Market Structure and Monetary Non-neutrality*

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Abstract

Canonical macroeconomic models of pricing under nominal rigidities assume markets consist of atomistic firms. Most US retail markets are dominated by a few large firms. To bridge this gap, I extend an equilibrium menu cost model to allow for a continuum of sectors with two large firms in each sector. Compared to a model with monopolistically competitive markets, and calibrated to the same good-level data on price dynamics, the dynamic duopoly model generates output responses to monetary shocks that are more than twice as large. Under duopoly, the response of low priced firms to an increase in aggregate marginal cost: a falling markup at its competitor reduces its value of a price increase and price conditional on adjustment. The dynamic duopoly model also implies (i) large first order welfare losses from nominal rigidities, (ii) smaller estimated menu costs, (iii) a U-shaped relationship between market concentration and price flexibility, for which I find empirical evidence.

Keywords: Oligopoly, menu costs, monetary policy, firm dynamics.

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1 Introduction

A standard assumption made for tractability in macroeconomic models is that firms behave competitively in the markets in which they sell their goods. This paper relaxes this assumption in a monetary business cycle model with nominal rigidity. I explore an oligopolistic market structure—familiar from other areas of economics—in which firms are large and markets are imperfectly competitive. Aggregating oligopolistic sectors, I find a range of new results relative to the monopolistically competitive benchmark. In particular, the real effects of shocks are substantially larger under oligopoly.

Figure 1 provides a simple motivation for this paper: product markets are highly concentrated. This is not a new fact. But Figure 1 documents this for a broad range of narrowly defined markets. Defining a market by a product category (e.g. ketchup, mayonnaise) in a specific state and month, I construct measures of concentration using weekly revenue data from the IRI data.1 The median number of firms in a market is 37, while the effective number of firms—a measure of market concentration defined as the inverse Herfindahl index—has a median of around three, and the median revenue share of the two largest firms is just under 70 percent.2 The number of firms in markets may be large, but firms are not equally sized and most sales accrue to a few. This paper investigates the macroeconomic implications of oligopolistic markets which seem appropriate given these facts, and have been studied previously in a microeconomic context.

In this paper, I extend a menu cost model of price adjustment to accommodate a dynamic duopoly within each sector. Firms face persistent, idiosyncratic shocks, must pay a cost to change the price of good they produce, and compete according to the rules of Markov Perfect Equilibrium. I aggregate a continuum of such sectors and subject the economy to aggregate shocks to the money supply.

I compare this model to one with a monopolistically competitive market structure: each sector is populated with a continuum of price-taking firms. Importantly, I calibrate both models to

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1The IRI data is weekly good level data for the universe of goods in a panel of over 5,000 supermarkets in the US from 2001 to 2011. For more details on the IRI data see Bronnenberg, Kruger, and Mela (2008), for its treatment in this paper see Appendix A.

2The inverse Herfindahl index (IHI) admits an interpretation of ‘effective number of firms’ as follows. The IHI of a sector with $n$ equally sized firms is $n$. Therefore if a sector has an IHI of 2.4, then it has a Herfindahl index consistent with a market populated by between 2 and 3 equally sized firms. For more on this interpretation see Adelman (1969). For a recent paper that uses this measure of market concentration see Edmond, Midrigan, and Xu (2015).
account for the same size and frequency of adjustment found in the IRI good-level data, as well as the same average markup. Since prices change frequently and by large amounts on average, matching these facts strongly curtails the real effects of monetary shocks in a monopolistically competitive model. Since idiosyncratic shocks are large and aggregate shocks are small, this can be thought of as delivering two models that are observationally equivalent in terms of good level price flexibility with respect to good level shocks, then comparing aggregate price flexibility with respect to aggregate shocks.

The calibrated oligopoly model is quantitatively distinct from the calibrated competitive in four key dimensions. First, the real effects of monetary shocks—measured as the standard deviation of output in an economy with only money shocks—are more than twice as large in the duopoly model. Second, the welfare costs associated with nominal rigidities are first order, and five times larger under duopoly. This difference is not due to price dispersion: the focus of policy prescriptions in competitive sticky price models. Third, the model avoids issues that arise in models of monopolistic competition that use strategic complementarity to reduce monetary

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3 See papers following Golosov and Lucas (2007), which I discuss in Section 2.
4 The optimal rule for monetary policy in a standard New-Keynesian model is derived from a second order approximation of the household’s utility function around a flexible price zero-inflation steady-state (see Gali, 2008, chap. 4). It depends on inflation in so far as inflation causes sub-optimally large (small) amounts of labor to be used in the production of goods that have sub-optimally low (high) prices. That is, there is a second order welfare cost of inflation that emerges entirely due to price dispersion.
neutrality: lower menu costs and similar sizes of idiosyncratic shocks are required to deliver the empirical frequency and size of price change. Fourth, if comparing market structures at the same parameters, prices are around twice as sticky under oligopoly. Exploiting large within-product, across-market variation in market concentration and price flexibility, I find conditional correlations in the data support these predictions.

These results are due to the interaction of three features of the model. First, when firms are imperfectly competitive and households have a low ability to substitute across different sectors, complementarities in price setting arise: if a competitor’s price is high, a firm’s static best response is also a high price, understanding that increasing the sector’s average price leads to little substitution towards other sectors. In the absence of nominal rigidities, however, the frictionless Bertrand-Nash equilibrium obtains since equilibrium strategies feature undercutting. Although pass-through from idiosyncratic shocks will be incomplete, pass-through from aggregate shocks will be complete. The second feature, menu costs, reduce the value of undercutting a competitor’s price. The third feature, a dynamic model, implies that firms can post high prices understanding their competitor will—in equilibrium—follow them in future periods, with the cost of adjustment lending credibility to this strategy.

These features yield equilibrium strategies of firms that are codependent. As discussed first by Maskin and Tirole (1988b) (alternating pricing), and Jun and Vives (2004) (convex costs), the equilibrium features dynamic complementarities in pricing. Static complementarities that are otherwise mute in the realized equilibrium of the model—due to a lack of commitment—are, under frictions, reflected in the Markov Perfect equilibrium. How do these dynamic complementarities lead to the main results in the paper?

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5Specifically, the frequency of price change is 30 percent less, and the average size of price change is twice as large.

6Due to the endogeneity of market structure, building on these conditional correlations towards causal statements is beyond the scope of this paper.

7Throughout I leave out the ‘strategic’ when discussing complementarities in pricing. This is to reserve the term strategic for describing the interaction of firms: strategic as opposed to competitive. This is a broader point of confusion: in many models of strategic complementarity what we may have in mind is an approximation to strategic behavior whereas behavior is actually competitive.

8Since this paper focuses on frictions in price—rather than quantity—setting, then Bertrand—rather than Cournot—competition is natural.
1. Real effects of monetary shocks. The key contribution of the paper is to aggregate an economy of oligopolistic sectors into an equilibrium macroeconomic model and understand how these dynamic complementarities can be important for macroeconomic dynamics. Following an increase in the money supply, equilibrium aggregate nominal cost increases. Under monopolistic competition this leads more firms with already low prices increase their price. Firms increasing their price also increase them by more to accommodate the persistent increase in nominal costs. As articulated in Golosov and Lucas (2007), when the frequency and average size of price change are large—as they are in the data—these, respectively, extensive and intensive margin effects lead to a quick increase in the aggregate price index.\textsuperscript{9}

Dynamic complementarities dampen the response of low priced firms on both margins. Many of these firms face direct competitors with high prices that welcome the increase in nominal costs, reducing their probability and size of adjustment. With its nominal price becoming more rigid and aggregate price level increasing, the competitor’s relative price falls. These changes at the inframarginal firm effect the behavior of the marginal firm. The value of a price increase falls since for any price increase its competitor now has a lower relative price. The size of a price increase falls since the firm’s prices are dynamic complements. The aggregate shock has exogenously made the marginal firm’s competitor more competitive within the sector, dissuading the marginal firm from a price increase which would make it less competitive within the sector.\textsuperscript{10} In this way the price response of firms that most undo the real effects of monetary shocks is dampened in the duopoly model.

To account for these results I decompose the response of the aggregate price level in a similar manner to Caballero and Engel (2007).\textsuperscript{11} The decomposition reveals that—in the aggregate, and in comparison to the monopolistically competitive model—the extensive and intensive margins of adjustment are dampened equally. However this hides important variation between sectors. In sectors with two initially low prices, money is in fact closer to neutral. The lowest priced

\textsuperscript{9}Figure XX in Appendix E replicates the standard figure for understanding these responses.

\textsuperscript{10}An additional mechanism is as follows. In the sectoral equilibrium, one motive for a low priced firm to increase its price is to dissuade a high priced competitor from reducing theirs. In equilibrium such price increases balance the short-run loss of market share for the long-run gain of a higher sectoral price. With its competitor moving towards its optimal relative price, this incentive is weaker, also dampening the incentive of a price increase. Quantitatively these two mechanisms are hard to disentangle.

\textsuperscript{11}This decomposition has provided an accounting tool for this class of models and has been used by Midrigan (2011), Alvarez and Lippi (2014) and others. For a diagramatic representation see Figure E1.
firm increasing its price pushes its competitor to do so also, increasing the extensive margin response. Since both are adjusting, the firms take the opportunity to increase prices by more than the increase in nominal cost, increasing the intensive margin response. These types of sectors in which dynamic complementarity in pricing increases the price response contribute substantially towards smaller output responses in the duopoly model. Quantitatively, however, they are offset by sectors of the type previously described, in which prices are initially dispersed. The decomposition makes clear the value of an equilibrium exercise in which sectors are aggregated; extrapolating from a particular sector might bias forecasts of the aggregate response.

2. Welfare losses of nominal rigidity

In the presence of menu costs, firms attain substantially higher markups than the frictionless equilibrium. Menu costs wipe out the small gains associated with deviations in the direction of the firm’s best response: undercutting its competitor. Quantitatively, the average markup is 10ppt higher than in the economy with menu costs set to zero.\(^\text{12}\)

Markups are the relevant measure of ‘real prices’ in the economy, and so this leads to output that is approximately 10ppt lower. The model therefore has implications not only for the dynamics of output, but also its level. And—although not a subject studied here—invites thinking about how policies designed for cyclical stability may affect average output and vica versa.\(^\text{13} \quad \text{14}\)

An extension of this result is that—within a certain range—firms may prefer to face larger nominal rigidities. Under zero menu costs firms would realize the Bertrand-Nash equilibrium markup. As menu costs increase, the equilibrium can accommodate higher prices: average markups increase, increasing firm value. As menu costs increase further, the firm is less re-

\(^{12}\)To put this in context: in a frictionless collusive equilibrium, markups would be 200ppt higher than the frictionless non-cooperative equilibrium. The presence of menu costs allow non-cooperative firms to attain higher markups but this is limited by the size of the menu costs. If the menu cost is small, then downward deviations are again profitable and the equilibrium can not sustain high markups. The size of the menu cost is disciplined by the average frequency and size of adjustment.

\(^{13}\)As an example, increasing trend inflation, would weaken dynamic complementarity in pricing. In the limit, all firms in the economy would change their price every period. If all firms change their price every period, then the only equilibrium is the frictionless Nash-Bertrand equilibrium and there is no wedge between the frictionless and average markup. On the other hand, this would lead to no real effects of a monetary shock since prices are perfectly flexible.

\(^{14}\)Under both market structures nominal rigidity imply output losses from price dispersion. These are equal in both models because, roughly speaking, both have the same average size of price change. In both models this leads to 2ppt output losses from nominal rigidity. Output losses due to the level of the markup are five times larger than those from dispersion.
sponsive to shocks: prices spend more time far from their optimum, decreasing firm value. I show that in the empirically relevant range of menu costs the first effect dominates and the value of the firm is increasing in menu costs. The model therefore provides a novel rationale for investment in advertising and other such activities that increase the cost of price adjustment.

3. Pricing complementarities and menu cost models It has long been understood that some source of ‘real rigidity’ is needed to generate substantial real effects of monetary shocks (Ball and Romer, 1990; Woodford, 2003). Quantitatively, since the frequency of price change is large, then some other force must stagger the adjustment in prices. Recent papers have tested earlier ideas for generating complementarity, extending the Golosov and Lucas (2007) framework, while still constraining the models by the large size and frequency of price change in good-level data. Klenow and Willis (2016) add non-CES preferences, leading to variable markups through prices. Burstein and Hellwig (2007) add a decreasing returns to scale in (DRS) production technology, leading to variable markups through costs. Both lead to complementarity between the firm price and aggregate price. Their findings—summarized by elegantly by Nakamura and Steinsson (2010)—are that such complementarities can not be a source of propagation. If the parameter controlling the strength of these complementarities is large, then unreasonably large idiosyncratic shocks and menu costs are required to match the good-level data on price adjustment. A key result of this paper is to describe a situation where this is not the case: idiosyncratic shocks and menu costs are smaller in the duopoly model, yet amplification is still achieved through pricing complementarities. As I explain, this is due to the complementarities existing between two firms’ prices, rather than a firm’s price and the aggregate price.

4. Empirical support Lower menu costs are generate the same level of observed price rigidity in the duopoly model. Firms with low prices are reluctant to increase their price due to short-run market share incentives. Firms with high prices are reluctant to decrease their prices due to a long-run incentive to have its competitor also choose a price. This has empirical content.

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15For a quantitative comparison of different treatments of pricing complementarity in New-Keynesian models see Woodford (2003, chap. 4).

16A literature in international economics has employed the same Kimball (1995) demand specification to study pass-through of exchange rate, or foreign, shocks to domestic prices. See: Gopinath and Itskhoki (2011), Berger and Vavra (2013). Note that the model in this paper may be simply extended to study this question.
The model predicts that when comparing markets with the same costs of adjustment, strategic behavior leads to less flexible prices. This paper considers cases of one, two and infinitely many firms, where firms are strategic in the limiting cases and non-strategic in between. And predicts a \( U \)-shape (inverted \( U \)-shape) relationship between market concentration and the frequency (average size) of price adjustment.

It is beyond this paper’s scope to document a causal relationship between market concentration and price flexibility. Market structure is itself endogenous and I do not aim to address this here. However I document that, across markets that plausibly have the similar primitives, there is evidence of this correlation. In particular, I define a market by a product category-MSA-month, and use variation within product-months, across MSAs, controlling for market size. The structure of this correlation is consistent with the causal implications of the model: there is \( U \)-shape (inverted \( U \)-shape) relationship between market concentration and frequency (average size) of price adjustment.

**Structure**  The remainder of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the model. Section 4 develops intuition for the main results by studying simulations of individual sectors. Section 5 describes the calibration strategy that anchors the comparison of market structures. Section 6 describes the main result, how this is robust to different calibration strategies, and how non-neutrality under duopoly differs to existing models. Section 7 describes two additional results: welfare costs of nominal rigidity and endogenous price stickiness. Section 8 tests the model’s predictions for the cross-sectional empirical relationship between market concentration and price flexibility. Section 9 concludes.

### 2 Literature

This paper is situated in two distinct literatures: (i) the strand of monetary economics following Golosov and Lucas (2007) that has studied the ability of menu cost models of price adjustment to explain monetary non-neutrality, (ii) a literature on dynamic games of price setting. Additionally the paper contributes new empirical facts to a recent literature that studies heterogeneity in price flexibility.
Equilibrium models of monopolistic competition and menu costs of price adjustment

Golosov and Lucas (2007) show that in an equilibrium menu cost model of price adjustment that matches the large size and frequency of price change in good-level data, shocks to nominal demand lead to small output effects. A number of papers show that extensions of the Golosov and Lucas (2007) model can generate larger real effects while matching the same price data. Midrigan (2011) and Alvarez and Lippi (2014) show that once the model accounts for small price changes it can generate output responses similar to a Calvo model of price adjustment calibrated to the same moments.17 Nakamura and Steinsson (2010) show that in the Golosov and Lucas (2007) model, the degree of monetary non-neutrality is convex in the degree of price flexibility. Noting that different sectors have different degrees of price flexibility they calibrate a multi-sector model and through this Jensen’s inequality effect generate large real effects of monetary shocks. Furthermore, like Klenow and Willis (2016) and Burstein and Hellwig (2007) the authors conclude that macro strategic complementarities that come through slow responses of input prices are the most likely candidate for monetary non-neutrality relative to other sources of real rigidities.18 The source of strategic complementarity I study in this paper is different and uniquely derives from the interaction of (i) non-atomistic market structure, (ii) menu costs. I return to a more precise comparison with these papers following my results in Section 6.

Partial equilibrium dynamic models of oligopoly with nominal rigidities

The industrial organization literature has understood that nominal rigidities in price setting induce an intertemporal complementarity in price setting when markets are oligopolistic. Maskin and Tirole (1988b) first make this point. In a highly stylized price setting environment where firms have an exogenous short-run commitment to prices, Markov Perfect equilibrium policies of firms may accommodate higher prices than in the frictionless static Nash equilibrium. Jun and Vives (2004) confirm this result in a differential game with convex costs of price adjustment: prices are strategic complements and adjustment costs allow firms to commit to

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17 Both Midrigan (2011) and Alvarez and Lippi (2014) achieve this through multiproduct firms with economies of scope in price change. Midrigan (2011) shows that the precise way that one accounts for small price changes is inconsequential. He shows that a single-product firm facing random menu costs can match the distribution of price changes and also delivers large output responses.

high price strategies should their competitor follow them, which they do in equilibrium. In
an empirical partial equilibrium setting Kano (2013) makes a similar point in an environment
where firms face fixed costs of adjustment. However the fact that dynamic oligopoly generates
additional nominal rigidity is insufficient for this paper, in which I aim to compare duopoly
and monopolistically competitive market structures. An accurate comparison demands that
they account for the same observed level of nominal rigidity in prices.

Nakamura and Zerom (2010) and Neiman (2011) study partial equilibrium models of
oligopolistic market structures at the sectoral level in which firms face menu costs of price
adjustment. Nakamura and Zerom (2010) study the behavior of three firms subject to only sec-
toral shocks to the costs of inputs. To align with the monetary economics literature I assume
that firms face shocks to both idiosyncratic and aggregate demand and consider the model
in general equilibrium. Neiman (2011) considers a model of duopoly with only idiosyncratic
shocks, but does not bring the model to data on size and frequency of price adjustment and
does not compare the implications of the model against a monopolistically competitive bench-
mark. Moreover, neither paper focuses on the inter-temporal strategic complementarities and
their effects on average markups.

Cross-sectional facts regarding price stickiness

A recent literature has documented cross-sectional heterogeneity in price flexibility. I contribute
to this literature in two ways. First I show that even within a sector—in my case grocery items—
price flexibility varies substantially (i) across product categories within regions, (ii) across re-
gions within product categories. Second I show that this variation is systematic. Price flexibility
is related to market concentration in a $U$-shaped way. Markets with very few or very many
firms have high price flexibility, and those with a small number of firms have lower price flex-
ibility. I show that my model is consistent with this new correlation, predicting higher (lower)
price flexibility when firms are atomistic (large).

Existing models that incorporate cross-sectional heterogeneity in price flexibility assume
this heterogeneity comes through differences in the magnitude of the nominal rigidity. In their
multi-sector menu cost model, Nakamura and Steinsson (2010) account for the unconditional
dispersion in price flexibility with differences in menu costs. Studying New-Keynesian models
Weber (2016) and Gorodnichenko and Weber (2016) follow a similar approach, allowing the exogenous probability of price adjustment to vary across sectors. In an international menu cost model Berger and Vavra (2013) show that heterogeneity in the curvature of demand rather than menu costs can jointly explain the positive cross-sectional relationship between price flexibility and pass-through. For the same levels of menu cost I find that a menu cost model generates endogenously less flexible prices under duopoly.

3 Model

The model studied in this paper can be framed in three ways. A dynamic menu cost model of price adjustment following Golosov and Lucas (2007), extended to accommodate oligopoly at the sectoral level. A static model of imperfect competition with finitely many firms following Atkeson and Burstein (2008), with dynamic state variables induced by shocks and pricing frictions. Or as a model of dynamic oligopoly with nominal rigidity as in Nakamura and Zerom (2010), extended to accommodate persistent idiosyncratic firm level shocks and situated in an equilibrium monetary business cycle model.

3.1 Environment and timing

Time is discrete. The economy is populated with two types of agents: households and firms. Households are identical, consume goods, supply labor, and hold shares in a portfolio of all firms in the economy. Firms are organized in a continuum of sectors which produce differentiated goods and are indexed $j \in [0, 1]$. Each sector contains two firms indexed $i \in \{1, 2\}$, which produce further differentiated. Households have nested CES preferences for goods: across- and then within-sectors. Goods are produced by a technology with constant returns in labor. Each period each firm draws a menu cost $\xi_{ijt} \sim H(\bar{\xi})$ which is the private information of firm $i$, and may change their price $p_{ijt}$ conditional on paying $\bar{\xi}_{ijt}$. Aggregate uncertainty arises due to shocks to the growth rate of the money supply $M_t$, and idiosyncratic uncertainty arises due to shocks to the household’s preferences for each good $z_{ijt}$. Both follow persistent first order stochastic processes, with innovations that are public information and realized at the same time as draws of $\xi_{ijt}$. 
Regarding notation; I write agents’ problems recursively, such that the time subscript $t$ is redundant. The aggregate state is denoted $S \in \mathcal{S}$ and the vector of sectoral state variables $s \in S$. The distribution of firms over sectoral states is given by the measure $\lambda(s)$. Integrals over the continuum of sectors are expressed as integrals of $s$ over $\lambda(s)$ rather than integrating $j$ over $[0, 1]$.

3.2 Household

Problem  Given prices for all goods in all sectors $p_i(s, S)$, wage $W(S)$, price of shares in the stock-market $\Omega(S)$, aggregate dividends $\Pi(S)$, and law of motion for the aggregate state $S' \sim \Gamma(S'|S)$, households’ policies for consumption demand for both goods in each sector $c_i(s, S)$, labor supply $N(S)$ and share demand $X(S)$, solve

$$W(S, X) = \max_{c_i(s, N, X')} \log C - N + \beta \mathbb{E}[W(S', X')]$$

where $$C = \left[ \int c(s) \frac{\theta - 1}{\theta} d\lambda(s) \right]^{\frac{\theta}{\theta - 1}},$$

$$c(s) = \left[ \left( z_1(s)c_1(s) \right)^{\frac{\eta - 1}{\eta}} + \left( z_2(s)c_2(s) \right)^{\frac{\eta - 1}{\eta}} \right]^{\frac{\eta}{\eta - 1}},$$

subject to the nominal budget constraint

$$\int_0^1 \left[ p_1(s, S)c_1(s) + p_2(s, S)c_2(s) \right] d\lambda(s) + \Omega(S)X' \leq W(S)N + \left( \Omega(S) + \Pi(S) \right) X.$$

Households discount the future at rate $\beta$, have time separable utility and derive period utility from consumption adjusted for the disutility of work, which is linear in labor. Utility from consumption is logarithmic in a CES aggregator of consumption utility from the continuum of sectors. The cross-sector elasticity of demand is denoted $\theta > 1$. Utility from sector $j$ goods is given by a CES utility function over the two firms’ goods with within-sector elasticity of demand $\eta > 1$. These elasticities are ranked $\eta > \theta$ indicating that the household is more willing to substitute goods within a sector (Pepsi vs. Coke) than across sectors (Soda vs. Laundry de-

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19A parameter controlling the utility cost of labor (i.e. $U(C, N) = \log C - \psi N$) can be normalized to one, so is not included.
tergent). Finally, household preference for each good is subject to a shifter $z_i(s)$ which evolves according to a random-walk

$$\log z_i'(s) = \log z_i(s) + \sigma z_i \epsilon_i, \quad \epsilon_i \sim \mathcal{N}(0,1), \quad (1)$$

where innovations $\epsilon_i$ are independent over firms, sectors, and time.

**Solution** The solution to the household problem consists of demand functions for each firm’s output $d_i(s, S)$, a labor supply condition $N(S)$, and an equilibrium share price $\Omega(S)$ which will be used to price firm payoffs. Demand functions are given by

$$d_i(s, S) = z_i(s)^{\eta-1} \left( \frac{p_1(s, S)}{p(S)} \right)^{-\eta} \left( \frac{p(s, S)}{p(S)} \right)^{-\theta} C(S), \quad (2)$$

where

$$p(s, S) = \left[ \left( \frac{p_1(s, S)}{z_1(s)} \right)^{1-\eta} + \left( \frac{p_2(s, S)}{z_2(s)} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

$$P(S) = \left[ \int p(s, S)^{1-\theta} d\lambda(s) \right]^{\frac{1}{1-\theta}}.$$

Aggregate real consumption is $C(S)$. The allocation of $C(S)$ to sector $s$ depends on the level of the sector $s$ price index $p(s, S)$ relative to the aggregate price index $P(S)$. The allocation of expenditure to firm $i$ is then determined by $z_i(s)$, and the level of firm $i$’s price relative to the sectoral price $p(s, S)$.

The aggregate price index satisfies

$$P(S)C(S) = \int \left[ p_1(s, S)c_1(s, S) + p_2(s, S)c_2(s, S) \right] d\lambda(s),$$

such that $P(S)C(S)$ is equal to aggregate nominal consumption. I assume that aggregate nominal consumption must be paid for using money $M(S)$ such that $M(S) = P(S)C(S)$ in equilibrium.\(^\text{20}\) The supply of money is exogenous and evolves according to a growth rate $g' = M'/M$ which follows a first order autoregressive process in logs

$$\log g' = (1 - \rho_g) \log \bar{g} + \rho_g \log g + \sigma_g \epsilon', \quad \epsilon' \sim \mathcal{N}(0,1). \quad (3)$$

The nominal economy will be trend stationary around the steady-state level of money growth $\bar{g} > 0$. An intratemporal condition determines labor supply and intertemporal Euler equation

\(^\text{20}\) An alternative approach would be to assume that money holdings enter the utility function as in Golosov and Lucas (2007). As noted in that paper, as long as utility is separable, the disutility of labor is linear and the utility of money is logarithmic one obtains the same equilibrium conditions regarding the wage, nominal consumption and money supply as studied here.
prices shares, which under log utility give

\[
W(S) = P(S)C(S),
\]

\[
\Omega(S) = \beta E \left[ \frac{P(S)C(S)}{P(S')C(S')} \left( \Omega(S') + \Pi(S') \right) \right] S.
\]

The equilibrium discount factor applied to firm payoffs \(Q(S, S')\) can therefore be written \(Q(S, S') = \beta \frac{W(S)}{W(S')}\). The household discounts states where wage growth is high, since consumption is relatively high in these states.

### 3.3 Firms

**Timing and information** Consider the problem for firm \(i\), and denote its direct competitor \(-i\). At the beginning of the period, the sectoral state vector \(s\) contains both firms’ (i) previous prices \(p_i, p_{-i}\) (iii) and current preference shocks \(z_i, z_{-i}\). Firm \(i\) then draws a menu cost \(\xi_i \sim H(\xi)\) which is private information. Then, at the same time as its competitor, firm \(i\) chooses whether to adjust its price and conditional on adjustment a new price level \(p_i'\). Prices are then made public and each firm produces and sells the quantity demanded by household. At the end of the period, preference shocks evolve stochastically to \(z_i'\) and \(z_{-i}'\) according to (1). The new sectoral state is \(s' = (p_i', p_{-i}', z_i', z_{-i}')\).

In making these decisions firm \(i\) understands the equilibrium policies of its direct competitor. These are given by an indicator for price adjustment \(\phi_{-i}(s, S, \xi_{-i}) \in \{0, 1\}\) and price conditional on adjustment \(p_{-i}'(s, S)\) Note that since the menu cost is sunk, the price conditional on adjustment is independent of the menu cost. This description of the environment explicitly restricts firm policies to depend only on only pay-off relevant information \((s, S)\), that is they are *Markov strategies*.\(^{21}\) A richer dependency of policies on the history of firm behavior is beyond the scope of this paper.\(^{22}\)

**Problem** Let \(V_i(s, S, \xi)\) denote the present discounted expected value of nominal profits of firm \(i\) after the realization of the sectoral and aggregate states \((s, S)\) and its menu cost \(\xi\). Then

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\(^{21}\)In the words of Maskin and Tirole (1988a), “Markov strategies...depend on as little as possible, while still being consistent with rationality”.

\(^{22}\)ADD: References to Rotemberg and Woodford, Rotemberg and Saloner. Infinite histories, collusion, trigger strategies.
$V_i(s,S,\xi)$ satisfies the following recursion

$$V_i(s,S,\xi) = \max \left\{ V_i^{adj}(s,S) - W(S)\xi, \ V_i^{stay}(s,S) \right\},$$

$$V_i^{adj}(s,S) = \max_{p_i^*} \int \phi_i(s,S,\xi_{-i}) \left\{ \pi_i(p_i^*,p_{-i}(s,S),s,S) + \mathbb{E} \left[ Q(S,S')V_i(s',S',\xi') \right] \right\}$$

$$+ \left(1 - \phi_{-i}(s,S,\xi_{-i})\right) \left\{ \pi_i(p_i^*,p_{-i},s,S) + \mathbb{E} \left[ Q(S,S')V_i(s',S',\xi') \right] \right\} \right\} dH(\xi_{-i}),$$

$$\pi_i(p_i,p_{-i},s,S) = d_i(p_i^*,p_{-i},s,S) \left( p_i - z_i(s)W(S) \right),$$

$$s' = \phi_{-i}(s,S,\xi_{-i}) \times (p_i^*,p_{-i}(s,S),z_i',z_{-i}') + \left(1 - \phi_{-i}(s,S,\xi_{-i})\right) \times (p_i^*,p_{-i},z_i',z_{-i}')$$

$$S' \sim \Gamma(S'|S).$$

The first line states the extensive margin problem, where adjustment requires a payment of menu cost $\xi$ in units of labor. The value of adjustment is independent of the menu cost and requires choosing a new price $p_i^*$. When making these decisions the firm must integrate out the unobserved states of its competitor—the menu cost $\xi_{-i}$—and take account of how its competitor’s pricing decision affects current payoffs $\pi_i$ and the evolution of $s'$. The term in braces on the second (third) line gives the flow nominal profits plus continuation value of the firm if its competitor does (does not) adjust its price $p_{-i}$. The value of non-adjustment $V_i^{stay}(s,S)$ is identical to adjustment with the restriction $p_i^* = p_i$.

The flow payoff introduces a role for $z_i(s)$ in costs. As in Midrigan (2011) I assume that $z_i(s)$—which increases demand for the good with an elasticity of $(\eta - 1)$—also increases total costs with a unit elasticity. This technical assumption will allow me to reduce the state-space of the firm’s problem while still ensuring that the primary structural interpretation of the shock is as a demand shock.

The firm discounts future nominal dividends using the household’s nominal discount factor $Q(S,S')$, and expectations are taken with respect to the equilibrium transition density $\Gamma(S'|S)$ and firm level shocks. Nominal profit $\pi_i$ depends on aggregate consumption $C(S)$ and aggregate price-index $P(S)$ which the firm understands as functions of aggregate state variables.

Before proceeding, note that the fact that the competitor’s intensive margin pricing decision $p_{-i}^*$ is also independent of the menu cost means that we can integrate out $\xi_{-i}$ when comput-
ing firm $i$’s payoff. We can replace $\phi_{-i}$ with the probability that firm $-i$ changes its price, $\gamma_{-i}(s, S) = \int \phi_{-i}(s, S, \xi_{-i}) dH(\xi_{-i})$. Since $\xi_i$ is iid we can also integrate it out of firm $i$’s Bellman equation:

$$V_i(s, S) = \int \max \left\{ V_{i}^{\text{adj}}(s, S) - W(S)\xi, V_{i}^{\text{stay}}(s, S) \right\} dH(\xi),$$ (7)

$$V_{i}^{\text{adj}}(s, S) = \max_{p_i'} \gamma_{-i}(s, S) \left\{ \pi_i(p_i', p_{-i}(s, S), s, S) + \mathbb{E} \left[ Q(S, S') V_i(s', S') \right] \right\} + \left(1 - \gamma_{-i}(s, S) \right) \left\{ \pi_i(p_i', p_{-i}, s, S) + \mathbb{E} \left[ Q(S, S') V_i(s', S') \right] \right\}.$$ (8)

The solution to this problem is firm $i$’s price conditional on adjustment $p_{i}^*(s, S)$ and probability of price adjustment $\gamma_i(s, S)$ which is given by

$$\gamma_i(s, S) = \mathbb{P} \left[ \xi_i W(S) \leq V_{i}^{\text{adj}}(s, S) - V_{i}^{\text{stay}}(s, S) \right] = H \left( \frac{1}{W(S)} \left[ V_{i}^{\text{adj}}(s, S) - V_{i}^{\text{stay}}(s, S) \right] \right).$$ (8)

### 3.4 Equilibrium

Given the above, the aggregate state vector $S$ must contain the level of nominal demand $M$, its growth rate $g$, and the distribution of sectors over sectoral state variables $\lambda$. A recursive equilibrium consists of

(i) Household demand functions $d_i(s, S)$

(ii) Firm value functions $V_i(s, S)$ and policies $p_{i}^*(s, S)$, $\gamma_i(s, S)$

(iii) Wage, labor supply, price index, consumption and discount factor functions

$$W(S), N(S), P(S), C(S), Q(S, S')$$

(iv) Law of motion $\Gamma(S, S')$ for the aggregate state $S = (z, g, M, \lambda)$ such that

(a) Household demand functions are consistent with household optimization condition (2)

(b) The functions in (iii) are consistent with household optimality conditions (4)

(c) Given household demand functions, competitor policies, wage, price, consumption, discount factor functions, and $\Gamma$: $p_{i}^*$, $\gamma_i$ are consistent with firm $i$ optimization and value function $V_i$ (7)
(d) The price function is equal to the household price index under firm policies

\[
p(s, S) = \left[ \left( \frac{p_1^*(s, S)}{z_1(s)} \right)^{1-\eta} + \left( \frac{p_2^*(s, S)}{z_2(s)} \right)^{1-\eta} \right]^{1-\eta},
\]

\[
P(S) = \left[ \int \mathbb{E}_{\gamma_1(s, S), \gamma_2(s, S)} \left[ p(s, S)^{1-\theta} \right] d\lambda (s) \right]^{1-\theta}
\]

(e) The household holds all shares \( X(S) = 1 \) and the price of shares is consistent with (4)

(e) The stochastic law of motion for \( g \) and path for \( M \) are determined by (3), and nominal demand satisfies \( P(S)C(S) = M(S) \)

(f) The law of motion for \( \lambda \) is consistent with firm policies. Let \( X = P_1 \times P_2 \times Z \times Z \in \mathbb{R}_+^4 \), and the corresponding set of Borel sigma algebras on \( X \) be given by \( \mathcal{X} = \mathcal{P}_1 \times \mathcal{P}_2 \times \mathcal{Z}_1 \times \mathcal{Z}_2 \). Then \( \lambda : \mathcal{X} \to [0, 1] \) and obeys the following law of motion for all subsets of \( \mathcal{X} \)

\[
\lambda' (\mathcal{X}) = \int_X \mathbb{E}_{\gamma_1(s, S), \gamma_2(s, S)} \left[ 1_{[p_1(s, S) \in P_1, p_2(s, S) \in P_2]} \right] G(z_1, Z_1)G(z_2, Z_2) d\lambda (s),
\]

\[
G(z, Z) = P [z' \in Z | z]
\]

This definition of equilibrium is an extension of the standard definition of a recursive competitive equilibrium found in Ljungqvist and Sargent (2012). Firms are atomistic and behave competitively with respect to firms in other sectors of the economy. However with respect to its competitor in sector \( j \), firm \( i \) understands the affect of firm \( j \)'s actions on current period payoffs and the evolution of the sectoral state \( s \). Condition (c) requires that the policies of firms within each sector are Markov Perfect. The relevant measure in the economy is then the measure over sectoral state variables which include both firm prices and sectoral demand shocks.

### 3.5 Monopolistically competitive model

The monopolistically competitive model is identical to the above except with infinitely many firms in each sector. In this model firm \( i \) in sector \( j \) belongs to a continuum of firms \( i \in [0, 1] \). These firms behave atomistically with respect to their sector and so the relevant state variables

\[\text{In this definition } \mathbb{E}_{\gamma_1(s, S), \gamma_2(s, S)} \left[ f(s, S) \right] \text{ is the expectation of } f \text{ under the sector } s \text{ probabilities of price adjustment.}\]
for the firm is limited to its own level of demand $z_i$ and past price $p_i$. The demand system from the household’s problem is then

$$d(s, S) = (s)^{\eta-1} \left( \frac{p(s, S)}{p_j(S)} \right)^{-\eta} \left( \frac{p_j(S)}{P(S)} \right)^{-\theta} C(S)$$

(9)

$$p_j(S) = \left[ \int \left( \frac{p(s, S)}{z(s)} \right)^{1-\eta} d\lambda_j(s) \right]^{\frac{1}{1-\eta}}$$

where $P(S) = \left[ \int_0^1 p_j(S)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$.

In the monopolistically competitive model changes in the firm’s price are correctly perceived to not affect the sectoral price $p_j$. Moreover, since the parameters of the firm environment are the same in all sectors and idiosyncratic shocks wash out at the sectoral level, the distribution of firms $\lambda_j$ is the same in all sectors. This implies that $p_j(S) = p_k(S)$ for all $j$ and $k$, such that $P(S) = p_j(S)$, and the cross-sector elasticity of demand $\theta$ appears nowhere in the firm problem or equilibrium conditions.

Note the connection between monopolistic competition and another market structure: sectoral monopoly. With one firm in each market the sectoral price index is simply the monopolist’s price and the within-sector elasticity of demand $\eta$ is redundant. A model of sectoral monopolistic competition under $\eta = \eta_{MC}$ will therefore be identical in firm and aggregate dynamics to a model of sectoral monopoly with $\theta = \eta_{MC}$. I return to this point when discussing the model’s implications for the empirical relationship between market concentration and price flexibility in Section 8.

3.6 From nominal prices $p$ to markups $\mu$

As stated, the sectoral MPE problem is computational infeasible with four state variables. In Appendix B I show that the firm’s problem can be stated as one in which the sectoral state vector is reduced to the firms’ two real prices. Since households are paid in terms of nominal wages and nominal profits it makes sense that the equilibrium can also be restated only in terms of real prices and quantities.

A firm’s real price is the ratio of its nominal price to nominal cost, or its markup $\mu_{ij} =$
Additionally, define the sectoral markup \( \mu_j = p_j/W \) and aggregate markup \( \mu = P/W \). Using the previous results for sectoral and aggregate price indexes these will be equal to
\[
\mu_j = \left( \mu_{1j}^{1-\eta} + \mu_{2j}^{1-\eta} \right)^{\frac{1}{1-\eta}}, \quad \text{and} \quad \mu = \left[ \int_0^1 \mu_j^{1-\theta} dj \right]^{\frac{1}{1-\theta}},
\]
respectively. All equilibrium conditions can therefore be stated in markups rather than prices. Note also that equilibrium conditions \( W = PC \) (labor supply), and \( PC = M \) (money demand) along with the definition of the aggregate markup imply that \( \mu = 1/C \): an increase in money which causes the aggregate markup to fall, causes real output to increase.

The profit function of the firm can be normalized by the wage and expressed in markups:
\[
\frac{\pi(\mu_i, \mu_{-i}, S)}{W(S)} = \left( \frac{\mu_i}{\mu_j(\mu_i, \mu_{-i})} \right)^{-\eta} \left( \frac{\mu_j(\mu_i, \mu_{-i})}{\mu(S)} \right)^{-\theta} (\mu_i - 1) \frac{1}{\mu(S)} = \tilde{\pi}(\mu_i, \mu_{-i}) \mu(S)^{\theta-1}.
\]
The discount factor is \( Q(S, S') = \beta W(S)/W(S') \), making it straightforward to similarly normalize value functions by the wage \( v(s, S) = V(s, S)/W(S) \). This renders the firm problem stationary and leads to the following expression for the value of the adjusting firm
\[
v_{i}^{adj}(\mu_i, \mu_{-i}, S) = \max_{\mu_i} E_{\mu'_{-i}, \epsilon_i' \epsilon_{-i}' S'} \left[ \tilde{\pi}(\mu_i', \mu_{-i}') \mu(S)^{\theta-1} + \beta v_i \left( \frac{\mu_i^*}{\bar{g}^f_{\epsilon_i^*}} + \frac{\mu_{-i}'}{\bar{g}^f_{\epsilon_{-i}'}} \right) | \mu_i, \mu_{-i}, S \right].
\]
This formulation of the problem provides a clear interpretation of the two shocks hitting the firm. First, a random-walk shock to \( z_i \) is permanent iid shock to the markup of firm \( i \) should the firm not adjust its price. As most firms will not adjust prices each period, this leads to a distribution of real prices. Second, a single positive innovation to money growth causes \( g \) to increase then return to steady-state \( \bar{g} \) at rate \( \rho_g \). Absent adjustment both firms’ markups would decline on impact—as aggregate marginal cost increases—and then continue to decline at a decreasing rate until they reach some permanently lower level. Firms pay a real cost \( \xi \) to adjust their markup.

The solution to this problem requires a pricing function for the aggregate markup \( \mu(S) \), implying that the infinite dimensional distribution \( \lambda \) is a state variable—where \( \lambda \) is the distribution of sectors over markups. To make this problem tractable I follow the lead of Krusell and Smith (1998). Since I already need to specify a price function for \( \mu \), a convenient choice of moment to
characterize \( \lambda \) is last period’s aggregate markup, \( \mu_{-1} \). I use the approximate law of motion

\[ \mu = F(\mu_{-1}, g) = \exp(\beta_0 + \beta_1 \log \mu_{-1} + \beta_2 \log g). \]

Under this approximation \( S = (\mu_{-1}, g) \).\(^{24}\) More details on the solution of the firm problem and equilibrium can be found in Appendix B.

### 3.7 Comments on assumptions

**CES demand structure** An alternative formulation of the demand system could have been chosen. A pertinent example is a nested logit system commonly used in structural estimation of demand systems. However, as shown by Anderson, de Palma, and Thisse (1992), the nested CES structure is isomorphic to a nested logit with a population of consumers that each choose a single option at each stage.\(^{25}\) That is, consumers may, have identical preferences for Kraft and Hellman’s mayonnaise, up to an iid taste shock that push each consumer’s tastes towards one or the other each period. A CES structure with equal weights will deliver the same market demand functions under an elasticity of substitution that reflects the distribution of taste shocks and reduced form elasticity of indirect utility to price.\(^{26}\)

**Random menu costs** Random menu costs serve two purposes in the model. First, they generate some small price changes. Some firms, having recently changed their price and accumulating little change in sectoral productivity, draw a small menu cost and again adjust their price. Below I show that a monopolistically competitive model with random menu costs gives a distribution of price changes that appear as smoothed versions of bimodal spikes of

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\(^{24}\)To verify this statement regarding the aggregate state, note that the approximation implies

\[
\bar{v}_i^{adj}(\mu_i, \mu_{-i}, \mu_{-1}, g) = \max_{\mu_{-i}', g} \left[ \frac{\mu_i^\prime - \mu_{-i}^\prime}{g' e_{-i}'}, F(\mu_{-1}, g)^{\theta - 1} + \beta v_i \left( \frac{\mu_i^\prime - \mu_{-i}^\prime}{g' e_{-i}'}, F(\mu_{-1}, g)^{\theta - 1} \right) \right].
\]

\(^{25}\)I thank Colin Hottman for making this point, and take its presentation from Hottman (2016).

\(^{26}\)For estimation of alternative static demand systems using scanner data similar to that used in this paper see: Lein and Beck (2015) (nested logit), Dossche, Heylen, and den Poel (2010) (AIDS), Hottman, Redding, and Weinstein (2014) (nested CES). Only the latter studies an equilibrium, imperfectly competitive model.
Second, and most importantly, random menu costs that are private information allow me to avoid solving for mixed-strategy equilibria, a technique I borrow from Doraszelski and Satterwaite (2007). One could imagine solving the model under mixed strategies with fixed menu costs. Given the values of adjustment and non-adjustment and a fixed menu cost $\xi$, the firm may choose its probability of adjustment

$$\gamma_i(s, S) = \arg \max_{\gamma_i \in [0,1]} \gamma_i \left[ v^\text{adj}_i(s, S) - \xi \right] + (1 - \gamma_i) v^\text{stay}_i(s, S).$$

If firm $-i$ follows a mixed strategy such that $v^\text{adj}_i(s, S) - \xi = v^\text{stay}_i(s, S)$, then a mixed strategy is a best-response of firm $i$. If one believes that menu costs are fixed, then this provides an alternative rationale for small price changes. Some firms may not wish to adjust prices this period, yet their mixed strategy over adjustment leads them to change prices nonetheless. However the solution of this model would be vastly more complicated and at this stage infeasible.

**Information** I assume that the evolution of product demand within the sector $(z_{1j}, z_{2j})$ is known by both firms at the beginning of the period and only menu costs are private information. An arguably more realistic case, is that menu costs are fixed but firms know only their own productivity and past prices of both firms. This would add significant complexity to the problem. First, if productivity is persistent then firms’ would face a filtering problem and a state vector that includes a prior over their competitor’s productivity. Second, computation is still complicated even if productivity is iid. From firm one’s perspective $z_{2j}$ would be given by a known distribution, which it must integrate over when computing expected payoffs. Integrating over firm two’s policy functions—which depend on $z_{2j}$—would be computationally costly.

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27 Midrigan (2011) explicitly models multiproduct firms and shows that the implications for aggregate price and quantity dynamics are—when calibrated to the same price-change data—the same as a model with random menu costs. What is important for these dynamics is that the model generates small price changes, leading to the statement that the conclusions drawn are not sensitive to the exact mechanism used to generate small price changes. In this sense one can think of the random menu costs in my model standing in for an unmodeled multiproduct pricing problem.
Idiosyncratic shocks  The assumptions that idiosyncratic shocks follow a random walk and move both marginal revenue and marginal product schedules of the firm are made for tractability, but are not unrealistic. The first seems plausible given that the model is being solved at a monthly frequency. It achieves tractability in that future states can be expressed as functions of growth rates of $z_{ij}$, which are iid under a random walk assumption. An alternative assumption in similar studies is a random walk in money growth is an AR(1) in firm level shocks—for example Nakamura and Steinsson (2010)—, which would reduce the dimension of the aggregate state, but leave the total sectoral state vector with four elements. The second seems acceptable if one does not hold a strong view on whether demand or productivity shocks drive firm price changes, a reasonable stance given that only revenue productivity is most commonly estimated from the data. Midrigan (2011) interprets $\epsilon_{ij}$’s as shocks to “quality”: the good is exogenously demanded more, but more costly to produce.

4 Illustrating the main mechanism

To understand the dynamics of markups in the two models I consider a simple exercise which corresponds to the central experiment in Golosov and Lucas (2007). Aggregate shocks are shut down and I examine the response of markups following a one time unforeseen increase in money. Firms assume that the aggregate markup remains at its steady state level. Both models are studied at the parameter values which I estimate in the following section.

4.1 Monopolistic competition

Figure 2 describes the behavior of firms in the monopolistically competitive model. The red (green) lines describe a firm that has received a string of positive (negative) quality shocks. The thin lines in panel A plot the evolution of each firm’s markup absent the increase in money supply, and the thin lines in panel B plot the firm’s probability of adjustment $\gamma_i(\mu_i)$. The dashed lines in panel A describe the optimal reset markup of each firm $\mu_i^*(\mu_i)$. I assume that both firms draw sequences of large menu costs, so that their prices do not adjust. Since this is the

28This turns out to be a good approximation. In the monopolistically competitive model this is formalized by Proposition 7 of Alvarez and Lippi (2014). This explained by the fact that the aggregate markup has only a second order effect on the policies of the firm.
monopolistically competitive model these firms are independent.

Now consider the thick lines in Figure 2 which describe behavior under a permanent increase in the money supply $\Delta M > 0$ in period 40 which, absent adjustment, reduces both firms markups. The low markup firm’s probability of adjustment increases as its markup moves away from its reset value. The size of its optimal adjustment increases to accommodate the entire increase in aggregate nominal cost, which is precisely $\Delta M$. At the high markup firm, the shock moves it closer to its reset value, its probability of adjustment falls, and its size of adjustment falls by $\Delta M$. Note that the firms’ optimal markups are equal and unaffected by the shock.

As detailed by Golosov and Lucas (2007), this behavior sharply curtails the real effects of the monetary expansion. The distribution of adjusting firms shifts towards those with already low prices. These are firms that are increasing their prices and now by larger amounts. Monetary neutrality is due to the behavior of these firms with low markups and a high probability of adjustment that are marginal with respect to the shock.

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Footnote 29: The increase in the money supply leads to an increase in nominal demand which, in the presence of sticky prices increases real consumption demand. To produce more goods requires more labor, and as labor demand increases the economy moves out along the household’s labor supply curve. This increases the wage, which lowers firms’ markups.
4.2 Duopoly

Figure 3 repeats this exercise in the duopoly model. A crucial departure from Figure 3 is that I now consider two firms in the same sector. The firms differ both in their policies absent the shock, and in their response to the shock. These differences are due to the interaction of menu costs and complementarity in prices that arise in the duopoly model.

**Static complementarity** What I will call static complementarity in pricing means that the cross-partial derivative of a firm's profit function is positive ($\pi_{12} > 0$): if firm two’s price is high, then firm one also desires a high price. Economically, this is the case for two reasons: (i) the household has a lower ability to substitute across sectors than within sectors, (ii) each firm is non-atomistic and understands how its price moves the sectoral price. If firm two’s price is high, then high within sector substitutability means that firm one can post a price not much lower than its competitor and achieve a high market share. Because of (ii) the firm understands that increasing its price also increases the sectoral price. This dampens the increase in its relative price within the sector, while low substitutability across sectors leads to a small impact on sectoral demand. Since $\eta > \theta$, then the first effect dominates.

In Appendix C I make precise these static complementarities in the duopoly model under CES preferences. I show that the best response functions in a static, frictionless model are upward sloping with a slope less than one: if $\mu_{-i}$ is greater than the frictionless Nash equilibrium markup $\tilde{\mu}$, then the best response of firm $i$ is to undercut $\mu_{-i}$ with $\mu_i \in (\tilde{\mu}, \mu_j)$. Figure E2(B), plots this best response function for the calibrated parameters of the model and comparative statics with respect to $\eta$.

**Dynamic complementarity** In a dynamic model with zero menu costs of price adjustment, static complementarity in pricing has zero impact on the equilibrium of the model other than the level of the markup. The unique Markov Perfect equilibrium of the model would consist of both firms playing the static Nash equilibrium markup in all periods. In other words the MPE policy function $\mu_i^*(\mu_i, \mu_{-i}) = \tilde{\mu}$ is independent of $\mu_i$ and $\mu_{-i}$. An aggregate monetary shock which moves both firms’ markups would therefore be immediately passed through into prices. As I detail below, the presence of menu costs will cause this static complementarity to be reflected
in the MPE policies of the firms, that is $\mu^*_i$ and $\gamma_i$ will depend on both firm’s initial markups. This I will call dynamic complementarity.\textsuperscript{30}

**Policy functions without the shock** The first notable departure from the monopolistically competitive model is in the markup policies of the two firms. In particular, the optimal markups $\mu^*_i(\mu_i, \mu_j)$ are no longer equal. In response to its competitor having a high markup, the static complementarities in pricing, and the fact that menu costs make a downward adjustment from its competitor costly, the low markup firm resets its markup to below, but near, its competitor. This serves two purposes. First, choosing a higher reset markup encourages the high markup firm to not undercut the low markup firm: in the short run the policy maintains market share. Second, when it does adjust its price, the high markup firm’s best response will also be high: in the long run the policy maintains a high average markup. This is to the advantage of both firms due to the static complementarity in pricing.

\textsuperscript{30}I take this language from Jun and Vives (2004) which differentiate between static and dynamic complementarity in the MPE of dynamic oligopoly models of both Cournot and Bertrand competition where convex adjustment costs are variously in terms of prices and quantities. This distinction is useful when, for example, under Bertrand competition prices are static strategic complements, but adjustment costs in quantities lead prices to by dynamic strategic substitutes.
Menu costs support this behavior. In a frictionless economy, the high markup firm’s best response would be to undercut the low markup firm. The fact that the low markup firm has a high reset price reduces the high markup firm’s gains associated with the static best response. If these gains are reduced sufficiently with respect to the expected menu cost, then in equilibrium the high markup firm’s price becomes stickier. In this way the menu cost grants firms with high prices some short run commitment to not undercut their competitor, conditional on accommodative high price policies of low priced firms.

In the non-cooperative Markov-Perfect equilibrium of the model, these policies attain markups substantially above the frictionless Bertrand-Nash equilibrium. However the size of this wedge is limited by the size of the menu cost. Initial markups that are too high invite undercutting policies from competitors which increase the competitor’s firm value by more than the menu cost. I quantify this wedge and its implications for welfare in Section 6. Appendix D discusses a one period game with menu costs that further strengthens this intuition for the role of menu costs in delivering high markups, as well as showing the importance of random menu costs in eliminating multiple equilibria.

### Policies following the shock

The second departure from the monopolistically competitive model is in the response to the monetary shock. The desired price increase at the marginal (low markup) firm still jumps to cover the increase in aggregate nominal cost, but this is tempered by the decline in its competitor’s markup. The falling markup at its competitor reduces the marginal firm’s market share for any price increase, implying a higher elasticity of demand. With a higher elasticity of demand its optimal markup conditional on adjustment falls, as to does the marginal value of any price increase. Integrated over the desired price increase, this tempers the jump in the probability of price adjustment.\footnote{If we were to assume a fixed price of firm $j$, and an optimal markup of firm $i$ that depends only on the markup of firm $j$ we could express the change in value delivered by the optimal desired price increase of firm $i$ as

$$\Delta_i(\mu_j) = v_i(\mu_i^*(\mu_j), \mu_j) - v_i(\mu_i, \mu_j) = \int_{\mu_i}^{\mu_i^*(\mu_j)} \frac{\partial v_i(x, \mu_j)}{\partial x} dx.$$} In the example of Figure 3, the

$$\Delta_i(\mu_j) = \frac{\partial \mu_i^*(\mu_j)}{\partial \mu_j} \left( \left. \frac{\partial v_i(x, \mu_j)}{\partial x} \right|_{x=\mu_i^*(\mu_j)} \right) + \int_{\mu_i}^{\mu_i^*(\mu_j)} \frac{\partial v_i(x, \mu_j)}{\partial \mu_j} \frac{\partial \mu_i^*(\mu_j)}{\partial x} dx.$$
frequency and size of price adjustment at the marginal firm increase by half as much as they do in the monopolistically competitive model.

The above logic assumes that the inframarginal firm’s nominal price remains fixed following the shock. This is a fairly good approximation since the inframarginal firm’s markup has moved closer to its reset markup, lowering its probability of adjustment. We do, however, observe a small increase in the optimal markup of the high markup firm. Understanding that its marginal competitor has a higher likelihood of changing its price, the inframarginal firm aims to encourage its competitor to choose a high markup conditional on adjustment.

This analysis shows that the key to understanding monetary non-neutrality in the duopoly model is the following: The behavior of the marginal firm, is now affected by the behavior of inframarginal firms. The exercise above provides a particularly stark example, considering firms with markups below and above their reset markups. I later show that this is the quantitatively relevant case in generating aggregate price stickiness relative to the monopolistically competitive model. However I note here that if both firms’ markups are initially low, then it will be the case that firms respond with a higher probability of adjustment and larger desired price change than a firm with a correspondingly low initial markup under monopolistic competition. With both firms adjustment likelihood increasing, the firms enthusiastically pursue a price increase larger than the increase in costs. Quantitatively though, sectors of this type are fewer and their force towards a less sticky aggregate price index is outweighed by sectors like the one described in this example.

Negative monetary shock   For the sake of repetition and completeness, consider the symmetric case of a negative shock to the money supply. In this case the nominal wage falls and—conditional on non-adjustment—markups increase. The inframarginal firm is now the high markup firm which is considering decreasing its price, while the shock has increased the real price of its competitor. With their competitor’s markup increasing the firm’s demand elasticity falls, reducing the desired size of price decrease and the value of a price decrease. Again, the frequency and absolute size of price change at the marginal firm increase by less relative to their monopolistically competitive counterpart.

If we assume that $\mu^*_i(\mu_j)$ is defined by the markup that maximizes $v_i(\mu_i, \mu_j)$, then the first term is zero. The second term is positive due to pricing complementarity, which results in a positive cross-partial derivative of the value function.
In these examples firms are not correctly forecasting the aggregate markup and—for illustrative purposes—the shock in period forty is extremely large. Moreover the monetary shock was an unforeseen and permanent rather than an understood part of the economic environment and persistent. I now return to the full model for a quantitative comparison of monetary non-neutrality under both market structures.

5 Calibration

In this section I calibrate both models to a standard set of good-level pricing moments considered in the quantitative menu cost model literature. Section 6 gives the main results regarding aggregate dynamics and confirms the intuition from the previous section. Section 7 discusses additional results which relate to how menu costs induce allow duopolists to sustain a wedge between average and frictionless markups and endogenous price rigidity in the duopoly model.

Externally fixed

Both models are calibrated at a monthly frequency with $\beta = 0.95^{1/12}$. I follow the same procedure as Midrigan (2011) for calibrating the persistence and size of shocks to the growth rate of money: $\rho_g = 0.61, \sigma_g = 0.0019$. I set $\bar{g}$ such that the model generates annual inflation of 2.5 percent. The final parameter set externally is the cross-sector elasticity $\theta$ which I set to 1.5, in the range of those estimates from micro-econometric studies of grocery store data (Dossche, Heylen, and den Poel, 2010).

Internally estimated

Both models have the same set of remaining parameters (i) the within-sector elasticity of substitution $\eta$, (ii) the size of shocks to $z$ denoted $\sigma_z$, (iii) the distribution of menu costs. I assume menu costs are uniformly distributed $\xi \sim U \left[0, \bar{\xi}\right]$ and refer to $\bar{\xi}$ as the menu cost. In choosing

\[32\] Specifically, I take monthly time-series for $M1$ and regress $\Delta \log M1_t$ on current and 24 lagged values of the monetary shock series constructed by Romer and Romer (2004). I then estimate an AR(1) process on the $\Delta \log M1_t$ predicted values from this regression. The coefficient on lagged money growth is $\rho_g = 0.608$, with standard error 0.045. The standard deviation of residuals from this second regression gives $\sigma_g$.

\[33\] Average US inflation from 1985 to 2016 is equal to 2.7 percent.
these parameters the moments I match are the average absolute size and frequency of price change in the IRI data, as well as a measure of the average markup.

As shown by Golosov and Lucas (2007), matching these first two moments severely constrains the ability of the monopolistically competitive menu cost model to generate sizeable output fluctuations. A high size of price change implies low markup firms adjusting following a monetary shock will have larger positive price changes. A high frequency of price change means the increase in nominal cost is more quickly incorporated into the aggregate price index. The average absolute log size of price change is 0.10, the average frequency of price change is 0.13. Appendix A details the construction of these measures. Here I simply note that I exclude sales and small price changes that may be deemed measurement error.

The third moment, the average markup, is motivated two ways. First, note that if the within-sector elasticity was the same in both models, then the effective elasticity of demand faced by the duopolist would be lower. The duopolist is not price-taking when it comes to the sectoral markup. Since the cross-sector elasticity $\theta < \eta$, then the effective elasticity of the duopolist lies between $\theta$ and $\eta$, the demand elasticity of the monopolistically competitive firm. A lower demand elasticity leads to lower price flexibility. Calibrating to the average markup means the elasticity of demand faced by firms in both models are approximately the same. In the following section I show that the main result of higher aggregate price stickiness under duopoly is robust to the value of $\eta$ chosen in the monopolistically competitive model once the monopolistically competitive model is recalibrated to match the frequency and size of price adjustment.

Second, matching the average markup implies that the average level of profits are the same in both models. A ranking of estimated menu costs is therefore preserved when transformed into the ratio of menu costs to profits, which is an economically more meaningful measure. This will allow me to make statements regarding the firm-level price stickiness endogenously generated by each model by simply comparing the estimated menu costs. It will turn out that the duopoly model does generate endogenously stickier prices with respect to firm-level shocks. Calibrating both models to match the same frequency of price change means that this feature will not play a direct role in a comparison of aggregate dynamics. Again, the spirit of the experiment is to control for price flexibility with respect to idiosyncratic shocks and examine the differential response of the two economies in response to aggregate shocks.

I choose a value of the average markup of 1.30. In their estimation of markups across 50
sectors Christopoulou and Vermeulen (2008) find an average markup in the US of 1.32. The average markup estimated by Berry et al. (1995) for the US auto industry is 1.31. The average markups estimated by Hottman (2016) using retail goods are in the range of 1.29 to 1.33. For comparison with macro models, a markup of 1.30 would be chosen by a monopolistically competitive firm—absent frictions—facing a demand elasticity of 4.33. This is a little low with respect to values chosen in macroeconomic models, but is in line with recent demand elasticity estimates for grocery goods from Beck and Lein (2015), Dossche, Heylen, and den Poel (2010) and Hottman, Redding, and Weinstein (2014). Since I take evidence for markups from the literature instead of the data, Section 6.4 discusses robustness of my results to this choice of average markup.

Calibrated parameter values are given in Table 1. The baseline calibration of the monopolistically competitive model is given by the column Base. The remaining columns give alternative calibrations of the monopolistically competitive model, which I will use to describe other results in the following sections.

For now, compare the Base calibration to Alt I which takes the monopolistically competitive model at exactly the same parameters as those calibrated in the duopoly model. The Base calibration calls for a lower elasticity of substitution and higher menu cost in the monopolistically competitive model to match the same moments. With a higher elasticity and lower menu cost

Table 1: Parameters in monopolistically competitive and duopoly models

<table>
<thead>
<tr>
<th>A. Parameter</th>
<th>Duopoly</th>
<th>Monopolistic competition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of demand</td>
<td>$\eta$</td>
<td>10.5</td>
</tr>
<tr>
<td>Size of menu cost</td>
<td>$\bar{\xi}$</td>
<td>0.17</td>
</tr>
<tr>
<td>Size of shocks</td>
<td>$\sigma_z$</td>
<td>0.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Moments matched</th>
<th>Duopoly</th>
<th>Monopolistic competition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average markup</td>
<td>$E[\mu_{it}]$</td>
<td>1.30</td>
</tr>
<tr>
<td>Frequency of price change</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>Size of price change</td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. deviation consumption</td>
</tr>
<tr>
<td>Average - Frictionless markup</td>
</tr>
</tbody>
</table>
Alt I generates a substantially higher frequency and lower average size of price change. With more flexible firm-level prices, output fluctuations—as measured by the standard deviation of consumption $\sigma(C_t)$—are less than half the size of Base.$^{34}$ This comparison expresses the sense in which the calibration strategy has been designed to undo features of the models that would lead to larger real effects in the duopoly model were one not to match these facts.

6 Aggregate dynamics

This section (i) presents the main result comparing monetary non-neutrality in both models, (ii) explores the quantitative relevance of the mechanism just described, (iii) considers the robustness, and (iv) distinguishes the mechanism from existing methods for generating larger real effects of monetary shocks in menu cost models.
6.1 Monetary non-neutrality

Table 1 also delivers the main result of the paper, which is that fluctuations in output are around 2.4 times larger in the duopoly model (0.31 vs 0.13). Figure 4 expands on these results. Panel A gives the comparative statics of $\sigma(C_t)$ with respect to the menu cost, with the calibrated values of the menu cost given by the vertical dashed lines. An alternative measure of monetary non-neutrality is the cumulative response of consumption; given by the area under the impulse response function of consumption. Panel B plots this IRF computed via a local projection approach for both models. The cumulative response is twice as large in the duopoly model.

These results can be compared with other papers that study the neutrality of money in extensions of the Golosov and Lucas (2007) model. Output fluctuations are slightly larger than in the multiproduct model of Midrigan (2011) ($\sigma(C_t) = 0.29$). The ratio of $\sigma(C_t)$ under duopoly to monopolistic competition is also larger than Nakamura and Steinsson (2010) find when comparing single and multi-sector menu cost models (a ratio of 1.82 compared to 2.38 here). In this sense, this paper adds realism—markets are concentrated—and moves the models towards the large real effects of monetary shocks we find in the data.

34 The standard deviation of log deviations of consumption from steady-state is a common summary statistic for the real effects of shocks to the money supply in menu cost models cited in Section 2. Specifically $\sigma(C_t)$ is equal to the standard deviation of HP-filtered deviations of log of consumption from its steady state value—its value in an economy where $g_t = \bar{g}$.

35 Random menu costs imply that the monopolistically competitive model generates larger output fluctuations than under a fixed menu cost model calibrated to the same data. In such a model I find that $\sigma(C_t) = 0.08$. This difference is for the reason discussed extensively in Midrigan (2011): random menu costs generate some small price changes dampening the selection effect of the aggregate shock.

36 The local projection IRFs computed in this section are econometrically equivalent to the approach used by Jorda (2005). To the best of my knowledge this paper is the first to use them in the quantification of a heterogeneous firms model so I discuss their construction briefly. The economy is simulated for 5,000 periods with aggregate and idiosyncratic shocks. Given the known time-series of aggregate shocks to money growth $\epsilon^g_t$, the horizon $\tau$ IRF is $IRF_\tau = \sum_{s=0}^{\tau} \hat{\beta}_\tau$, where $\hat{\beta}_\tau$ is estimated by

$$\Delta \log C_t = \alpha + \beta_\tau \epsilon^g_{t-\tau} + \eta_t.$$  

The benefits of computing the IRF in this manner are (i) it is exactly what one would compute in the data if the realized path of monetary shocks was known, which is consistent with the approach that uses identified monetary shocks from either a narrative or high-frequency approach (Gertler and Karadi, 2015), (ii) it avoids the time consuming approach of simulating the model many times as is usually done in heterogeneous agents models with aggregate shocks.

37 See their Table VI (first row, first two columns). When comparing single and multi-sector menu cost models of their Calvo+ model—a menu cost model where the menu cost is $\xi > 0$ with probability $\alpha$ and zero with probability $(1 - \alpha)$—this ratio is 1.63.

38 In a VAR study in which nominal shocks are identified by long run restrictions on Shapiro and Watson (1988) attribute 28% of the variation in output to nominal shocks. The standard deviation of HP-filtered log consumption

"
### 6.2 Verifying the mechanism

To check whether the intuition from Section 4 holds in the full model we can study the response of the size and frequency of price change for low and high markup firms following a positive monetary shock. Figure 5 shows that the intuition from Figures 2 and 3 holds.\(^{39}\)

In both models the broad dynamics are the same. The marginal, low markup firms increase their adjustment (panel A), and the size of their price change increases (panel B) to account for the persistent rise in aggregate marginal cost. The inframarginal, high markup firms, decrease their adjustment and size of price change as their prices—which were initially too high relative to marginal cost—are more appropriate given the increase in aggregate marginal cost. However both the frequency and size of price change of low markup firms respond by less in the duopoly model. The falling markups of their inframarginal high markup competitors reduces the marginal firm’s value of a price increase and also the optimal price conditional on adjustment.

Observe that the size of price changes at high markup firms falls by less in duopoly model. This is consistent with high priced firms’ value of a price decrease falling now that their com-

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\(^{36}\)Notes: Impulse response functions are computed by local projection. For details see footnote 36. To isolate the effect of a positive monetary shock, only positive innovations to money growth \(\varepsilon_g > 0\) are included in the regressions used to compute the impulse response function. Green (Red) lines correspond to High (Low) markup firms. Solid (Dashed) lines correspond to the Duopoly (Monopolistically competitive) model.

\(^{39}\)Low and High markup firms in the duopoly model (solid lines) are defined by ranking firms by markup within each sector. In the monopolistically competitive model (dashed lines) I pair firms randomly and assign and then assign them to the Low and High categories according to their ranking within this pair.
petitor has a higher probability of increasing their price. Maintaining a higher price incentivizes a higher price from their competitor and so a higher sectoral markup, preferred by both firms given the household’s relative inability to substitute across sectors. As previously discussed, this is a force toward greater aggregate price flexibility in the duopoly model: high priced firms decreasing prices by less increases the average size of price change in the economy. However, the falling probability of adjustment at high markup firms implies that this differential response in terms of desired size of price decrease does not have large effects.

### 6.3 Decomposition

I can more formally decompose the response of the economy by splitting changes in the aggregate price index into its extensive and intensive margin, and examining how this margins behave varies across sectors of the economy. This follows the spirit of Caballero and Engel (2007)’s theoretical decomposition of a wide class of sticky price models.40

Consider two simulations of the model, where the model has been solved in the presence of aggregate shocks. In one simulation, aggregate shocks are set to zero such that there is only trend inflation. A second simulation features identical draws of idiosyncratic shocks, but includes a single shock to the money growth at date $t$. Denote by $\Delta \bar{p}_t$ the log change in the aggregate price index in the first simulation and $\Delta \hat{p}_t$ the same statistic in the simulation with the shock. We are interested in decomposing the inflation generated by the shock $\pi_t = \Delta \hat{p}_t - \Delta \bar{p}_t$. Let $x_{it} = \log p^*_i - \log p_{it-1}$ denote the desired log price change of firm $i$, and $\gamma_{it}$ denote the probability of price change. Then approximately $\Delta p_t \approx N^{-1} \sum_{i=1}^{N} \gamma_{it} x_{it}$. Using this we obtain the following decomposition of inflation:

$$\pi_t \approx N^{-1} \sum_{i=1}^{N} \gamma_{it} (\hat{x}_{it} - \bar{x}_{it}) + \bar{x}_{it} (\hat{\gamma}_{it} - \bar{\gamma}_{it}) + (\hat{\gamma}_{it} - \bar{\gamma}_{it}) (\hat{x}_{it} - \bar{x}_{it}).$$

(11)

Panel A of Table 2 computes this decomposition for each of the two models. The first and second lines shows that in both models inflation is generated roughly equally by intensive

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40See Figure E1 for a diagramatic representation of this decomposition in a monopolistically competitive model with fixed menu costs.
and extensive margin adjustment of prices. The main result from the previous section was that inflation responds more to an increase in money growth in the duopoly model. The third line shows that the difference in inflation is roughly equally accounted for by decreases in all margins of adjustment.

Panel C then accounts for these differences across the distribution of sectors. For example, the bottom-left entry states that 9% of the lower intensive margin of adjustment in the duopoly model can be accounted for by sectors in which both firms have markups above the median markup. These results support the earlier statement that it is the sectors like those discussed in Section 4—in which one firm has a low markup, the other a high markup—that account for the difference between the two models.

The behavior across sectors is, however, much richer. Sectors in which both firms have low markups actually contribute towards greater aggregate price flexibility. Consider two firms both with low markups, but one higher than the other. With both firms moving away from their desired markup the probability of both firms adjusting their price increases. Conditional on both firms changing their price, the equilibrium desired price increase accounts for the increase in marginal cost and more, as firms establish a new, high, sectoral price, knowing that deviations in future periods will be costly. Complementarity implies such a strategy is valuable to both firms, increasing the probability of adjustment.

Quantitatively the additional flexibility from such sectors is offset by those discussed earlier. First, there are twice as many sectors with low-high markups than low-low. Second, sectors where firms begin with two high—so inframarginal—markups behave similarly in both models as both firms move away from their adjustment thresholds.

To add to appendix: Results for the calibrated model with a fixed cost of adjustment as in Golosov-Lucas. This model will load almost entirely on the extensive margin of adjustment. Recall that the idea of muting inflation responses by introducing more kurtosis into the distribution of price changes is that in doing so the extensive margin falls by reducing the mass pushed into adjustment following a shock. In these experiments, the realizations of random numbers used to generate the simulations are the same across models. This means that two firms in one sector in the duopoly model have two corresponding, but unrelated, firms in the monopolistically competitive model. The different parameters of each model map random numbers into different (i) idiosyncratic shocks, (ii) menu costs, but the underlying random numbers are the same for each of these pairs. In each model, these pairs of firms are then assigned to the quadrants of the distribution of markups as in Panel C according to whether their markups are above or below the median of all firms in the simulation. Comparison figures—like the ones in Section 4—for a sector with two low markups.
Table 2: Market structure and the composition of monetary non-neutrality

6.4 Robustness

Greater monetary non-neutrality under a duopoly market structure depends on the price setting technology, but not by alternative calibration strategies for the elasticity of demand.

State vs. time dependent price setting

A motivation for studying state-dependent menu cost models of price adjustment is that they realistically allow firms to choose when to change their prices, as opposed to time-dependent Calvo models of price adjustment which assume that adjusting firms are randomly chosen. First, replicating previous exercises in the literature I compare the monopolistically competitive menu cost model in this paper to its Calvo counterpart, parameterized to match the same set of moments. Output fluctuations as measured by $\sigma(C_t)$ are three times larger in the Calvo model (0.38 vs. 0.13).\(^\text{44}\) Second, I compare a Calvo configurations of both the monopolistically competitive and duopoly models, again recalibrating the size of the shocks to match the average size of price changes (Figure 66B) and choosing the probability of price change $\alpha$ to exactly replicate

\(^{44}\)In this sense the duopoly model model accounts for around three quarters of the difference between monopolistically competitive time and state-dependent models. This comparison may seem unwarranted. However, a feature of the literature has been to ask whether menu cost models can deliver real effects as large as Calvo models. In Midrigan (2011) the main result is that a Golosov-Lucas model delivers $\sigma(C_t) = 0.07$, a Calvo model $\sigma(C_t) = 0.35$, and the author’s benchmark multiproduct model $\sigma(C_t) = 0.29$. The result being that the model generates real effects of money that are 80% as large as in the Calvo model. In my case this number is a little more than 80%.
Figure 6: Market structure and monetary non-neutrality in a Calvo model of price rigidity

Notes: Vertical dashed lines mark the empirical frequency of price adjustment $\alpha = 0.13$. In both duopoly and monopolistically competitive models the size of shocks $\sigma_z$ is set to 0.05 in order to match the average size of price changes at $\alpha = 0.13$ (panel B). In both models $\theta = 1.5$ and the elasticity of demand is chosen to obtain a frictionless markup of 1.20: $\eta_d = 10.5, \eta_m = 6$.

The observed frequency of price change. Figure 6A shows that in a time-dependent setting output fluctuations are only 10 percent larger in the duopoly model (0.41 vs 0.38). Compare this to the main result in the state-dependent model: output fluctuations were nearly 250 percent larger in the duopoly model.\textsuperscript{45}

There are two reasons why, in the Calvo model, the real effects of monetary shocks are less dependent on market structures. First, under Calvo, the extensive margin effect is eliminated as the frequency of price adjustment is exogenous. The value of a price change still falls at a marginal firm with an inframarginal competitor, but this does not affect its probability of a price change. Recall that the lower extensive margin and covariance term—which is also eliminated

\textsuperscript{45}Solving the menu cost model for very low values of $\bar{\xi}$ is infeasible (owing to technical issues relating to the approximation of the probability of price adjustment). Such issues do not arise in a Calvo setting and we can study the two models as prices become perfectly flexible. Figure 6 reassuringly demonstrates that both models behave in the same way as the nominal rigidity disappears ($\alpha \to 1$). Figure 6D shows that as nominal rigidity increases ($\alpha$ decreases from 1), the value of the firm increases under duopoly—nominal rigidity allows accommodates higher markups—and decreases under monopolistic competition—nominal rigidity means prices spend more time away from the optimum. As $\alpha$ approaches zero this second force dominates even in the duopoly model, and values fall in both models.
under Calvo—accounted for the majority of the difference in inflation responses (see Table 2).

Second, dynamic complementarity is weaker, leading to a larger intensive margin response under Calvo than under menu costs. With random adjustment, a high priced firm facing a competitor with a slightly lower price can not choose when to lower its price. This reduces the incentive of a low priced firm to choose a price close to, but slightly below, their competitor. Menu costs allow firms some commitment to their prices, so accommodate an equilibrium that inherits the static complementarity in pricing of the model. Random adjustment weakens this. The falling markup at the inframarginal firm following an increase in the money supply, therefore, has a smaller impact on the optimal price of the marginal firm.\textsuperscript{46}
Within sector demand elasticity $\eta$

An alternative strategy for calibrating the elasticity of demand would have been to choose $\eta$ such that markups in a frictionless economy coincided exactly.\(^{47}\) In Appendix C I derive the result that the frictionless markup in each model is

$$\mu_d^* = \frac{\frac{1}{2} (\eta_d + \theta)}{\frac{1}{2} (\eta_d + \theta) - 1}, \quad \mu_m^* = \frac{\eta_m}{\eta_m - 1}.$$  

The baseline calibration with $\eta_d = 10.5$ implies $\mu_d^* = 1.20$, which is less than the observed average markup, a point I return to below. Setting $\mu_d^* = \mu_m^*$ requires $\eta_m = 6$. Column Alt III of Table 1 uses this value of $\eta_m$ and a new value of the menu cost to match the data on size and frequency of price change.\(^{48}\) Once recalibrated, the model generates business cycles of the same magnitude as Base. Column Alt II shows that this also holds if one takes $\eta = 10.5$ from the duopoly model and again recalibrate the menu cost.

Figure 7 shows that this holds across all values of $\eta_m \in [2, 10]$, or equivalently $\mu_m^* \in [1.11, 2.00]$. Solid lines describe the monopolistically competitive model under different values of $\eta_m$, each time recalibrating the size of the menu cost (panel B) to deliver the same size and frequency of price change (panel A). Dashed lines describe the same economy but with menu costs fixed at the baseline value of 0.29 from Table 1 (Alt III). In all cases $\sigma(C_t) \approx 0.13$. It does not matter which monopolistically competitive model—indexed by $\eta_m$—I compare the duopoly model to, so long as it is calibrated to match the size and frequency of price adjustment. Put differently, larger output fluctuations are not obtained by simply ‘giving more market power to monopolistically competitive firms’, reducing the substitutability of their goods.\(^{49}\)

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\(^{46}\)To add: Take two firms with markups $\mu_i, \mu_j$. Decrease the markup of $\mu_j$ by some percentage $\Delta$, then compute the percentage change in the optimal markup of $\mu_i$. This elasticity can be computed across the distribution of markups for any $\mu_i < \mu_j$. This elasticity is zero in the monopolistically competitive model. The above conjectures that the elasticity is smaller in the Calvo duopoly model than the menu cost duopoly model.

\(^{47}\)The motivation for such an approach would be to keep the models identical in the absence of nominal rigidity, and then observing how nominal rigidity differentially effects each model. This reflects more the second way of framing the model—discussed at the beginning of Section 3—in which nominal rigidity is added to a model like Atkeson and Burstein (2008). The baseline calibration strategy reflects the first way of framing the model, in which a sticky price model like Golosov and Lucas (2007) is the baseline, to which oligopoly is added.

\(^{48}\)With a higher demand elasticity than Base, prices far from their optimum are more costly to the firm, leading to more frequent price changes for any given level of menu cost. This requires a larger menu cost both in levels, and as a ratio of profits.

\(^{49}\)Here one could simply cite the results of Alvarez, LeBehin, and Lippi (2016), who show that, theoretically, to an appropriate degree of approximation, the real effects of monetary shocks in a monopolistically competitive
The irrelevance of $\eta_m$ for aggregate dynamics of the monopolistically competitive model does not, however, carry over to the duopoly model. A lower $\eta_d$ weakens complementarities and firms’ static best response functions flatten. In the limit $\eta_d = \theta$, and behavior becomes monopolistically competitive.\(^{50}\) As per Figure 7D and the above discussion, such a model will, once recalibrated, imply $\sigma(C_t) = 0.13$. A higher $\eta_d$ strengthens complementarities, increasing output fluctuations as the behavior of inframarginal prices have a larger impact on marginal firm adjustment. Changing $\eta_d$ also monotonically changes the average markup, which was used in the calibration.

Figure 8 draws out this intuition. For a given average markup one can recalibrate the model and determine the real effects of monetary shocks. In the monopolistically competitive model, the real effects of monetary shocks are independent of the average markup in recalibrated models.\(^{51}\)

These results have implications for the importance of demand elasticity estimates. Monetary policy is quantitatively unaffected by the value of the demand elasticity if one believes markets to be monopolistically competitive. The model is misleadingly robust to misspecification. However if markets are imperfectly competitive, then correct estimates of demand elasticities are necessary for understanding the effects of monetary policy.

6.5 Comparison to menu cost models that amplify shocks

The duopoly model stands in contrast to extensions of the monopolistically competitive model that reduce monetary neutrality by altering the microeconomics of the model. The mechanism here does not operate through (i) higher kurtosis in the distribution of desired price changes, (ii) complementarities due to the second-order properties of demand or decreasing returns to scale.\(^{52}\)

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\(^{50}\) This is verified by noting that the sectoral price index—which contains a firm’s director competitor’s price—drops out of the firm’s demand function when $\eta = \theta$ (see equation (2)).

\(^{51}\) Due to the computational burden of recalibrating the duopoly model, robustness with respect to the average markup $E[\mu_t]$ is forthcoming.

\(^{52}\) Since the macroeconomics of the duopoly and monopolistically competitive model are the same, I do not compare the model to those that alter the macroeconomics of the model to slow the pass-through of the monetary
Neutrality in response to a monetary shock is determined by the shift in the distribution of firms adjusting their price from high markup firms decreasing their prices to low markup firms increasing their prices. In the case of an increase in the money supply this depends on the slope of the distribution of firms near the adjustment thresholds. In a model with Gaussian shocks, this slope is steep.

In a model with more small price changes, or more kurtosis in the distribution of price changes, this slope is shallower. In Midrigan (2011) and following work by Alvarez and Lippi (2014) this is achieved by modelling multiproduct firms with economy of scope in price changes. Firms have $n$ products and adjust all markups when one good hits a threshold of adjustment despite other goods’ markups being close to their optimum. In Gertler and Leahy (2008) this is achieved through large infrequent shocks that throw $\mu_{it}$ beyond an adjustment threshold, forcing the firm to adjust while its markup is still not far from its reset value. Alvarez, LeBehin, and Lippi (2016) formalize these types of results by showing that—within this class of models—the frequency and kurtosis of price changes are sufficient statistics for the real effects of monetary shocks. If kurtosis is high then many price changes are interior to adjustment thresholds, which implies movements in adjustment thresholds sweep fewer firms into adjusting. The models of Gertler and Leahy (2008) and Midrigan (2011) deliver precisely this large kurtosis.

shock to movements in nominal cost. The most relevant case being Nakamura and Steinsson (2010).
Figure 9: Distributions of markup gaps (solid) and markup changes (dashed)

Notes: The markup gap $x_{it} = \log \mu_{it} - \log \mu_{it}^*$ is defined with respect to the markup at the beginning of the period following the realization of shocks $\mu_{it}$, and the coincident optimal markup $\mu_{it}^*$. Firms are binned in 0.025 intervals of $x_{it}$. The top set of lines plot the fraction of firms in each bin. The bottom set of lines plot the fraction of firms in each bin multiplied by the fraction of firms in that bin changing prices. First, note that in a Calvo model of price adjustment, the dashed lines would be a multiple $\alpha < 1$ of the solid lines, where $\alpha$ is the exogenous probability of price adjustment. Second, summing points on the dashed lines obtains the total fraction of firms changing prices which—under the calibration of Table 1—is equal to 0.13 in both models.

Figure 9 shows that changing market structure—while keeping size and frequency of price change the same—does not change the kurtosis of the distribution of price changes. The distribution of desired price changes is similar in both models, with some additional left skewness under duopoly due to the lower frequency of price change at low markup firms. That the duopoly model generates larger real effects confirms that it does not belong to the class of models for which these sufficient statistics apply.

What differentiates the duopoly model from the class of models studied by Alvarez, LeBehin, and Lippi (2016) is the presence of complementarities in price setting. The results from Alvarez and Lippi (2014) and Alvarez, LeBehin, and Lippi (2016) apply when—to a first order—a firm’s optimal markup is independent of all other prices. Once strategic complementarities enter the model, this is no longer the case. In the duopoly model a competitor’s price enters the first order conditions of the firm, breaking the application of these sufficient statistics. Similarly, models with complementarities between the firm price and aggregate price, as will be discussed next, do not fit into the class of models studied in these papers.

Models of strategic complementarity and monopolistic competition

As noted by Nakamura and Steinsson (2010), “monetary economists have long relied heavily on complementarity in price setting to amplify monetary non-neutrality generated by nominal rigidities.”
In monopolistically competitive models such complementarities can be introduced between
the firm’s price and the aggregate price. A Kimball (1995) demand aggregator, as studied by
Klenow and Willis (2016) and Beck and Lein (2015), can flexibly introduce such complementari-
ties. Demand curves feature an elasticity of demand which is increasing in a firm’s relative price
\( \bar{\mu}_{it} = (\mu_{it}/\mu_t) \). Let \( \mu^*_\text{CES} \) be the optimal markup under CES demand. Under CES demand an
adjusting firm would choose \( \mu^*_\text{CES} \) both before and after \( \mu_t \) falls (recall Figure 2). Under Kimball
demand the higher relative markup associated with \( \mu^*_\text{CES} \) increases the firm’s demand elasticity
at that point, leaving \( \mu^*_\text{CES} \) sub-optimal and leading the firm to reduce the size of its desired
adjustment.53 The value of an adjustment also falls, reducing the frequency of adjustment.54

Rather than supporting these demand-side real rigidities, the work of Klenow and Willis
(2016) and Beck and Lein (2015) lead us to reject them. As Nakamura and Steinsson (2010) con-
tinue, “introduction of such strategic complementarities render the models unable to match the average
size of price changes for plausible parameter values...requiring] massive idiosyncratic shocks and large
menu costs”55 The duopoly model features such strategic complementarities, large amplification
of monetary shocks, matches the same micro-data, and does so with (i) lower menu costs as a fraction of profits, (ii) the same size of idiosyncratic shocks. My results differ from the
consensus that has formed, correctly, around competitive models. Why is this the case?

First, why do these quantitative issues arise when introducing strategic complementarity in
the monopolistically competitive model? In the case of Kimball demand, the goal is to generate
a pro-cyclical elasticity of demand due to movements in \( \mu_{it} \) relative to \( \mu_t \). Since monetary shocks
and movements in \( \mu_t \) are small, large real effects occur only if firms’ demand elasticity increases
a lot when \( \mu_t \) falls by a little. This requires a large super-elasticity of demand—the elasticity of
the elasticity of demand. By symmetry, however, this implies that a large decline in \( \mu_{it} \), due
to a negative productivity shock, will cause a huge decrease in the elasticity of demand. For a
firm with a low markup this sharply increases the value and optimal size of a price increase.
This latter case approximates the day-to-day workings of the firm: aggregate shocks are small,

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53Simply put, the firm would like to keep \( \mu_{it} \) closer to \( \mu_t \).

54Studying pass-through in an international setting Gopinath and Itskhoki (2008) and Berger and Vavra (2013)
also use a Kimball demand aggregator in a menu cost model. Both papers conclude that having a demand elasticity
that is increasing in the firms markup is important for capturing imperfect exchange rate pass-through.

55As Nakamura and Steinsson (2010) continue in Section VB, the results of Klenow and Willis (2016) “cast doubt
on strategic complementarity as a source of amplification in menu cost models with idiosyncratic shocks”
and the firm most frequently responds to idiosyncratic shocks.\footnote{For completeness: A positive idiosyncratic shock increases $\mu_{it}$, the firm’s elasticity of demand increases, which increases the value of a price decrease. In this sense the Kimball structure with a positive super-elasticity is symmetric in increasing the frequency of adjustment in response to both positive and negative shocks.} As such, increasing the super-elasticity, \textit{ceteris paribus}, makes the profit function more concave, sharply increasing firm level price flexibility (higher frequency of adjustment, smaller size of price changes).

An alternative source of complementarity is studied by Burstein and Hellwig (2007), who introduce decreasing returns to scale. As above, a decline in $\mu_t$ will leave a firm with relatively lower sales if it adjusts to the optimal markup it was considering before the aggregate shock. With decreasing returns to scale, lower sales translate into lower marginal cost, dampening the previously desired price increase. Yet the same issue arises. To generate large output effects a small decline in $\mu_t$ must significantly lower marginal cost at any desired markup. If this is the case then marginal cost will shoot up following an exogenous decrease in the markup, pushing the firm to increase its price. And again, this is the most relevant case when idiosyncratic shocks are large relative to aggregate shocks. As such, increasing the decreasing returns to scale faced by the firm, \textit{ceteris parabus}, also sharply increases firm level price flexibility.

The rejection of these sources of complementarity as a source of amplification of monetary shocks is then based on three observations. Here I consider only the arguments of Klenow-Willis but similar remarks apply to Burstein-Hellwig. First, a specification of $\epsilon$ that generates large responses, requires the size of shocks be increased from 11% to 28% at a monthly frequency, and menu costs double as a fraction of revenue, in order to bring firm level price flexibility back down to its empirical levels.\footnote{Beck and Lein (2015) find that even for (i) low values of the super-elasticity of demand ($\epsilon = 1.5$), and (ii) highly persistent firm-level productivity, the size of shocks must still double to match the size of price change data.} Second, the shape of the demand function implies that firms shut-down production in 15% of months. Third, recent empirical studies by Dossche, Heylen, and den Poel (2010) and Beck and Lein (2015) document low values of $\epsilon$ in grocery store data.

The duopoly model generates a pro-cyclical elasticity of demand at marginal firms that causes large real effects, but with lower menu costs, the same size of shocks, and no shut-downs. How is this so? Lower menu costs as a fraction of profits are due to endogenous price stickiness and coordination around high markups, which I discuss in Section 7. Firms never shut down since the profit function is always positive for $\mu_{it} \in [1, \infty)$. Why are the shocks the
same size in both models? After a monetary shock it is true that the fall in an inframarginal competitor’s price increases the marginal firm’s elasticity of demand, as in the Kimball model. But on a day-to-day basis shocks to its competitor’s costs may mean its competitor’s markup decreases, increases, or—as it does on average—remains unaffected. That is, if we were to draw a firms at random from different markup bins, then the average elasticity of demand faced by a firm should not change as we move left to right across bins. This average elasticity of demand is well approximated by the frictionless model’s elasticity of demand. Targeting the average markup ensures these are similar for both models, so the required size of shocks is the same.58

Figure 10 highlights this point, plotting the profit function of the firm under a Kimball aggregator with a super-elasticity equal to ten, as used in Klenow and Willis.59 The concavity in demand results in a more concave profit function. This figure dispels the notion that the duopoly profit function has additional curvature similar to the Kimball profit function. The duopoly model does have excess curvature relative to the monopolistically competitive demand func-

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58 Another way of putting this is as follows. In KW and BH, complementarities are between the firm’s price, which fluctuate a lot, and the aggregate price, which fluctuate a little. In the duopoly model, complementarities are between two prices that fluctuate a lot. So while these extensions of monopolistically competitive model binds aggregate and idiosyncratic price adjustment through the ratio of an aggregate and idiosyncratic price, this relationship is broken in the latter.

59 For further comparison, Gopinath and Itskhoki (2008) consider a baseline value of $\varepsilon = 4$ and Berger and Vavra (2013) a value of $\varepsilon = 2.5$. 
tion despite having the same elasticity at the frictionless markup. However the additional curvature is small and is what one would get from a Kimball aggregator with a super-elasticity of $\varepsilon \in [0.3, 0.7]$.

Figure 10 shows the firm two’s profit function when its competitor’s markup is at the average value of 1.30. Comparing this to profits under monopolistic competition it is clear the first order gains that the MPE policies of attain. If both markups were initially at 1.30, the static best response would be 1.26. The second order gains from this policy are visibly small, requiring only small menu costs render them unprofitable. This establishes an ability to commit to high markups. The first order increase in profits relative to the frictionless Nash equilibrium are visibly large.

### 7 Additional results

Two further results abstract from the implications of the duopoly model for aggregate price flexibility. First, there is a substantial wedge between the average and frictionless markup, implying first order losses from nominal rigidities. Second, prices are endogenously sticky, which empirical predictions that I test in the following section.

**Result 1 - Markups and the welfare cost of nominal rigidity**

As noted above, the pricing policies of firms in the duopoly model are able to sustain markups that are higher than the frictionless markup. Table 3 quantifies the implications of this wedge for output. The second row gives output in both economies in the counterfactual case that firms’ markups were equal to the average markup in the economy. By construction these are

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60Beck and Lein (2015) estimate a median super-elasticity of around 1 using European retail goods.
equal since both economies are calibrated to the same average markup. The first row computes output in the economy, the difference between rows one and two captures the second order output losses due to price dispersion. The third row computes output for counterfactual economies with perfectly flexible prices. Comparing rows (3) and (1), the total output losses due to nominal rigidities are around 10 percent in the duopoly model and only 2.5 percent in the monopolistically competitive model. Moreover, more than three quarters of this difference is due to the difference in the level, rather than dispersion, in markups.\footnote{Total menu costs paid are, however, lower as a fraction of output in the duopoly economy, since prices are endogenously stickier. However since menu costs are a small fraction of output anyway, they would not affect Table 3 at two decimal places.}

Similar to the stylized model of Maskin and Tirole (1988b), price frictions bestow short run commitment to high prices. One firm’s high price encourages high prices from its competitor. Such dynamic complementarity in prices in the presence of adjustment costs have also been studied by Jun and Vives (2004) in a model with no idiosyncratic shocks and convex costs of adjustment. Here I can quantify the wedge between frictionless and average markups in a model that matches the salient features of firm level price adjustment. I also avoid two features of these environments that push toward a larger wedge: in the first case an exogenous timing assumption, and in the second case a cost of price change that is increasing in the size of the deviation in prices.

Notes: Figures plot the comparative statics of the average markup and average firm value given by Bellman equation (7), with respect to the menu cost $\xi$. In each case the other parameters of the model are as in Table 1. Vertical dashed lines give the baseline values of $\xi$ from Table 1. Since the figure is a comparative static across menu costs, the calibration Alt III has been used since this benchmarks the markup across market structures when $\xi = 0$.
Figure 11A provides comparative statics of the average markup with respect to the size of the menu cost. The average markup is sharply increasing in the menu cost in the duopoly model. In the monopolistically competitive model the increase is slight due to the precautionary motive induced by a positive third derivative of the firm’s profit function (see Figure E2C), which implies it is less profitable to sell large quantities at low prices than small quantities at high prices.

Figure 11B quantifies a related result: the value of the firm may be increasing in the size of the menu cost. A higher menu cost implies greater dynamic complementarity, which accommodates higher markups but at the expense of price flexibility. In the duopoly model, local to the estimated menu cost, the gains from the former offset losses from the latter. While the monopolistically competitive firm always prefers lower menu costs, duopolists have a positive optimal menu cost. At $\xi = 0.29$ the value of the firm is maximized and 6 percent higher than at the estimated value of 0.17. Figure 6D shows that this value maximizing nominal rigidity exists independent of the price change technology.

Four potentially interesting implications for future research are as follows. First, the fact that firms desire some, but not too much, nominal rigidity may rationalize why firms in tight oligopolies engage in investments that increase the cost of price changes. Second, if policies such as higher trend inflation weaken the ability of firms to commit to higher markups, then such policies can have first order welfare implications. Third, these results imply a systematically bias estimates of markups in the industrial organization literature. A common practice following Berry, Levinsohn, and Pakes (1995) is to estimate a demand system under Nash-Bertrand or Nash-Cournot and use the estimated parameters to infer markups. If markups are higher under nominal rigidity than in a frictionless model, then estimates will be biased downward. Finally, these results may distort our understanding of the welfare implications of frictions in macroeconomics. The standard intuition holds in the monopolistically competitive model: firms and households both dislike frictions. In an oligopoly there is a range over which higher frictions are redistributive: profits increase but output falls. This may rationalize costly lobbying by incumbents to increase procedural frictions for investment.

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62 For example, firms print brochures with prices fixed for some period of time.
63 Indeed, in the limit, high trend inflation would cause firms to reset their prices every period and the frictionless Nash equilibrium markup would be obtained, eliminating these first order welfare losses of nominal rigidity.
Notes: Figures plot the comparative statics of the average frequency and size of log price change for low and high markup firms, with respect to the menu cost $\xi$. Firms are split into low and high markup firms by the median markup in the economy. Vertical dashed lines give the baseline values of $\xi$ from Table 1. Since the figure is a comparative static across menu costs, the calibration $Alt III$ has been used since this benchmarks the markup across market structures when $\xi = 0$.

Result 2 - Lower price flexibility

Table 1 reveals that the duopoly model requires a smaller menu cost to match the data on price adjustment. Prices decreases are less valuable due to a long-run incentive to maintain a high sectoral markup. Price increases are less valuable due to a short-run incentive to maintain a high market share. Nominal prices therefore change less often for any $\xi$.

Figure 12 shows that the first case dominates: lower price flexibility at low price firms. This feature brings the model closer to another feature of the data. In the monopolistically competitively model, a precautionary motive leads to a higher frequency of adjustment at low priced firms. The ratio of high priced firm frequency of price adjustment to low priced firm frequency of price adjustment is 0.79. In the duopoly model, a reduced incentive to increase prices delivers a of 0.95. In data similar to that used in this paper Burstein and Hellwig (2007) document that low and high priced goods have approximately the same frequency of price change (a ratio of 0.97). The monopolistically competitive model is unable to generate this kind of symmetry in adjustment. Table 4 summarizes this and a number of other results from the comparison of the two models.
<table>
<thead>
<tr>
<th>Moment</th>
<th>M.C. (1)</th>
<th>Duopoly (2)</th>
<th>Ratio* / Diff+ (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average markup</td>
<td>1.22</td>
<td>1.31</td>
<td>0.09+</td>
</tr>
<tr>
<td>Standard deviation of consumption</td>
<td>0.13</td>
<td>0.29</td>
<td>2.23*</td>
</tr>
<tr>
<td>Cumulative consumption response</td>
<td>2.02</td>
<td>4.36</td>
<td>2.15*</td>
</tr>
<tr>
<td>Average firm value</td>
<td>59.9</td>
<td>81.8</td>
<td>1.37*</td>
</tr>
<tr>
<td>... at max $\zeta^*$</td>
<td>-</td>
<td>87.0</td>
<td>1.06*</td>
</tr>
<tr>
<td>Frequency - Low $\mu$</td>
<td>0.146</td>
<td>0.128</td>
<td>-0.018+</td>
</tr>
<tr>
<td>Frequency - High $\mu$</td>
<td>0.116</td>
<td>0.121</td>
<td>0.005+</td>
</tr>
<tr>
<td>Size - Low $\mu$</td>
<td>0.090</td>
<td>0.097</td>
<td>0.007+</td>
</tr>
<tr>
<td>Size - High $\mu$</td>
<td>-0.084</td>
<td>-0.073</td>
<td>-0.011+</td>
</tr>
</tbody>
</table>

Table 4: Summary of statistics

Notes: Summary of statistics from monopolistically competitive (column 1) and duopoly model (column 2), at the parameter values given in Table 1. Data points marked with a * (±) in column 3 give the ratio of (difference between) moments from the duopoly and monopolistically competitive models. The entry of 1.06 for ‘Value at max $\zeta^*$’ is the ratio of the value of the firm in the duopoly model under the value maximizing menu cost to the value under the estimated menu cost.

8 Price flexibility and market concentration in the data

The second result in the previous section—that oligopoly can be a force towards less flexible prices—motivates studying the relationship between market concentration and price flexibility.

Motivation from the model Prices are endogenously more rigid when firms act non-atomistically with respect to their sector: lower menu costs are required to match the empirical frequency of price adjustment. I also noted that a monopolistically competitive market structure is mathematically identical to a model with a monopolist in each sector, subject to $\eta = \theta$. This gives intuition for price flexibility under one, two and infinitely many firms: prices are more flexible in the two limiting cases, and less so under duopoly.64

What to test? Suppose firms in all markets faced an economic environment determined by the same parameters (ie $\bar{z}$ and $\sigma_z$ are constant across markets). What should we expect as we compare markets with one and two firms? There are two off-setting forces. First, the elasticity effect discussed in Section 6.4 would lead the adjustment to increase. Competing with more

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64 The case of three and four firms, and so on, I leave to future work. I note briefly that the computational complexity of solving the model with more firms comes not with (i) integrating over more firms actions when computing payoffs, or (ii) adding state variables, which increases the dimensionality of the value function problem, but with converging on the MPE policy functions which are problematic to approximate in higher dimensions.
firms, any one firms’ revenue share is lower, so their elasticity of demand is higher.\textsuperscript{65} Second, the strategic forces studied in this paper would lead adjustment to decrease. As we consider markets with more firms, we might expect this second force to dissipate and the elasticity effect to dominate: firms increasingly behave atomistically and the frequency of price change increases.

This thought experiment leads me to test for a \textit{U}-shaped (hump-shaped) relationship between frequency (size) of price change and market concentration. Note that increasing price flexibility as firms are added does not suggest that these oligopoly forces are not present, only that they are weaker than the elasticity effect. In this sense the right tail of a \textit{U}-shape is confounded. However, decreasing price flexibility as firms are added indicates that the oligopoly effect is both present and strong enough to offset the elasticity effect.

\textbf{Variation in concentration} To carry out these tests I return to the IRI data and exploit two separate sources of variation in the concentration of markets. The first uses variation across \textit{states}, within \textit{product categories}. The second uses variation across \textit{products}, within \textit{states}. Figure 13 provides examples. Panel A describes the time-series of the effective number of firms in the market for Mayonnaise in four different states. Clearly there is very little variation in the time dimension, whereas variation across states is large. Panel B describes the same time-series but for different product categories within the state of New Jersey. Here most of the variation is across products.

What is striking and useful, about this kind of variation in the data is that cases arise where the market for product \textit{X} may be very concentrated in state \textit{A} and less in state \textit{B}, however the market for product \textit{Y} is more concentrated in \textit{B} than \textit{A}. Market concentration is location-good specific. Furthermore, it appears to be an almost permanent feature of markets.\textsuperscript{66}

Any systematic relationship between concentration and price flexibility that is consistent across both sources of variation would be difficult to explain using variation in menu costs or the stochastic process for costs. This has been the primary approach to modelling variation in

\textsuperscript{65}Figure 7A shows this relationship between elasticity of demand and frequency of price change in the monopolistically competitive model.

\textsuperscript{66}This variation in market concentration has been studied using the same data by Bronnenberg, Dhar, and Dubé (2009) and Bronnenberg, Dube, and Gentzkow (2012). The latter points to the migration of individuals—who carry with them brand preferences—as a major determinant of market shares.
Figure 13: Empirical variation in market concentration

Notes For construction of the *Effective number of firms* measure see the notes to Figure 1. Each series gives effective number of firms for a given product-state market, computed using revenue shares within a quarter.

price flexibility in structural models (Nakamura and Steinsson (2010), Weber (2014)). Such an explanation would require variation that is neither consistent across goods or locations. This would seem a tall order, compounded by the additional fact that the products in this data are (i) all non-durable goods, (ii) sold in similar stores.

**Variation in flexibility** Figure 14 describes the variation in price flexibility found in the data. Comparable to Figure I in Nakamura and Steinsson (2010), panel 1A describes heterogeneity in the frequency of price change across product $p$, region $r$, month $t$ markets. This paper contributes to the study of heterogeneity in price flexibility by showing that even *within* product groups there is substantial variation. Panels B and C of each row show the substantial variation found when comparing the same products at the same time across states, and comparing different products within the same state at the same time. In percentage terms, the average absolute deviation of frequency (average absolute size) of price change from its across-state within-product-month mean is 31 (11) percent.\textsuperscript{67} These statistics are similar, but in all cases a little larger when considering across-product within-state variation.\textsuperscript{68} I now quantify the ex-

\textsuperscript{67}Specifically, let $x_{prt}$ be the price flexibility measure. Then the statistic reported is $(PT)^{-1} \sum_{p=1}^{P} \sum_{t=1}^{T} \left| \frac{x_{prt} - \bar{x}_{pt}}{\bar{x}_{pt}} \right|

\textsuperscript{68}Average deviation of frequency 36 percent, and 11 percent for size. The average absolute deviation of frequency (average absolute size) of price change from its across-product within-region-month mean is 33 (12) percent.
Figure 14: Empirical variation in (1) frequency, and (2) absolute log size of price change

Notes: The first (second) row of figures refers to the average monthly frequency of price change (log absolute size of price change), $y_{pst}$, in market $pst$. In each row the histograms are as follows. Panel A: Histogram of $y_{pst}$. Panel B: Histogram of the ratio of deviations of $y_{pst}$ from its across-state within-product-month average: $(y_{pst} - \bar{y}_{pt}) / \bar{y}_{pt}$. Panel C: Histogram of the ratio of deviations of $y_{pst}$ from its across-product within-state-month average: $(y_{pst} - \bar{y}_{st}) / \bar{y}_{st}$.

tent to which the variation in price flexibility in panels B and C can be explained using the previously described variation in market concentration.

Estimating equations Let $y_{pst}$ be a measure of market concentration in a product $p$, state $s$, month $t$ market. Let $x_{pst}$ be a measure of price flexibility, and $X_{pst}$ some other data at the market level. The across-state within-product-month regression specification is

\[
(y_{pst} - \bar{y}_{pt}) = \alpha + \beta (x_{pst} - \bar{x}_{pt}) + \delta (x_{pst} - \bar{x}_{pt})^2 + \gamma X_{pst} + \epsilon_{pst}
\]  

(12)
<table>
<thead>
<tr>
<th>Across-state w/in product</th>
<th>Across-product w/in state</th>
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</thead>
<tbody>
<tr>
<td><strong>Size (%)</strong></td>
<td><strong>Frequency</strong></td>
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<tr>
<td>Eff. number of firms</td>
<td>-0.912***</td>
</tr>
<tr>
<td></td>
<td>(0.161)</td>
</tr>
<tr>
<td>Eff. number of firms²</td>
<td>-0.048***</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
</tr>
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<td>R-squared</td>
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</tr>
<tr>
<td>Rev prostitution control</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 5: Regression results - Cross-product regression

Notes: Results for the estimation of equation (12) (first two columns) and (13) (last two columns). Data-points in the regression consists of product-month-state level observations. Size of price change is the product-month-state average of monthly log absolute price changes for all products conditional on price change. For example, for each calendar month in 2005-Q2 I compute the average log price change of all shampoo products in New Jersey. I then take the average of these observations. Freq is frequency of price change, computed at the same level and is the fraction of goods changing price. Effective number of firms is given by the inverse Herfindahl index $h_{pst}^{-1}$, where the Herfindahl index is the revenue-share weighted average revenue-share of all firms in the market, $h_{pst} = \sum_{i \in \{pst\}} (rev_{pst} / rev_{pst})^2$. Errors are clustered at the ps-level.

where $\bar{y}_{pt}$ is the across-state mean for product $p$ in month $t$. The across-product within-state-month regression specification is

$$
(y_{pst} - \bar{y}_{st}) = \alpha + \beta (x_{pst} - \bar{x}_{st}) + \delta (x_{pst} - \bar{x}_{st})^2 + \gamma X_{pst} + \epsilon_{pst}
$$

(13)

where $\bar{y}_{st}$ is the across-product mean for state $s$ in month $t$.

In the main results the effective number of firms is used as a measure of market concentration, and frequency and average size of price change as measures of price flexibility. I include an additional control for revenue in the market $pst$.\(^{69}\) Errors are clustered at the state-product level. Results are described in Table 5.

Consistent with the theory, the quadratic terms are negative (positive) in the case of size (frequency) of price changes. Coefficient estimates are remarkably similar across both regression specifications, despite each using very different sources of variation in the data.\(^{70}\)

\(^{69}\) This controls for the fact that if there is economy of scope in the cost of price change then flexibility will be higher when revenues are higher.

\(^{70}\) Appendix E show that the results are robust to different specifications. Weighting by the number of different goods in each market (Table E1) or uniform weights (Table E2) does not affect results. Neither does removing the control for total market revenue (Table E3). Using the revenue share of the largest firm as a measure of concentration results in significant quadratic terms only in the across-state within-product case (Table E4).
A. Frequency of price change

B. Size of price change

Figure 15: Variation in market concentration

Notes: Solid (dashed) lines are medians (25th/75th percentiles) of fitted values from regression (12), where averages for both effective number of firms and the dependent variable are taken within bins of effective number of firms of width one.

Figure 15 displays these results graphically. Solid lines denote the average fitted values of frequency and size of price change from the across-state within-product regression (12). Dashed lines denote the lower and upper quantiles. The model’s interpretation of these plots would be that oligopolistic forces are strong, counteracting the elasticity-effect, but weaken at around five equally sized firms. Consistent with the model, price flexibility is similar in markets with very low and very high levels of concentration in which firm behavior may be described as atomistic. This suggests a promising route for future research in which models with more than two firms per sector can be used to understand when and how these oligopoly forces peak.

9 Conclusion

This paper establishes that the competitive structure of markets can be quantitatively important for the transmission of macroeconomic shocks. In particular, in a menu cost model of firm level price setting—which aggregates to a monetary business cycle model—I showed that a monopolistically competitive market structure and a duopoly market structure can generate different levels of monetary non-neutrality. Even when calibrated to match the same salient features of price flexibility in the data, the duopoly model generates larger output responses. Following a monetary expansion the incentive for low priced firms to respond to the shock
increases less sharply as a lower sectoral price reduces the incentive to adjust. Nominal rigidity plus the ability to time price changes are shown to be crucial in allowing firms to commit to these policies which lower monetary neutrality and increase markups in equilibrium.

More broadly, this paper aims to bridge an inconsistency between data and macroeconomic models that aggregate idiosyncratic firm behavior. Recently, macroeconomic models with heterogeneous firms have been used to answer questions of the following type: Micro-data suggests a friction of type-$x$ at the firm level, does incorporating this friction affect the aggregate dynamics of the economy with respect to aggregate shocks? Examples of such frictions include fixed costs of investment, equity issuance costs, collateral constraints on borrowing, and—in the model studied in this paper—menu costs of price adjustment. These models are used to interpret data that has a key feature: the size distribution is fat tailed. Yet in these models firms are assumed to behave atomistically, regardless of their size. This paper extends the structure of models used to answer these questions to allow for non-atomistic behavior, and found—in the case of nominal rigidities and monetary shocks—that this can be important for aggregate dynamics.

The structure of the model studied in this paper also allows one to study a larger set of microeconomic behavior and its implications for macroeconomic outcomes. One could draw motivation from simple, well studied, models of sectoral strategic interaction that may either amplify or moderate macroeconomic shocks. Do firms accumulate excess capacity as a threat against the expansion of competitors, and if so, does this have implications for the response of aggregate investment following technology shocks? Can oligopsony power that may arise in a labor market with a few large firms help to explain why wages do not fall sharply in a recession? Returning to the model at hand, one could also ask whether changing market concentration over time could help explain missing inflation following the 2008 Great Recession in the US? These and other questions can be asked with modifications of the existing model, while being consistent with a salient feature of the micro-data usually studied through heterogeneous agent models: fat tails of size distributions.

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71 For example, in the US, around half of all employment is in the largest 0.4 percent of firms.
References


APPENDIX

This Appendix is organized as follows. Section A further describes the IRI data and its treatment. Section B describes the computational methods used to solve the model in Section 3. Section C describes the properties of the static Nash equilibrium of the duopoly model with no menu costs. Section D studies a static game with menu costs and exogenously specified initial markups. Section E includes additional figures and tables.

A Data description

The data used throughout this paper come from the IRI Symphony data (web-link). Details on this data can be found in the summary paper by Bronnenberg, Kruger, and Mela (2008).\textsuperscript{72} The data are at a weekly frequency from 2001 to 2011 and contain revenue and price data at the good level, where a good is defined by a unique barcode number (UPC). Data is collected in over 5,000 stores covering 50 metropolitan areas.\textsuperscript{73} For each store, data is recorded for all UPCs within each of 31 different product categories determined by IRI, for example toothpaste, or laundry detergent. The measures that I construct from this data and use in the paper relate to (i) market concentration, (ii) price changes.

To measure market concentration I define a market by product category $p$, state $s$ and month $t$. I then construct revenue for each firm within market $pst$ by summing revenue for all UPCs within that market. To identify a firm I use the five digits of a good’s UPC which uniquely identify the company. For example, the five digits 21000 in the barcode 00012100064595 identify Kraft within a market for Mayonnaise, 48001 identifies Hellman’s.

To compute measures of price changes first requires a measure of price. To obtain prices I simply divide revenue by quantity. I compute price change statistics monthly, using observations of prices in the third week of each month. I focus only on regular price changes and deem a price to have been changed in month $t$ if it (i) changes by more than 0.1 percent, considering price changes smaller than this to be due to rounding error from the construction of the price,

\textsuperscript{72}Other recent papers to use this data include Stroebel and Vavra (2014) and Coibion, Gorodnichenko, and Hong (2015).
\textsuperscript{73}Details on the identification of stores is removed from the data, replaced with a unique identifying number. Walmart is not included in the data.
(ii) was neither on promotion in month \( t - 1 \), or on promotion in month \( t \). The IRI data includes indicators for whether a good is on promotion and so I use this directly rather than using a sales filter. This second requirement means I exclude both goods that go on promotion and come off promotion. The frequency of price change in market \( pst \) is the fraction of goods that change price. The size of price change in market \( pst \) is the average log change in prices for all regular price changes within market \( pst \).

When computing moments for use in the calibration of the model I first take a simple average across \( s \) and \( t \) for each product \( p \), then take a revenue weighted average across products.

**B Computation**

First I show that the initially stated Bellman equation (7) corresponds to the restated one (10), since the latter is used in computation. Second, I describe the numerical methods used in computing the equilibrium of the model.

**Price indices** Denote the first firm’s markup \( \mu_{ij} = \frac{p_{ij}}{z_{ij}W} \). Using this, the sectoral price index \( p_j \) can be written

\[
p_j = \left[ \left( \frac{p_{1j}}{z_{1j}} \right)^{1-\eta} + \left( \frac{p_{2j}}{z_{2j}} \right)^{1-\eta} \right]^{1-\eta} = W \left[ \mu_{1j}^{1-\eta} + \mu_{2j}^{1-\eta} \right]^{1-\eta}
\]

Defining the sectoral markup \( \mu_j = p_j/W \), implies \( \mu_j = \left[ \mu_{1j}^{1-\eta} + \mu_{2j}^{1-\eta} \right]^{1-\eta} \). Using the sectoral markup, the aggregate price index \( P \) can be written

\[
P = \left[ \int_0^1 p_j^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}} = \left[ \int_0^1 \mu_j^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}} W.
\]

Defining the aggregate markup \( \mu = P/W \), implies \( \mu = \left[ \int_0^1 \mu_j^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}} \).
Profits These expressions for markups can be used to rewrite the firm’s profit function. Starting with the baseline case

\[
\pi_{ij} = z_{ij}^{\eta-1} \left( \frac{p_{ij}}{p_j} \right)^{-\eta} \left( \frac{p_j}{P} \right)^{-\theta} (p_j - z_{ij}W) C,
\]

and using \( C = M/P = 1/\mu \), I obtain

\[
\pi_{ij} = \left( \frac{\mu_{ij}}{\mu_j} \right)^{-\eta} \left( \frac{\mu_j}{\mu} \right)^{-\theta} (\mu_{ij} - 1) \frac{W}{\mu} = \hat{\pi} \left( \mu_{ij}, \mu_{-ij} \right) \mu^{\theta-1} W,
\]

where the function \( \hat{\pi} \) depends on the aggregate state only indirectly through the policies of each firm within the sector. This step makes clear the use of the technical assumption that \( z_{ij} \) also increases average cost, allowing for an expression for profits only in terms of markups.

Markup dynamics Suppose that a firm sells at a markup of \( \mu_{ij} \) today. The relevant state tomorrow is the markup that it will sell at tomorrow if it does not change its price \( \mu'_{ij} = p_{ij}/z'_{ij}W' \). Replacing \( p_{ij} \) with \( \mu_{ij} \) we can write \( \mu'_{ij} \) in terms of today’s markup and the growth rates of the aggregate wage and idiosyncratic demand

\[
\mu'_{ij} = \mu_{ij} \frac{z_{ij} W}{z_{ij} W'} = \mu_{ij} \frac{1}{\epsilon_{ij} + \gamma}.
\]

Under the random walk assumption for \( z_{ij} \), we have \( z'_{ij}/z_{ij} = \exp (\epsilon_{ij}) \). Under the equilibrium condition that \( W = M \) and the stochastic process for money growth we have \( W'/W = \exp (\gamma) \).

Bellman equation Using these results in the firm’s Bellman equation reduces the value of adjustment from (7) to the following, where for clarity I am assuming that the competitor’s markup \( \mu_{-i} \) is fixed

\[
V_{i}^{adj} \left( \mu_i, \mu_{-i}, S \right) = \max_{\mu'_{i}} \hat{\pi} \left( \mu'_{i}, \mu_{-i} \right) \mu(S)^{\theta-1} W(S) + \mathbb{E} \left[ Q(S, S') V_i \left( \frac{\mu'_{i}}{\epsilon_{i} + \gamma}, \frac{\mu'_{-i}}{\epsilon_{-i} + \gamma}, S' \right) \right].
\]
Under the equilibrium discount factor $Q(S, S') = \beta W(S) / W(S')$, all values can be normalized by the wage such that $v_i = V_i / W$:

$$v_i^{adj}(\mu_i, \mu_{-i}, S) = \max_{\mu_i'} \hat{\pi}(\mu_i, \mu_{-i}) \mu(S)^{\theta - 1} + \beta E \left[ v_i \left( \frac{\mu_i'}{e^{\epsilon_i + g'}, \frac{\mu_{-i}'}{e^{\epsilon_{-i} + g'}, S'} \right) \right].$$

Replacing the aggregate state $S = (g, \lambda)$ with that used in the approximation $S = (g, \mu_{-1})$, we then have the following

$$v_i^{adj}(\mu_i, \mu_{-i}, g, \mu_{-1}) = \max_{\mu_i'} \hat{\pi}(\mu_i, \mu_{-i}) \hat{\mu}(g, \mu_{-1})^{\theta - 1} + \beta E \left[ v_i \left( \frac{\mu_i'}{e^{\epsilon_i + g'}, \frac{\mu_{-i}'}{e^{\epsilon_{-i} + g'}, g', \hat{\mu}(g, \mu_{-1})} \right) \right],$$

where $\hat{\mu}$ is given by the assumed log-linear function: $\log \hat{\mu} = \alpha_0 + \alpha_1 g + \alpha_2 \log \mu_{-1}$.

The equilibrium condition requiring the price index be consistent with firm prices has also been restated in terms of markups, which implies the entire equilibrium is now restated in terms of markups. Note that in order to determine quantities I need to also simulate paths for $M_t$ and $z_{ijt}$. To simulate changes in prices it is sufficient to known a path for markups $\mu_{ijt}$, innovations $\epsilon_{ijt}$ and money growth $g_t$.

**To be added**
- Solution method for Bellman equation
- Solution method for Krussell-Smith algorithm

### C Frictionless Nash equilibrium

Here I verify the statements in the main text regarding (i) the level of the frictionless Nash equilibrium markup, (ii) the properties of the static best response functions. Numerically, these can be verified by examining Figure E2.
Frictionless markup

I state the problem from the perspective of firm one, with the objective function

\[ \pi_1(\mu_1, \mu_2) = \mu_1^{-\eta} \mu_2^{\eta-\theta} (\mu_1 - 1) X, \]

where

\[ \mu = \left[ \mu_1^{1-\eta} + \mu_2^{1-\eta} \right]^{\frac{1}{1-\eta}}, \]

and \( X \) is due to aggregate variables which the firm takes as given and drop out of its first order condition. In what follows it is useful to note that given the CES structure of demand

\[ \frac{\partial \mu}{\partial \mu_1} = \left[ \mu_1^{1-\eta} + \mu_2^{1-\eta} \right]^{\frac{1}{1-\eta}} \mu_1^{-\eta} = \left( \frac{\mu_1}{\mu} \right)^{-\eta}. \]

We can also write the revenue of the firm \( r_1 = p_1 d(p_1, p(p_1, p_2)) \) in terms of markups

\[ r_1 = \mu_1^{1-\eta} \mu_2^{\eta-\theta} W \]

where the wage \( W \) is taken as given by both firms. This implies that the revenue share of the firm is

\[ s_1 = \frac{r_1}{r_1 + r_2} = \frac{\mu_1^{1-\eta}}{\mu_1^{1-\eta} + \mu_2^{1-\eta}} = \left( \frac{\mu_1}{\mu} \right)^{-\eta} \frac{\mu_1}{\mu} = \frac{\partial \mu}{\partial \mu_1} \frac{\mu_1}{\mu}. \]

Under a CES demand system a firm’s revenue share is equal to the elasticity of the sectoral markup with respect to its own price.

The first order condition of the firm’s problem is

\[ \left[ \mu_1^{-\eta} - \eta \mu_1^{-\eta-1}(\mu_1 - 1) \right] \mu_2^{\eta-\theta} + (\eta - \theta) \mu_1^{-\eta} \mu_2^{\eta-\theta-1}(\mu_1 - 1) \frac{\partial \mu}{\partial \mu_1} = 0, \]

where the term in square brackets gives the first order condition of a monopolistically competitive firm facing elasticity of demand \( \eta \). The second term gives the effect of the firm’s markup on the sectoral markup. Since \( \eta > \theta \), for any given price of firm two, firm one derives a positive marginal benefit from a higher sectoral price. Substituting in the above result and simplifying we obtain

\[ \mu_1 - \eta(\mu_1 - 1) + (\eta - \theta)(\mu_1 - 1)s_1 = 0. \]
Rearranging, the firm’s equilibrium markup can be expressed

\[ \mu_1 = \frac{s_1 \theta + (1 - s_1) \eta}{s_1 \theta + (1 - s_1) \eta - 1}. \]

Since the unique equilibrium is symmetric, then \( s_1 = s_2 = 0.5 \). This implies that markups are consistent with those chosen by a monopolistically competitive firm facing a log-linear demand curve with elasticity of demand \( \varepsilon = \frac{1}{2} (\theta + \eta) \).

Figure C1 traces out properties of the static Nash equilibrium for different preferences weights of the household for each firm’s good, taking \( \eta \) and \( \theta \) from Table 1. The parameter \( \omega_1 \) controls preferences within the sector-level CES function

\[ c = \left[ \omega_1 c_1^\eta + (1 - \omega_1) c_2^\eta \right]^{\frac{\eta}{\eta - 1}}. \]

The figure makes clear that in the limit of \( \omega_1 = 1 \), firm one acts like a monopolist facing a cross-sector elasticity of substitution \( \theta \). When \( \omega_1 = 0 \), the firm behaves monopolistically competitively with respect to its sector facing elasticity of demand \( \eta \) and with a vanishing revenue share. When \( \omega = 0.5 \) we are in the case studied in this paper with equal revenue shares and a frictionless markup governed by the average of the two elasticities.
Best response functions

The key property of the static best response function $\mu_1^*(\mu_2)$ is that its upwards sloping with a slope less than one. To prove this take firm one’s first order condition: $\pi_1^1(\mu_1, \mu_2) = 0$, where the superscript refers to the firm, and the subscript refers to the derivative. By the implicit function theorem, then local to $(\mu_1^*, \mu_2^*)$, the derivative of $\mu_1^*(\mu_2)$ can be obtained by re-arranging the total derivative of the first order condition

$$\frac{\partial \mu_1^*(\mu_2)}{\partial \mu_2} = -\frac{\pi_{12}^1(\mu_1, \mu_2)}{\pi_{11}^1(\mu_1, \mu_2)}.$$

The second order conditions of the Nash equilibrium require that the principal minors of the Jacobian of the first order conditions alternate in sign,

$$\pi_{11}^1(\mu_1^*, \mu_2^*) < 0,$$

$$\pi_{11}^1(\mu_1^*, \mu_2^*) \pi_{22}^2(\mu_1^*, \mu_2^*) - \pi_{12}^1(\mu_1^*, \mu_2^*) \pi_{21}^2(\mu_1^*, \mu_2^*) > 0.$$

By symmetry, the second condition implies that $|\pi_{11}^1(\mu_1^*, \mu_2^*)| > |\pi_{12}^1(\mu_1^*, \mu_2^*)|$. Combined with the first condition, we have the result that $\partial \mu_1^*(\mu_2)/\partial \mu_2 \in (0,1)$.

In terms of comparative statics, when strategic complementarities in price setting are larger—that is, $\pi_{ij}^i$ is large—the slope of the best response function is steeper. In the nested CES model this depends positively on $\eta - \theta$. To see this, combine a slightly simplified version of the first order condition of firm 1,

$$\left[\mu_1^{-\eta} - \eta \mu_1^{-\eta-1}(\mu_1 - 1)\right] + (\eta - \theta)\mu_1^{-\eta} \mu_1^{-1}(\mu_1 - 1) \frac{\partial \mu}{\partial \mu_1} = 0,$$

with the observation that the cross-partial derivative of the sectoral markup is positive,

$$\frac{\partial^2 \mu}{\partial \mu_1 \partial \mu_2} = \frac{\eta}{\mu} \left(\frac{\mu_1}{\mu}\right)^{-\eta} \left(\frac{\mu_2}{\mu}\right)^{-\eta} > 0.$$

It is clear, then, that when $\eta - \theta$ is large, $\pi_{12}^1(\mu_1^*, \mu_2^*)$ is also larger and the slope of the best response function increases.
D Static game

In this appendix I solve a static price setting game which shows how menu costs can lead to higher prices than obtain in a frictionless setting.

Consider two firms that start with prices $p_1 = p_2 = \bar{p}$. Assume that these prices are greater than the frictionless Nash equilibrium price $p^*$. The profit function of firm one is $\pi_1(p_1, p_2)$ and the symmetric profit function of firm two is $\pi_2(p_1, p_2)$. These are consistent with the main text

$$\pi_1(p_1, p_2) = p_1^{-\eta} p(p_1, p_2)^{\eta-\theta} (p_1 - 1),$$
$$p(p_1, p_2) = \left[p_1^{1-\eta} + p_2^{1-\eta}\right]^{1/1-\eta}.$$

Let $p^*(p_j)$ denote the symmetric best response function, so that $p^* = p^*(p^*)$.

The rules of the game are that both firms move simultaneously and pay a menu cost $\xi$ to change their price. Consider three types of equilibrium: (i) both firms change their price, (ii) one firm changes its price, (iii) neither firm changes its price.

If both firms change their price, then it must be that the prices chosen are $(p_1^*, p_2^*)$. Given that both firms are changing their prices, then the price chosen by each firm must be a best response to its competitor. This is only satisfied at $(p_1^*, p_2^*)$. If menu costs are zero, then this is the only equilibrium. If menu costs are positive, however, then we also require the following condition to hold

$$\xi \leq \Delta \pi^* = \pi_1(p_1^*, p_2^*) - \pi_1(\bar{p}_1, p_2^*). \quad (\text{D1})$$

This states that given that firm two changes its price to $p_2^*$, then firm one would also change its price to $p_1^*$. This condition may not hold when either (i) $\xi$ is large, or (ii) $\bar{p}_1$ is close to $p^*$, since $\pi_1(p_1, p_2^*)$ is decreasing in $p_1$ when $p_1 > p^*$.

Now consider requirements for $(\bar{p}_1, \bar{p}_2)$ to be an equilibrium. Clearly if $\xi = \infty$ then this is the only equilibrium. If $\xi < \infty$, then we would also require that

$$\xi \geq \Delta \pi = \pi_1(p_1^*(\bar{p}_2), \bar{p}_2) - \pi_1(\bar{p}_1, \bar{p}_2). \quad (\text{D2})$$

74In the figures below I use the values of $\eta = 1.5, \bar{\eta} = 10.5$ as in Table 1, such that $p^* = 1.20$. 

68
Figure D1: Comparative statics of conditions for the static game

Panel A. plots the left hand side of the conditions (D1) and (D2). Panel B. plots the left hand side of the conditions (D3) and (D4) that are required for one firm changing its price to be an equilibrium.

That is, given $\bar{p}_2$, the most profitable deviation for firm one does not increase its value by more than the menu cost.

Figure D1(A) shows that for a given value of $\bar{p}$ (on the x-axis), and a high value of $\xi$ (on the y-axis) we start with only the $\bar{p}$-equilibrium. As $\xi$ decreases, we reach a point where $\Delta \pi^* > \xi$ (given $p_1^*$, firm two would want to respond with $p_2^*$), but also $\Delta \hat{\pi} < \xi$ holds (given $\bar{p}_1$, firm two has no response that increases its value by more than the menu cost). This implies that both equilibria exist. As we further decrease $\xi$, then $\Delta \hat{\pi} > \xi$, the no price change equilibrium disappears, and both firms choose $p^*$.

One firm changing its price is not an equilibrium. If it were, then two conditions must hold. First, firm two must find it profitable to change its price given that firm one’s price remains at $\bar{p}$

$$\xi \leq \Delta \hat{\pi}_2 = \pi_2(\bar{p}_1, p_2^*(\bar{p}_1)) - \pi_2(\bar{p}_1, \bar{p}_2).$$  \hspace{1cm} (D3)

Second, firm one leaving its price fixed must be its best response. The best it can do if it were to change its price would be to choose $p^*(p_2^*(\bar{p}_1))$. Therefore we require that

$$\xi \geq \Delta \hat{\pi}_1 = \pi_1(p_1^*(p_2^*(\bar{p}_1)), p_2^*(\bar{p}_1)) - \pi_1(\bar{p}_1, p_2^*(\bar{p}_1)) \leq \xi.$$ \hspace{1cm} (D4)

Figure D1(B) shows that these two conditions never hold simultaneously, since $\Delta \hat{\pi}_2 < \Delta \hat{\pi}_1$. If
firm two chooses its best response to firm one not changing its price, then this greatly increases the profitability of firm one’s next iteration of best responses. Any case where firm two finds it profitable to pay $\xi$ and increase its price, firm one would find it profitable to also pay $\xi$ and undercut firm two. Figure D2 depicts graphically these changes in values.

From this static game we observe that for a given menu cost $\xi$, high prices $\bar{p}$ can be sustained so long as they are not too high. When the initial price is too high, one firm has a profitable deviation, even when they pay the menu cost. If the value of one firm’s undercutting strategy exceeds the menu cost, then the value of an iterative undercutting strategy from its competitor also exceeds the menu cost. This leads both firms to change their prices. Once this occurs, only the low frictionless Nash price is achievable. If initial prices are not too high, then the menu cost is enough to wipe out the small value of profitable deviations which make the high priced strategy credible. Quantitatively, the gains from static best responses are second order, requiring small menu costs to wipe them out. The net result is an equilibrium with a higher sectoral markup: a first order term in the firm’s profit function.

A helpful way of thinking about the dynamic model is that the firms have an average real price at a $\bar{p}$ in a region where menu costs successfully serve this role. Idiosyncratic shocks force real prices apart, but adjustment strategies keep $\bar{p}$ from becoming too high as to induce strategic undercutting, or too low as to reduce long run firm profitability.
E Additional figures and tables

Figure E1: Price changes in a monopolistically competitive, fixed menu cost model

A. Intensive: Still increase prices, and increase them by more
B. Extensive + Covariance: More firms increase prices, and increase them by more
C. Extensive: No longer decrease prices
D. Intensive + Covariance: Less firms decrease prices, and decrease them by less

Figure E2: Properties of firm profit functions

Notes: Panels A, C, D, display features of the duopoly profit functions under $\theta = 1.5, \eta = 10.5$ as in Table 1. Given these parameters, the frictionless Nash-Bertrand markup is 1.20 due to an effective elasticity of demand of $\epsilon = \frac{1}{2}(\theta + \eta)$ and a symmetric equilibrium. Panel B plots that static best response function $\mu_i^*(\mu_j)$ under $\theta = 1.5$ and different values of $\eta$. Higher values of $\eta$ reduce the Nash equilibrium markup (given by the intersection of the best response with the 45-degree line), and increase the slope of the best response function.
### Table E1: Regression results - Number of goods weighted regression

Notes: See notes for Table 5. This table provides results for the same regressions except where the number of goods sold in each market are applied as weights in estimation of equations (12) (first two columns) and (13).

<table>
<thead>
<tr>
<th></th>
<th>Across-state w/in product</th>
<th>Across-product w/in state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Size (%)</td>
<td>Frequency</td>
</tr>
<tr>
<td>Eff. number of firms</td>
<td>0.186***</td>
<td>-0.516***</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.134)</td>
</tr>
<tr>
<td>Eff. number of firms²</td>
<td>-0.024***</td>
<td>0.026</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Observations</td>
<td>32,016</td>
<td>32,016</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.061</td>
<td>0.078</td>
</tr>
<tr>
<td>Rev pst control</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

### Table E2: Regression results - Unweighted regression

Notes: See notes for Table 5. This table provides results for the same regressions except where uniform weights are applied in estimation of equations (12) (first two columns) and (13).

<table>
<thead>
<tr>
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<th>Across-state w/in product</th>
<th>Across-product w/in state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Size (%)</td>
<td>Frequency</td>
</tr>
<tr>
<td>Eff. number of firms</td>
<td>0.205***</td>
<td>-0.664***</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.088)</td>
</tr>
<tr>
<td>Eff. number of firms²</td>
<td>-0.018***</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>Observations</td>
<td>32,016</td>
<td>32,016</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.061</td>
<td>0.078</td>
</tr>
<tr>
<td>Rev pst control</td>
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<td>✓</td>
</tr>
<tr>
<td>Across-state w/in product</td>
<td>Across-product w/in state</td>
<td></td>
</tr>
<tr>
<td>--------------------------</td>
<td>--------------------------</td>
<td></td>
</tr>
<tr>
<td>Size (%)</td>
<td>Size (%)</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>Frequency</td>
<td></td>
</tr>
</tbody>
</table>

| Eff. number of firms     | 0.244***                 |
|                         | -0.956***                |
|                         | (0.038)                  |
|                         | (0.181)                  |
| Eff. number of firms$^2$| -0.048***                |
|                         | 0.183***                 |
|                         | (0.010)                  |
|                         | (0.050)                  |
| Observations            | 32,016                   |
| R-squared               | 0.100                    |
| Rev$_{pst}$ control     | ✗                         |

Table E3: Regression results - No control for revenue

Notes: See notes for Table 5. This table provides results for the same regressions except where no additional controls are used in estimating (12) (first two columns) and (13).

<table>
<thead>
<tr>
<th>Across-state w/in product</th>
<th>Across-product w/in state</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size (%)</td>
<td>Size (%)</td>
</tr>
<tr>
<td>Frequency</td>
<td>Frequency</td>
</tr>
</tbody>
</table>

| Rev. share top firm      | -1.411***                |
|                         | 7.727***                 |
|                         | (0.428)                  |
|                         | (2.015)                  |
| Rev. share top firm$^2$  | -9.739***                |
|                         | 16.939**                 |
|                         | (1.964)                  |
|                         | (7.817)                  |
| Observations            | 32,016                   |
| R-squared               | 0.133                    |
| Rev$_{pst}$ control     | ✓                         |

Table E4: Regression results - Alternative concentration measure - Revenue share of largest firm

Notes: See notes for Table 5. This table provides results for the same regressions except where the revenue share of the largest firm in the market is used as the control variable when estimating (12) (first two columns) and (13).