 339,000 lives.

Output loss vs loss of lives. We summarize the loss of lives, as well as the loss of output under various scenarios, in Table 2. Letting the epidemic progress without the government intervention results in a 4.4 percent loss of output during the first year of the pandemic and, the calibrated value of two thirds of one percent of the population. The output loss under rigid wages is somewhat larger, as is the loss of lives. The efficiency dictates even larger loss of output: 7.2 percent without a quarantine, and 11.8 percent with quarantine, with proportionally larger savings of lives, as discussed before. The recession produced under quarantine is thus almost three times as large as it would be in the absence of the government intervention. The loss of output during the worst week of the pandemic is much larger: 11.1 percent under flexible wages, and 38.8 percent under quarantine.

[12] We assume that US population is 327 million



(a) Probability of getting infected. (b) Fraction of recovered and dead.

Figure 8: Pandemics: equilibrium vs. efficient allocation

9 Policy Implications

The goal of the optimal tax policy is to reduce both the value of the match, as well as the value of the unemployment, by the amount ψ_{t+1} that corresponds to the size of the externality. How is that achieved? The most direct way is a tax on those, whose behavior creates the externality. But who creates the externality in our model, the susceptible workers, or the infected individuals? Ultimately, the infected people are the source of the externality by infecting the others, but it is the behavior of the susceptible individuals that needs to be corrected in the first place. It turns out that the externality can be corrected by both means: the tax burden can be shifted from the

Implementation #1: Infection Tax. Conceptually the easiest way to correct the externality is a Pigouvian tax on the infected people, at the amount equal to the net social harm. From the first-order conditions for the planning problem, the harm is equal for both employed and unemployed, and is equal to

$$\tau_t^I = \psi_{t+1}.$$

Table 2: Loss of Output and Lives

	Loss of output annual	Loss of output peak	Loss of lives
Equilibrium, flexible wages	4.4 %	11.1 %	0.67 %
Equilibrium, rigid wages	5.6 %	12.4 %	0.66 %
Efficient, no quarantine	7.2 %	17.3 %	0.63 %
Efficient with quarantine	11.8 %	38.8 %	0.56 %

Loss of output and loss of lives in the four scenarios. Annual loss of output is the fraction of output lost during the first year. Peak loss of output is the largest weekly loss of output. One tenth of one percent loss of lives represents approximately 327 000 lives.

The infection tax is paid by the unemployed workers, and either the employed workers or employers. The model does not determine whether the employed workers or employers pay the tax; both options are possible, and if the tax burden is shifted from one party to the other, the equilibrium wage adjusts appropriately. We will for simplicity assume that it is the employed worker who pays the infection tax. Then the value functions for the workers in the infected stage change from (6) and (9) to

$$\begin{aligned} K_t^{UI} &= b - c - \tau_t^I + \beta \left[\pi_R K^{UR} + (1 - \pi_R - \pi_D) K_{t+1}^{UI} \right] \\ K_t^{EI}(w) &= w - c - \tau_t^I + \beta\lambda \left[\pi_R K^{UR} + (1 - \pi_R - \pi_D) K_{t+1}^{UI} \right] \\ &\quad + \beta(1-\lambda) \left[\pi_R K^{ER} + (1 - \pi_R - \pi_D) K_{t+1}^{EI} \right]. \end{aligned}$$

The infection tax $\{\tau_t^I\}$ *implements* the efficient allocation in a sense that the equilibrium value of the match F_t^S equals the optimal value of the match $\mu_t^{ES} - \mu_t^{US}$, and the equilibrium market tightness in the susceptible stage equals the optimal market tightness in the susceptible stage.[13]

What are the properties of the infection tax? First, the tax varies with time, but is the same for infected employed and infected unemployed. This is a consequence of the fact that the externality is the same regardless of the worker's labor market status. This means that the tax does not affect the value of the match F^I. Second, the infection tax does not converge to zero over time. As long as there is a small probability that

[13] The value of the match and market tightness in the recovered stage is always equal to their corresponding optimal values. The value of the match in the infected stage is also equal, but that, by itself, is not important, because no decisions are made in the infected stage.

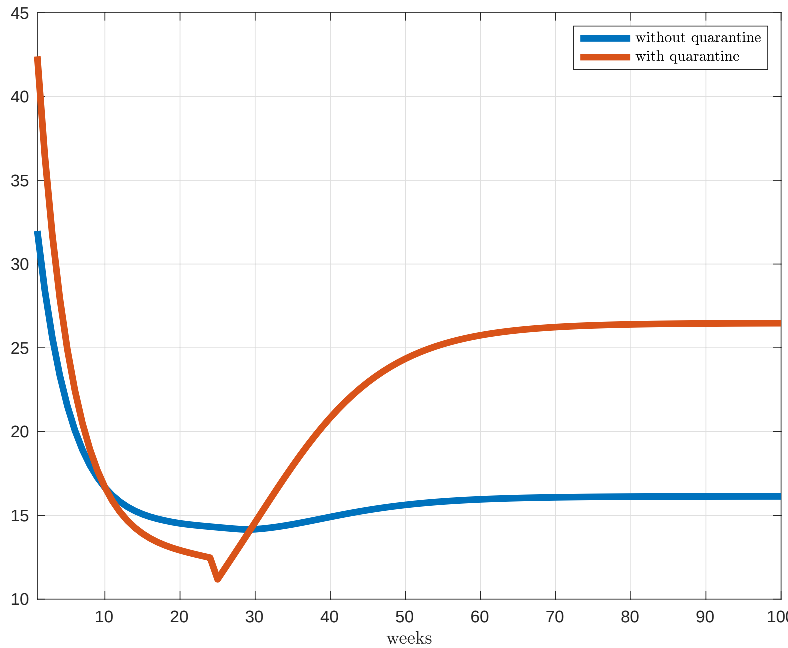


Figure 9: Infection tax, multiples of weekly weekly wages.

someone will get infected, the tax is imposed. Its long run value can be computed easily. In the steady state, the fraction of unemployed susceptible is $US = uS$, while the fraction of employed susceptible is $ES = (1-u)S$, where $u = \lambda/(\lambda + p^R)$ is the steady state unemployment rate. One can show that $\tau^I = \kappa S$, where κ is a positive constant. The infection tax is thus proportional to the fraction of the population that remains susceptible to the infection. This means that the long run infection tax is *complementary* to quarantine. Since quarantine reduces the number of infected people, the fraction of susceptibles remains higher in the steady state, and the infection tax must be higher.

Figure 9 shows the optimal infection tax, with and without quarantine. The infection tax is substantial. It is U-shaped, and the highest at the beginning of the epidemic, when it is equal to 43 weekly wages with quarantine, and to 32 weekly wages without quarantine. With weekly wages around 1000 dollars, the infection tax is around 32 and 43 thousand dollars in the two cases.

The infection tax consists of three components, corresponding to the three terms in the externality: a decrease in the job finding rate, an infection of the employed people, and infection of the unemployed people. The first component, a decrease from the job finding rate, is insignificant. By far the largest component is the second one, a correction for infecting the employed people. This is because the employed people are the most significant group. While the loss from infecting one employed person, $\mu_t^{ES} - \mu_t^{EI}$, is comparable in magnitude to the loss from infecting one unemployed person, $\mu_t^{US} - \mu_t^{UI}$, there is more employed workers in the economy than unemployed ones. The reason, why the infection tax is U-shaped is the following. The tax is the highest at the beginning because there is a lot of susceptible workers. The decrease, and subsequent increase, is driven by the difference $\mu_t^{ES} - \mu_t^{EI}$: it is the smallest when the emidemic peaks, since susceptible workers are likely to get infected soon anyway, and increases afterwards.

Implementation #2: Taxing Susceptibles. The tax on the infected may have its disadvantages, not captured in our simple model. It may be considered unfair since it is paid by those who are sick, or people may not take it into account properly when making decisions in the susceptible stage. Thus, we now investigate how the tax can be "shifted" to the susceptible stage, where all the relevant decisions are made.

We derive the equivalent optimal tax in the susceptible stage in two steps. First, consider replacing the infection tax τ_t^I, which is paid every period as long as the worker remains sick, by its expected present value, which is paid only once at the beginning of the infection. Since the workers are risk neutral, they are indifferent between both options. The expected present value of the infection tax is, for both employed and unemployed,

$$\bar{\tau}_t^I = \sum_{j=0}^{\infty} \beta^j (1 - \pi_R - \pi_D)^j \tau_{t+j}^I.$$

In step two, consider shifting the present value of the infection tax from the beginning of the infection to the susceptible stage. The expected value of $\bar{\tau}_{t+1}^I$ is

$$\tau_t^{US} = \beta \pi_{t+1}^{UI} \bar{\tau}_{t+1}^I \tag{32}$$

$$\tau_t^{ES} = \beta \pi_{t+1}^{EI} \bar{\tau}_{t+1}^I. \tag{33}$$

The present value of the tax is multiplied by the probability of getting infected, and is discounted to the current period. It now differs for the employed and unemployed workers. If the tax τ_t^{ES} is again paid by the worker and not by the firm, the value function

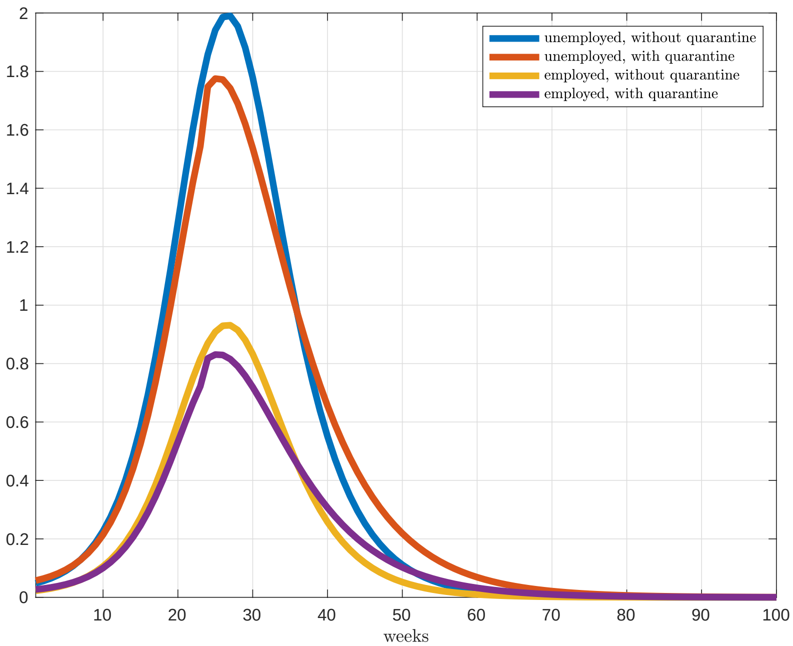


Figure 10: Tax on susceptible workers, multiples of weekly weekly wages.

in the suspected stage becomes

$$K_t^{US} = b - \tau_t^{US} + \beta \left[p_{t+1}^S (1 - \pi_{t+1}^{UI}) K_{t+1}^{ES} + \pi_{t+1}^{UI} K^{UI} + (1 - p_{t+1}^S)(1 - \pi_{t+1}^{UI}) K_{t+1}^{US} \right]$$

$$K_t^{ES} = w - \tau_t^{ES} + \beta \left\{ \lambda \left[(1 - \pi_{t+1}^{EI}) K_{t+1}^{US} + \pi_{t+1}^{EI} K^{UI} \right] + (1 - \lambda) \left[(1 - \pi_{t+1}^{EI}) K_{t+1}^{ES} + \pi_{t+1}^{EI} K^{EI} \right] \right\},$$

where K^{UI} and K^{EI} are the value functions in the infected stage in the absence of the infection tax, given by (6) and (9). One can show that if the infection tax $\{\tau_t^I\}$ implements the efficient allocation, then the tax on susceptibles $\{\tau_t^{US}, \tau_t^{ES}\}$ implements the efficient allocation as well.

The tax on susceptibles is proportional to the probability of getting infected, and so it copies the dynamic profile of the epidemic, as Figure 10 shows. The ratio of the tax on employed and unemployed equals s^E/s^U by construction, and so the unemployed always pay less. The tax is also significantly smaller than the infection tax, and peaks at around 2 weekly wages at the height of the epidemic for the employed. Since $\tau_t^{ES} > \tau_t^{US}$, the tax on susceptible workers discourages employment by increasing the utility from unemployment relative to the utility from employment. During the peak of the epidemic, the effective unemployment benefits $b - \tau_t^{US}$ exceed the return from employment $b - \tau_t^{ES}$, with or without quarantine. This reduces labor market tightness to zero, as shown in

Figure 7a.

The fact that the tax on susceptible workers reduces the return from work looks very much like a temporary increase in the unemployment benefits. However, the increase in the unemployment benefits is not sufficient to implement the optimum, and must be accompanied by a tax on employed workers as well. To see why, consider an alternative tax system, where the employed workers are not taxed and unemployed workers receive benefits $\tau_t^{ES} - \tau_t^{US} > 0$. This preserves the difference between the return from production and return from unemployment $y - \tau_t^{ES} - b + \tau_t^{US}$. It does, however, increase the loss from getting infected Δ_t^S, increases the the reservation and equilibrium wage, and reduces labor market tightness too much. A tax reform based purely on an increase in the unemployment benefits is thus not efficient. the result holds regardless of whether a quarantine is implemented or not.

The tax on employment of susceptibles τ_t^{ES} can be again paid either by the employer or the employee, without affecting the equilibrium allocations or after tax wages. If the tax is paid by the employer, the equilibrium wage will decrease by τ_t^{ES} to offset the transfer of the tax liability to the employer.

Implementation #3: Tax on Vacancy Creation. The tax τ_t^{ES} is paid by while the susceptible match lasts. We now consider another equivalent representation of the optimal tax, by shifting the tax burden forward to the vacancy creation stage, to be paid by any firm that posts a vacancy. Consider again. a two-stage transformation. Let $\bar{\tau}_t^{ES}$ be the expected present value of the tax at time t.[14] The firm is indifferent between paying the expected present value of the tax τ_t^{VS} once the match has been formed, and paying the tax on susceptible match τ_t^{ES} as long as the match lasts and the worker is susceptible. In step two, the tax τ_t^{VS} is shifted to the time of a vacancy creation, as an additional cost. At the time the vacancy is created, the expected value of the tax on a susceptible match, denoted by τ_t^{VS}, is

$$\tau_t^{VS} = \beta q_t^S (1 - \pi_t^{UI}) \bar{\tau}_t^{ES}.$$

The equilibrium condition for the vacancy creation is now

$$(1 - \phi) F_t^S = \frac{k + \tau_t^{VS}}{\beta (1 - \pi_t^{UI}) q(\theta_t^S)}.$$

[14]The expected present value does not have a closed form solution. Its computation is relatively straightforward, however, and is not shown.

One can again show that if the tax on susceptibles $\{\tau_t^{US}, \tau_t^{ES}\}$ implements the efficient allocation, then the tax on susceptible workers combined with the tax on vacancy creation $\{\tau_t^{US}, \tau_t^{VS}\}$ implements the efficient allocation as well.

Implications for policies. There may be multiple policies that achieve the efficient allocation. The goal of all of them is to align the private and social value of a match. One of the possible policies is a tax on vacancy creation. Suppose that θ_t^{*S} is the efficient labor market tightness in the susceptible stage, and F_t^{*S} and pi^{*UI} is the corresponding social value of the match and infection probability. Then a tax on vacancy creation $\{\tau_t\}$ must satisfy

$$(1-\phi)F_t^{S^*} = \frac{k+\tau_t}{\beta(1-\pi_{t+1}^{UI*})q(\theta_t^{S^*})}.$$

Since the value of matches is always lower in the efficient allocations (as implied, for example, by figure 7a), the tax on vacancy creation is always positive, and the largest throughout the peal of the epidemics. The policy recommendation contrasts here with Guerrieri et al. (2020), who advocate policies that reduce the number of business shutdowns and exits. In their case, the externality from keeping businesses afloat is positive, because it increases aggregate demand. In our model, the externality is negative, because it increases the spread of the pandemic.

10 Conclusions

We use a standard search and matching theory to study the impact that a COVID-19 pandemic has on the behavior of labor markets, and on the optimal policies during the pandemic. We find that the labor market separates the recovered and susceptible individuals. The epidemics has two opposing effects on wage formation of the people who are susceptible to the infection: the reservation wage increases because labor market activities make one more likely to get infected, but the value of the match decreases because job tenure is shorter and potentially less productive. We find that the second effect dominates before the pandemic peaks, while the first one dominates during and after the peak, and so the wage first decreases and then substantially increases. The unemployment rate increases substantially among the susceptible, decreases among the recovered, and increases overall by about 3 percentage points.

Since the equilibrium is not efficient because of an infection externality, we also study the efficient allocations. We consider two scenarios, one where the government is not allowed to destroy matches, and one, where the government can quarantine people and destroy matches. In both cases, the government wants to tax vacancies creation, especially around the peak of the epidemics, and slow down the spread of the epidemics. We find that the, unlike its private value, the social value of the match becomes negative before the epidemics peaks, and the government optimally quarantines about a quarter of the employed individuals. This has substantial output cost, almost 40 percent of the output during the weeks when the epidemics peaks, but saves about 339 000 lives.

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Appendix A: The Efficient Allocations

We assume that the matching function takes the form assumed in the calibration. The problem is to choose the flows $\{US_t, ES_t, UI_t, EI_t, UR_t, ER_t, D_t\}$ and $\{VS_t, VR_t\}$ maximize

$$\sum_{t=0}^{\infty} \beta^t \left[(ES_t + ER_t)y + (US_t + UI_t + UR_t)b - (UI_t + EI_t)c - (VS_t + VR_t)k\right]$$

subject to the constraints

$$\begin{aligned}
US_{t+1} &= [1 - s^U(EI_t + UI_t)]\,[US_t - m(US_t, VS_t)] + \lambda[1 - s^E(EI_t + UI_t)]ES_t + Q_t \\
ES_{t+1} &= (1-\lambda)ES_t - (1-\lambda)s^E(EI_t + UI_t)ES_t + [1 - s^U(EI_t + UI_t)]m(US_t, VS_t) - Q_t \\
UI_{t+1} &= (1 - \pi_R - \pi_D)UI_t + s^U(EI_t + UI_t)US_t + \lambda s^E(EI_t + UI_t)ES_t + \lambda(1 - \pi_R - \pi_D)EI_t \\
EI_{t+1} &= (1-\lambda)(1 - \pi_R - \pi_D)EI_t + (1-\lambda)s^E(EI_t + UI_t)ES_t \\
UR_{t+1} &= UR_t - m(UR_t, VR_t) + \pi_R UI_t + \lambda \pi_R EI_t + \lambda ER_t \\
ER_{t+1} &= (1-\lambda)ER_t + (1-\lambda)\pi_R EI_t + m(UR_t, VR_t),
\end{aligned}$$

and a nonnegativity constraint $Q_t \geq 0$. Let $\mu_t\beta^{ER}_{t+1}$, $\mu_t\beta^{UR}_{t+1}$, $\mu_t\beta^{ES}_{t+1}$, $\mu_t\beta^{EI}_{t+1}$, $\mu_t\beta^{US}_{t+1}$, $\mu_t\beta^{UI}_{t+1}$ be the Lagrange multipliers on the corresponding constraints. We proceed recursively, first solving the recovered stage and then the jointly the susceptible and infected stage.

Recovered stage. The first-order conditions for the recovered stage are

$$
\begin{aligned}
k &= \beta\left(\mu^{ER}_t - \mu^{UR}_t\right) m_V(1, \theta^R_t) \\
\mu^{UR}_t &= b + \beta\left[\mu^{ER}_{t+1} m_U(1/\theta^R_{t+1}, 1) + \mu^{UR}_{t+1}\left[1 - m_U(1/\theta^R_{t+1}, 1)\right]\right] \\
\mu^{ER}_t &= y + \beta\left[\mu^{ER}_{t+1}(1-\lambda) + \mu^{UR}_{t+1}\lambda\right]
\end{aligned}
$$

The recovered stage is again independent of time and the values of θ^R and the multipliers μ^{ER} and μ^{UR} are independent of time.

Infected stage. The first-order conditions in the employment and unemployment flows for the infected stage are:

$$
\begin{aligned}
\mu^{UI}_t = b - c + \beta\Big[&(1-\pi_R-\pi_D)\mu^{UI}_{t+1} + \pi_R\mu^{UR} \\
&- (\mu^{ES}_{t+1} - \mu^{US}_{t+1}) s^U m(1, \theta^S_{t+1}) US_{t+1} - (\mu^{ES}_{t+1} - \mu^{EI}_{t+1})(1-\lambda) s^E ES_{t+1} \\
&- (\mu^{US}_{t+1} - \mu^{UI}_{t+1})(s^U US_{t+1} + \lambda s^E ES_{t+1})\Big] \qquad (34)
\end{aligned}
$$

$$
\begin{aligned}
\mu^{EI}_t = -c + \beta\Big[&\lambda\left((1-\pi_R-\pi_D)\mu^{UI}_{t+1} + \pi_R\mu^{UR}_{t+1}\right) + (1-\lambda)\left((1-\pi_R-\pi_D)\mu^{EI}_{t+1} + \pi_R\mu^{ER}_{t+1}\right) \\
&- (\mu^{ES}_{t+1} - \mu^{US}_{t+1}) s^U m(1, \theta^S_{t+1}) US_{t+1} - (\mu^{ES}_{t+1} - \mu^{EI}_{t+1})(1-\lambda) s^E ES_{t+1} \\
&- (\mu^{US}_{t+1} - \mu^{UI}_{t+1})(s^U US_{t+1} + \lambda s^E ES_{t+1})\Big] \qquad (35)
\end{aligned}
$$

The second and third lines of the conditions represent new terms that reflect the externality of the problem. Unlike the equilibrium value functions in the infected stage, the values μ^{UI}_t and μ^{EI}_t depend on time, because the size of the externality is time varying. However, it is easy to show, that its difference, $\mu^{EI}_t - \mu^{UI}_t$, is independent of time. This is so because the probability of getting infected depends only on the total number of infected people and so the externality from one additional employed infected person and one additional unemployed infected person is the same. Hence, the difference $\mu^{EI}_t - \mu^{UI}_t$ is independent of the externality, and time independent.

Susceptible stage. The optimal allocations in the susceptible stage solve

$$\mu_t^{ES} = y + \beta \left[(1-\lambda) \left((1-\pi_{t+1}^{EI})\mu_{t+1}^{ES} + \pi_{t+1}^{EI}\mu_{t+1}^{EI} \right) + \lambda \left((1-\pi_{t+1}^{EI})\mu_{t+1}^{US} + \pi_{t+1}^{EI}\mu_{t+1}^{UI} \right) \right]$$
$$\mu_t^{US} = b + \beta \Big[(1-\pi_{t+1}^{UI}) m_U(1/\theta_{t+1}^S, 1)\mu_{t+1}^{ES} + (1-\pi_{t+1}^{UI}) \left(1 - m_U(1/\theta_{t+1}^S, 1) \right) \mu_{t+1}^{US} + \pi_t^{UI}\mu_{t+1}^{UI} \Big],$$

where we have used $\pi_t^{EI} = s^E I_t$, $\pi_t^{UI} = s^U I_t$. The first-order conditions are analogous to the corresponding equilibrium conditions. When the matching function is given by (31), the expected difference is that the bargaining weight ϕ is now replaced by the elasticity of the matching function α.

The first-order condition in the susceptible vacancies V_t^S is

$$k = \beta \left(\mu_t^{ES} - \mu_t^{US} \right) (1 - s^U I_t) m_V(1, \theta_t^S).$$

In the planning problem without quarantine, Q_t is set equal to zero. The first-order conditions are solved recursively, together with the laws of motion (1) and (2). In the planning problem with quarantine, Q_t is chosen by the planner, but must be nonnegative. This yields the complementarity-slackness condition in Q_t:

$$\mu_t^{ES} - \mu_t^{US} \geq 0, \quad Q_t \geq 0, \quad (\mu_t^{ES} - \mu_t^{US})Q_t = 0.$$

Efficiency of equilibrium. There is a close correspondence between the Lagrange multipliers in the planning problem, and the value functions of the workers, and of the firm. The Lagrange multipliers μ_t^{ES}, μ_t^{EI} and μ_t^{ER} represent the social values of employment in the three stages, and correspond to the total value of the match for the firm and worker together, $J_t^{ES} + K_t^{ES}$, $J_t^{EI} + K_t^{EI}$ and $J_t^{ER} + K_t^{ER}$. The Lagrange multipliers μ_t^{US}, μ_t^{UI} and μ_t^{UR} represent the social value of unemployment and have their counterparts in the private value of unemployment K_t^{US}, K_t^{UI} and K_t^{UR}.

The optimal value of θ^R in the recovered stage satisfies

$$k = \beta \frac{y - b}{1 - \beta \left[1 - \lambda - m_U(1/\theta^R, 1) \right]} m_V(1, \theta^R).$$

This condition is identical to the equilibrium condition for θ^R if the matching function takes the form in (31) and $\phi = \alpha$, which is the Hosios condition.

Appendix B: Proofs

Lemma 1. *Suppose that $\{\tau_t^I\}$ implements the efficient allocation. Then $\{\tau_t^{US}, \tau_t^{ES}\}$ implements the efficient allocation as well.*

Proof. It is enough to show that the value of the match in the susceptible stage is the same as under the infection tax. Let K_t^{UI} be the loss from getting infected under the infection tax, and $\tilde{K}_t^{UI}$ be the loss in the absence of the infection tax. Then by construction $K_t^{UI} = \tilde{K}^{UI} - \bar{\tau}_t^{UI}$. Similarly, let F_t^S and $\tilde{F}_t^S$ be the value of the susceptible match under the infection tax, and in the absence of the infection tax but with the tax on susceptibles as defined above. Finally, Δ_t^S and $\tilde{\Delta}_t^S$ are defined analogously.

Suppose that $F_{t+1}^S = \tilde{F}_{t+1}^S$, and $\Delta_{t+1}^S = \tilde{\Delta}_{t+1}^S - \bar{\tau}_{t+1}^{UI}$. It is easy to verify that the property holds in the steady state. The value of the susceptible match under the infection tax is

$$
\begin{aligned}
F_t^S &= y - b + \beta(1-\lambda)\left[\pi_{t+1}^{EI} F^I + (1-\pi_{t+1}^{EI}) F_{t+1}^S\right] - \beta\phi p_{t+1}^S (1-\pi_{t+1}^{UI}) F_{t+1}^S + \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\Delta_{t+1}^S \\
&= y - b + \beta(1-\lambda)\left[\pi_{t+1}^{EI} F^I + (1-\pi_{t+1}^{EI}) \tilde{F}_{t+1}^S\right] - \beta\phi p_{t+1}^S (1-\pi_{t+1}^{UI}) \tilde{F}_{t+1}^S \\
&\quad + \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)(\tilde{\Delta}_{t+1}^S - \bar{\tau}_{t+1}^I) \\
&= y + \beta(\pi_{t+1}^{UI} - \pi_{t+1}^{EI})\tau_{t+1}^I - b + \beta(1-\lambda)\left[\pi_{t+1}^{EI} F^I + (1-\pi_{t+1}^{EI}) \tilde{F}_{t+1}^S\right] - \beta\phi p_{t+1}^S (1-\pi_{t+1}^{UI}) \tilde{F}_{t+1}^S \\
&\quad + \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\tilde{\Delta}_{t+1}^S \\
&= y + \tau_t^{US} - \tau_t^{ES} - b + \beta(1-\lambda)\left[\pi_{t+1}^{EI} F^I + (1-\pi_{t+1}^{EI}) \tilde{F}_{t+1}^S\right] - \beta\phi p_{t+1}^S (1-\pi_{t+1}^{UI}) \tilde{F}_{t+1}^S \\
&\quad + \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\tilde{\Delta}_{t+1}^S \\
&= \tilde{F}_t^S.
\end{aligned}
$$

Hence the value of the match is preserved in period t as well. The loss from the infection in period t satisfies

$$
\begin{aligned}
\Delta_t^S &= K_t^{UI} - \beta K_{t+1}^{UI} - b + \tau_t^{US} - \beta(1-\pi_{t+1}^{UI})\left(\phi p_{t+1}^S F_{t+1}^S - \Delta_{t+1}^S\right) \\
&= \tilde{K}_t^{UI} - \bar{\tau}_t^{UI} - \beta(\tilde{K}_{t+1}^{UI} - \bar{\tau}_{t+1}^{UI}) - b + \tau_t^{US} - \beta(1-\pi_{t+1}^{UI})\left(\phi p_{t+1}^S \tilde{F}_{t+1}^S - \tilde{\Delta}_{t+1}^S + \bar{\tau}_{t+1}^{UI})\right) \\
&= \tilde{K}_t^{UI} - \beta\tilde{K}_{t+1}^{UI} - b - \beta(1-\pi_{t+1}^{UI})\left(\phi p_{t+1}^S \tilde{F}_{t+1}^S - \tilde{\Delta}_{t+1}^S)\right) - \bar{\tau}_t^{UI} + \beta\pi_{t+1}^{UI}\bar{\tau}_{t+1}^{UI} + \tau_t^{US} \\
&= \tilde{\Delta}_t^S - \bar{\tau}_t^{UI},
\end{aligned}
$$

and so the assumed property holds in period t as well. □

Appendix C: Equilibrium with Taxes

Here we generalize the equilibrium conditions to allow for two types of taxes: the infection tax τ^{UI} and τ^{EI}, paid every period during the infection but possibly different for employed and unemployed, and τ_t^{US} and τ_t^{ES}, paid by the susceptible workers, again possibly different for employed and unemployed. There are no taxes in the recovered stage, and no taxes on vacancy creation. The recovered stage is the same as before, and has a value of the match F^R.

Infected stage. The value of the match is

$$F_t^I = \tau_t^{UI} - \tau_t^{EI} - b + \beta(1-\lambda)\left[\pi_R F^R + (1-\pi_R-\pi_D)F_{t+1}^I\right]$$

The equilibrium wage is

$$w_t^I = (1-\phi)\left(b - \tau_t^{UI} + \tau_t^{EI}\right),$$

and the value functions are

$$K_t^{UI} = b - \tau_t^{UI} - c + \beta\left[\pi_R K^{UR} + (1-\pi_R-\pi_D)K_{t+1}^{UI}\right]$$
$$K_t^{EI} = w_t^I - \tau_t^{EI} - c + \beta\lambda\left[\pi_R K^{UR} + (1-\pi_R-\pi_D)K_{t+1}^{UI}\right] + \beta(1-\lambda)\left[\pi_R K^{ER} + (1-\pi_R-\pi_D)K_{t+1}^{EI}\right].$$

Finally, for the firm,

$$J_t^I = -w_t^I + \beta(1-\lambda)\left(\pi_R J^R + (1-\pi_R-\pi_D)J_{t+1}^I\right).$$

Susceptible Stage. The value of the match and of $\Delta_t^S = K_t^{UI} - K_t^{US}$ is

$$F_t^S = y + \tau_t^{US} - \tau_t^{ES} - b + \beta(1-\lambda)\left(\pi_{t+1}^{EI}F_{t+1}^I + (1-\pi_{t+1}^{EI})F_{t+1}^S\right) - \beta\phi p_{t+1}^S(1-\pi_{t+1}^{UI})F_{t+1}^S$$
$$+ \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\Delta_{t+1}^S$$
$$\Delta_t^S = K_t^{UI} - \beta K_{t+1}^{UI} - b + \tau_t^{US} - \beta(1-\pi_{t+1}^{UI})\left(\phi p_{t+1}^S F_{t+1}^S - \Delta_{t+1}^S\right)$$

The wage in the susceptible stage is a solution to

$$w_t^S = y + (1-\phi)\left[\beta(1-\lambda)\left(\pi_{t+1}^{EI}F_{t+1}^I + (1-\pi_{t+1}^{EI})F_{t+1}^S\right) - F_t^S\right].$$

The value functions are

$$K_t^{US} = b - \tau_t^{US} + \beta\left[p_{t+1}^S(1-\pi_{t+1}^{UI})K_{t+1}^{ES} + \pi^{UI}K_{t+1}^{UI} + (1-p_{t+1}^S)(1-\pi_{t+1}^{UI})K_{t+1}^{US}\right]$$
$$K_t^{ES} = w_t^S - \tau_t^{ES} + \beta\lambda\left[(1-\pi_{t+1}^{EI})K_{t+1}^{US} + \pi_{t+1}^{EI}K_{t+1}^{UI}\right] + \beta(1-\lambda)\left[(1-\pi_{t+1}^{EI})K_{t+1}^{ES} + \pi_{t+1}^{EI}K_{t+1}^{EI}\right]$$

for the workers, and

$$J_t^S = y - w_t^S + \beta(1-\lambda)\left(\pi_{t+1}^{EI}J_{t+1}^I + (1-\pi_{t+1}^{EI})J_{t+1}^S\right)$$

for the firm.

10.1 Equivalence classes of tax systems

We consider several tax systems that differ in the absence of various tax components. Denote the value functions in the infected stage as a function of taxes, by $F_t^I(\{\tau_t^{UI}, \tau_t^{EI}\})$, $K_t^{UI}(\{\tau_t^{UI}, \tau_t^{EI}\})$, $K_t^{EI}(\{\tau_t^{UI}, \tau_t^{EI}\})$ and $J_t^I(\{\tau_t^{UI}, \tau_t^{EI}\})$. Then

$$\begin{aligned}
F_t^I\left(\{\tau_t^{UI}, \tau_t^{EI}\}\right) &= F_t^I(0,0) + \eta_t^I \\
K_t^{UI}\left(\{\tau_t^{UI}, \tau_t^{EI}\}\right) &= K_t^{UI}(0,0) - \eta_t^{UI} \\
K_t^{EI}\left(\{\tau_t^{UI}, \tau_t^{EI}\}\right) &= K_t^{EI}(0,0) + \phi\eta_t^I - \eta_t^{UI} \\
J_t^I\left(\{\tau_t^{UI}, \tau_t^{EI}\}\right) &= J_t^I(0,0) + (1-\phi)\eta_t^I,
\end{aligned}$$

where

$$\eta_t^{UI} = \textstyle\sum_{j=0}^{\infty} \beta^j (1-\pi_R-\pi_D)^j \tau_{t+j}^{UI} \tag{36}$$
$$\eta_t^{I} = \textstyle\sum_{j=0}^{\infty} \beta^j (1-\lambda)^j (1-\pi_R-\pi_D)^j (\tau_{t+j}^{UI} - \tau_{t+j}^{EI}) \tag{37}$$

compute present value of the tax liabilities in the unemployed state and in the match overall.

We now consider a tax reform that is *equivalent* to the tax system with only infection taxes $\{\tau_t^{UI}, \tau_t^{EI}\}$ in a sense that it produces, for given market tightness and infection probabilities $\{\theta_t^S, \pi_t^{UI}, \pi_t^{EI}\}$, the same value of the match $\{F_t^S\}$, but shifts some of the tax burden, possibly all, to the susceptible stage. The tax reform may change the value of the utility loss from getting infected by $\Delta_t^S(\{\tau_t^{UI}, \tau_t^{ES}\}) - \Delta_t^S(0,0) = \zeta_t$, or the value of the match in the infected stage. The value of ζ must satisfy

$$\zeta_t = -\tau_t^{US} - \eta_t^{UI} + \beta\eta_{t+1}^{UI} + \beta(1 - \pi_{t+1}^{UI})\zeta_{t+1}. \tag{38}$$

In addition, if the value of the match $\{F_t^S\}$ does not change in any period, the value function for the match implies

$$0 = \tau_t^{US} - \tau_t^{ES} + \beta(1-\lambda)\pi_{t+1}^{EI}\eta_{t+1}^{I} + \beta(\pi_{t+1}^{EI} - \pi_{t+1}^{UI})\zeta_{t+1}. \tag{39}$$

Any tax reform $\{\tau_t^{US}, \tau_t^{ES}, \tau_t^{UI}, \tau_t^{EI}\}$ that satisfies (38) and (39) with η_t^I and η_t^{UI} given by (36) and (36) yields the same equilibrium outcome.

One possible solution is to start with the optimal tax (which requires $\tau_t^{UI} = \tau_t^{EI}$ and so has $\eta_t^I = 0$), and move all the tax burden to the infected stage by setting $\tau_t^{US} = \beta\pi_{t+1}^{UI}\eta_{t+1}^{UI}$ and $\tau_t^{ES} = \beta\pi_{t+1}^{EI}\eta_{t+1}^{EI}$. This yields a solution $\zeta_t = -\eta_t^{UI}$.

Consider an alternative tax reform that sets the tax on the employed to be zero in both stages ($\tau_t^{ES} = \tau_t^{EI} = 0$). Then the tax on employed in

$$0 = \tau_t^{US} + \beta(1-\lambda)\pi_{t+1}^{EI}\eta_{t+1}^{I} + \beta(\pi_{t+1}^{EI} - \pi_{t+1}^{UI})\zeta_{t+1} \tag{40}$$

$$\zeta_t = -\tau_t^{US} - \eta_t^{UI} + \beta\eta_{t+1}^{UI} + \beta(1 - \pi_{t+1}^{UI})\zeta_{t+1} \tag{41}$$

The optimal infection tax has $\tau_t^{UI} = \tau_t^{EI} = \overline{\tau}_t^I$ and so has $\eta_t^I = 0$. The optimal value of η_t^{UI} is denoted by $\overline{\eta}_t^{UI}$.

$$\begin{aligned}
F_t^S &= y + \tau_t^{US} - \tau_t^{ES} - b + \beta(1-\lambda)\left(\pi_{t+1}^{EI}F_{t+1}^I\left(\{\tau_{t+1}^{UI}, \tau_{t+1}^{EI}\}\right) + (1-\pi_{t+1}^{EI})F_{t+1}^S\right) - \beta\phi p_{t+1}^S(1-\pi_{t+1}^{UI})F_{t+1}^S \\
&\quad + \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\Delta_{t+1}^S\left(\{\tau_{t+1}^{US}, \tau_{t+1}^{ES}, \tau_{t+1}^{UI}, \tau_{t+1}^{EI}\}\right) \\
&= y - b + \beta(1-\lambda)\left(\pi_{t+1}^{EI}F_{t+1}^I\left(\{\overline{\tau}_{t+1}^I, \overline{\tau}_{t+1}^I\}\right) + (1-\pi_{t+1}^{EI})F_{t+1}^S\right) - \beta\phi p_{t+1}^S(1-\pi_{t+1}^{UI})F_{t+1}^S \\
&\quad + \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\Delta_{t+1}^S\left(\{0, 0, \overline{\tau}_{t+1}^I, \overline{\tau}_{t+1}^I\}\right)
\end{aligned}$$

which can be rewritten as

$$\begin{aligned} F_t^S &= y + \tau_t^{US} - \tau_t^{ES} - b + \beta(1-\lambda)\left(\pi_{t+1}^{EI}(F_{t+1}^I(0,0) + \eta_{t+1}^I) + (1-\pi_{t+1}^{EI})F_{t+1}^S\right) - \beta\phi p_{t+1}^S(1-\pi_{t+1}^{UI})F_{t+1}^S \\ &+ \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\Delta_{t+1}^S\left(\{\tau_{t+1}^{US}, \tau_{t+1}^{ES}, \tau_{t+1}^{UI}, \tau_{t+1}^{EI}\}\right) \\ &= y - b + \beta(1-\lambda)\left(\pi_{t+1}^{EI}F_{t+1}^I(0,0) + (1-\pi_{t+1}^{EI})F_{t+1}^S\right) - \beta\phi p_{t+1}^S(1-\pi_{t+1}^{UI})F_{t+1}^S \\ &+ \beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\Delta_{t+1}^S\left(\{0, 0, \overline{\tau}_{t+1}^I, \overline{\tau}_{t+1}^I\}\right) \end{aligned}$$

Cancelling terms

$$\tau_t^{US} - \tau_t^{ES} = \beta\left[\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\zeta_{t+1} - (1-\lambda)\eta_{t+1}^I\right], \tag{42}$$

where we have defined $\zeta_t = \Delta_t^S\left(\{0, 0, \overline{\tau}_t^I, \overline{\tau}_t^I\}\right) - \Delta_t^S\left(\{\tau_t^{US}, \tau_t^{ES}, \tau_t^{UI}, \tau_t^{EI}\}\right)$ is the difference in the utility loss. It satisfies

$$\zeta_t = -\tau_t^{US} + \eta_t^{UI} - \overline{\eta}_t^{UI} + \beta\left(\overline{\eta}_{t+1}^{UI} - \eta_{t+1}^{UI}\right) + \beta(1-\pi_{t+1}^{UI})\zeta_{t+1}. \tag{43}$$

A tax reform $\tau = \{\tau_t^{US}, \tau_t^{ES}, \eta_t^I, \eta_t^{UI}\}$ implements the optimum if it satisfies (42) and (43). Clearly, the tax system that taxes optimally in the infected stage $\overline{\tau} = \{0, 0, 0, \overline{\eta}_t^{UI}\}$. Another tax system is $\tilde{\tau} = \{\tilde{\tau}_t^{US}, \tilde{\tau}_t^{ES}, 0, 0\}$, where $\tilde{\tau}_t^{US} = \beta\pi_{t+1}^{UI}\overline{\eta}_{t+1}^{UI}$ and $\tilde{\tau}_t^{ES} = \beta\pi_{t+1}^{EI}\overline{\eta}_{t+1}^{EI}$. This tax system shifts all the tax burden to the susceptible stage. It is easy to verify that it satisfies (42) and (43), with $\zeta_t = -\overline{\eta}_t^{UI}$.

Another tax system does not tax the employed and taxes the unemployed in the susceptible stage optimally: $\hat{\tau} = \{\hat{\tau}_t^{US}, 0, \hat{\eta}_t^I, \overline{\eta}_t^{UI}\}$, where $\hat{\eta}_t^I = \sum_{j=0}^{\infty}\beta^j(1-\lambda)^j(1-\pi_R - \pi_D)^j\tau_{t+j}^{UI}$. Then

$$\zeta_t = -\sum_{j=0}^{\infty}\beta^j\prod_{k=1}^{j}(1-\pi_{t+k}^{UI})\tau_{t+j}^{US} \tag{44}$$

and

$$\tau_t^{US} = -\beta\left(\pi_{t+1}^{EI} - \pi_{t+1}^{UI}\right)\sum_{j=0}^{\infty}\beta^j\prod_{k=1}^{j}(1-\pi_{t+1+k}^{UI})\tau_{t+1+j}^{US} - \beta(1-\lambda)\hat{\eta}_{t+1}^I \tag{45}$$