Firm and Worker Dynamics in a Frictional Labor Market

Adrien Bilal
Princeton University

Nikolas Engbom
Federal Reserve Bank of Minneapolis

Simon Mongey
University of Chicago

Gianluca Violante
Princeton Univeristy

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Job-to-job flows are important for understanding $F(n, z)$

Account for a lot of reallocation

- 50% of hires are from another job, 50% of quits are to another job

Play an important role in characterizing firm dynamics

... in cross-section

- Firms with higher growth rates have lower quit rates

  Davis Faberman Haltiwanger Rucker (2010), Faberman Nagypal (2008)

- Small, young & high productivity firms poach on net $(EE_{hires} > EE_{quits})$

  Haltiwanger Hyatt McEntarfer (2017), Haltiwanger Hyatt McEntarfer Kahn (2018)

... in response to shocks

- Reducing $\downarrow EE_{quits}$, increasing $\uparrow EE_{hires}$ accounts for 72% of net-growth

  Bagger Fontaine Galenianos Trapeznikova (2019)

What role does job-to-job mobility play in determining $F(n, z)$?

This paper

Model

- Multi-worker firms + Off/On-the-job (random) search
- Decreasing returns to scale + Convex vacancy costs
- $\underbrace{\{\text{Entry/Exit, Age, Size, Growth}\}}_{\text{Firm dynamics / misallocation literature, e.g. Hopenhayn Rogerson (1993)}} + \underbrace{\{\text{Unemployment, Job-to-job flows}\}}_{\text{OJS literature, e.g. Postel-Vinay Robin (2002)}}$

Questions

✓ Calibrated to aggregate data, can we match cross-sectional flows?

- Endogeneity of hiring costs important for allocation $F(n, z)$?

New tractable framework that can handle lots of (firm) heterogeneity

Model - Continuous time environment

Workers

- Measure $\bar{\mathtt{N}}$ of infinitely lived, homogeneous workers
- Risk neutral. Discount rate ρ, exog. separate at rate δ
- Supply one unit of labor. Produce b when unemployed.

Firms

- Technology $y(z, n) = zn^{\alpha}$, where $d \log z \sim GBM(\mu, \sigma)$. Scrap π
- M_e entrants pay ψ_e for n_e workers and draw of $z_e \sim F_e(\zeta)$

Search and matching

- Workers search for free on- and off- the job
- Firms search by posting costly vacancies $c(v, z, n)$
- Aggregate matching function $\mathtt{M} = \chi \mathtt{V}^{\beta}(\mathtt{U} + \xi \mathtt{E})^{1-\beta}$
- $G_v(z, n) = v$-weighted distribution, $G_n(z, n) = n$-weighted distribution,

Model - Assumptions

1. Two-sided limited commitment

- Firms can fire workers, workers can quit at will. Both at any time.

2. Mutual consent

- Wages can only be modified when an outside option to one party gives it a credible threat
  - $\pi(z, w_1, \cdot)$ meets w_2.
  - No credible threat to fire if $\pi(z, \cdot, w_2) < \pi(z, w_1, w_2)$ so keep w_1
  - Credible threat to fire if $\pi(z, \cdot, w_2) > \pi(z, w_1, w_2)$ so cut w_1

3. Bargaining protocol

- Firms make take-it-or-leave-it offers
- Poaching firms make first offer, competitor responds, worker decides

4. Transfers

- Transfers allowed between workers and firm

Model - $\Omega(x) = J(x) + \sum_{i=1}^{n} V(x,i)$

$$
\begin{aligned}
\rho\Omega(x) \quad &= \quad y(x) - c(v(x), x) \\
EU_{Layoff} \quad & + \sum_{i=1}^{n(x)} \delta \left[\mathbf{\Omega}(d(x,i)) - \Omega(x) + U\right] \\
UE_{Hire} \quad & + v(x) q(\theta) \phi \left[\mathbf{\Omega}(h_U(x)) - \Omega(x) - U\right] \cdot \mathbb{I}_{\{x \in \mathcal{A}\}} \\
EE_{Hire} \quad & + v(x) q(\theta)(1-\phi) \int_{x \in \mathcal{Q}^E(x', i')} \left[\mathbf{\Omega}\left(h_E(x', i', x)\right) - \Omega(x) - \mathbf{V}\left(h_E(x', i', x), i'\right)\right] dG_n(x' \\
EE_{Quit} \quad & + \xi f(\theta) \sum_{i=1}^{n(x)} \int_{x' \in \mathcal{Q}^E(x,i)} \left[\mathbf{\Omega}\left(q_E(x, i, x')\right) - \Omega(x) + \mathbf{V}\left(h_E(x, i, x'), i\right)\right] dG_v(x') \\
\overline{UE_{Hire}} \quad & + v(x) q(\theta) \phi \left[\mathbf{\Omega}(t_U(x)) - \Omega(x)\right] \cdot \mathbb{I}_{\{x \notin \mathcal{A}\}} \\
\overline{EE_{Hire}} \quad & + v(x) q(\theta)(1-\phi) \int_{x \notin \mathcal{Q}^E(x', i')} \left[\mathbf{\Omega}\left(t_E(x', i', x)\right) - \Omega(x)\right] dG_n(x', i') \\
\overline{EE_{Quit}} \quad & + \xi f(\theta) \sum_{i=1}^{n(x)} \int_{x' \notin \mathcal{Q}^E(x,i)} \left[\mathbf{\Omega}\left(r(x, i, x')\right) - \Omega(x)\right] dG_v(x') \\
\text{Shocks} \quad & + \mathbb{E}_\eta \left[\mathbf{\Omega}(g_z(x, \eta)) - \Omega(x)\right]
\end{aligned}
$$

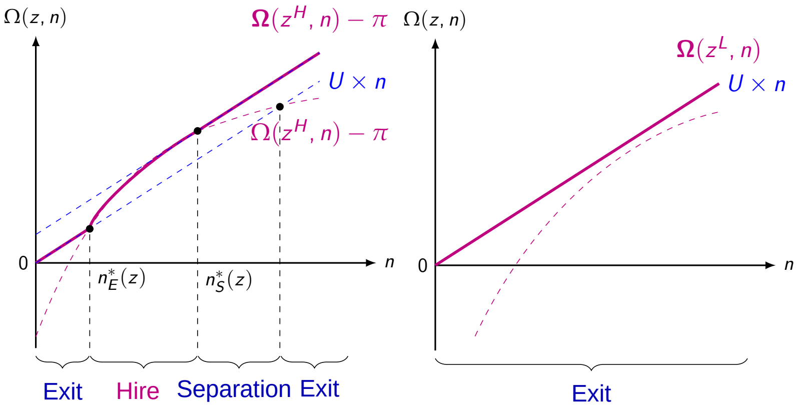
Model - $\Omega(x) = \Omega(z, n, \dots)$

$$
\begin{aligned}
\rho\Omega(z,n,...) \quad &= \quad y(z,n) - \underbrace{\delta\sum_{i=1}^{n}\left[\Omega(z,n,\dots) - \boldsymbol{\Omega}(z,n-1,\dots) - U\right]}_{EU_{Layoff}} \\
- \quad & c\big(v(z,n,\Omega(z,n,\dots),\dots),z,n\big) \quad + \quad v\big(z,n,\Omega(z,n,\dots),\dots\big)q(\theta)\Bigg[\\
+ \quad & \underbrace{\phi\left[\boldsymbol{\Omega}(z,n+1,...) - \Omega(z,n,...) - U\right]}_{UE_{Hire}} \\
+ \quad & \underbrace{(1-\phi)\int_{(z,n,...)\in\mathcal{Q}^{E}(z',n',\Omega(n',z',\dots),\dots)} \Big[\big\{\boldsymbol{\Omega}(z,n+1,...) - \Omega(z,n,...)\big\} - \big\{\Omega(z',n',...) - \boldsymbol{\Omega}(z',n'-1,...)\big\}\Big] dG_n(z',n',...)}_{EE_{Hire}}\Bigg] \\
+ \quad & \mathbb{E}_{\eta}\left[\boldsymbol{\Omega}\left(g_z\left(z,\eta\right),n,\dots\right) - \Omega\left(z,n,\dots\right)\right] \\
\boldsymbol{\Omega}(z,n,\dots) \quad &= \quad \max\left\{\pi_{Exit} + nU\,,\ \max_{s\in\{1,2,...,n-1\}}\Omega(z,n-s,\dots) + sU\,,\ \Omega(z,n,\dots)\right\}
\end{aligned}
$$

Model - $\Omega(x) = \Omega(n, z)$

$$
\begin{aligned}
\rho\Omega(z,n) \quad = \quad & y(z,n) - \underbrace{\delta \sum_{i=1}^{n} \Big[\Omega(z,n) - \boldsymbol{\Omega}(z,n-1) - U\Big]}_{EU_{Layoff}} \\
- \quad & c\big(v(z,n), z, n\big) \quad + \quad v\big(z,n\big) q(\theta) \Bigg[\\
+ \quad & \underbrace{\phi\Big[\boldsymbol{\Omega}(z,n+1) - \Omega(z,n) - U\Big]}_{UE_{Hire}} \\
+ \quad & \underbrace{(1-\phi) \int_{(z,n)\in\mathcal{Q}^E(z',n')} \Big[\big\{\boldsymbol{\Omega}(z,n+1) - \Omega(z,n)\big\} - \big\{\Omega(z',n') - \boldsymbol{\Omega}(z',n'-1)\big\}\Big] dG_n(z',n')}_{EE_{Hire}} \Bigg] \\
+ \quad & \mathbb{E}_\eta \Big[\boldsymbol{\Omega}\left(g_z\left(z,\eta\right), n\right) - \Omega\left(z,n\right)\Big] \\
\boldsymbol{\Omega}(z,n) \quad = \quad & \max\Big\{\pi + nU \,,\, \max_{s\in\{1,2,\ldots,n-1\}} \Omega(z,n-s) + sU \,,\, \Omega(z,n)\Big\}
\end{aligned}
$$

Characterization - Joint value and policies



$$\mathbf{\Omega}(z, n) - \pi = \max \left\{ nU \, , \, \Omega(z, n) - \pi \, , \, \max_{s \in \{1,2,\ldots\}} \Omega(z, n - s) + sU - \pi \right\}$$

Model - $\Omega(x) = \Omega(n, z)$

$$
\begin{aligned}
\rho\Omega(z,n) \quad = \quad & y(z,n) \underbrace{- \delta \sum_{i=1}^{n} \left[\Omega(z,n) - \boldsymbol{\Omega}(z,n-1) - U \right]}_{EU_{Layoff}} \\
- \quad & c\big(v(z,n), z, n\big) \quad + \quad v(z,n) q(\theta) \Bigg[\\
+ \quad & \underbrace{\phi \left[\boldsymbol{\Omega}(z,n+1) - \Omega(z,n) - U \right]}_{UE_{Hire}} \\
+ \quad & \underbrace{(1-\phi) \int_{(z,n) \in \mathcal{Q}^E(z',n')} \Big[\left\{ \boldsymbol{\Omega}(z,n+1) - \Omega(z,n) \right\} - \left\{ \Omega(z',n') - \boldsymbol{\Omega}(z',n'-1) \right\} \Big] dG_n(z',n')}_{EE_{Hire}} \Bigg] \\
+ \quad & \mathbb{E}_\eta \Big[\boldsymbol{\Omega}\left(g_z(z,\eta), n\right) - \Omega(z,n) \Big] \\
\boldsymbol{\Omega}(z,n) \quad = \quad & \max \Big\{ \pi_{Exit} + nU \,,\, \max_{s \in \{1,2,\ldots,n-1\}} \Omega(z,n-s) + sU \,,\, \Omega(z,n) \Big\}
\end{aligned}
$$

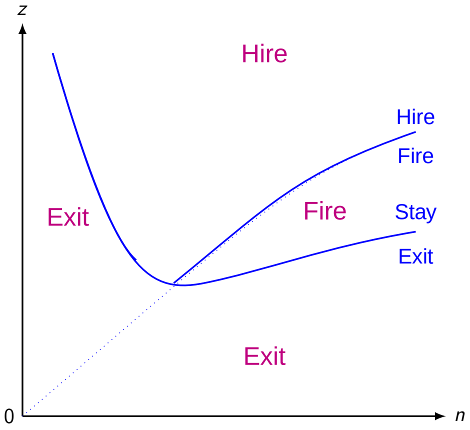
Model - $S(z,n) = \Omega(z,n) - nU$

$$
\begin{aligned}
\rho S(z,n) &= \max_{v \geq 0} \quad y(z,n) - nb \quad - \quad \delta n \mathbf{S}_n(z,n) \quad - \quad c(v,z,n) \\
&+ \quad vq(\theta) \underbrace{\left[\phi \mathbf{S}_n(z,n) + (1-\phi) \int_0^{\mathbf{S}_n(z,n)} \mathbf{S}_n(z,n) - S'_n \, dG_n(S'_n) \right]}_{\text{Return on a vacancy } R(\mathbf{S}_n)} \\
&+ \quad \mathbb{E}_\eta \left[\mathbf{S}(g_z(z,\eta), n) - S(z,n) \right] \\
\mathbf{S}(z,n) &= \max \left\{ \pi_{Exit} \, , \, \max_{n' \leq n} S(z,n') \, , \, S(z,n) \right\}
\end{aligned}
$$

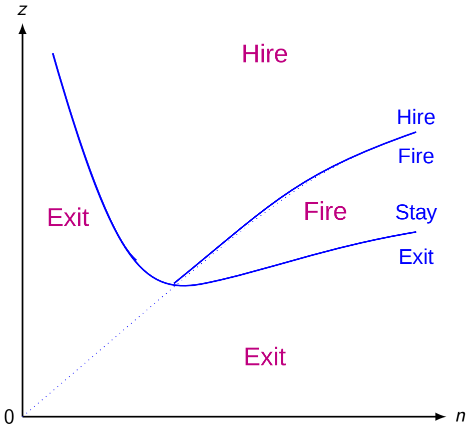
Model - $S(z,n) = \Omega(z,n) - nU$

$$\begin{aligned}
\rho S(z,n) &= \max_{v \geq 0} \quad y(z,n) - nb \quad - \quad \delta n S_n(z,n) \quad - \quad c(v,z,n) \\
&+ \quad vq(\theta) \underbrace{\left[\phi S_n(z,n) + (1-\phi) \int_0^{S_n(z,n)} S_n(z,n) - S_n' \, dG_n(S_n') \right]}_{\text{Return on a vacancy } R(S_n)} \\
&+ \quad \mu(z) S_z(z,n) + \frac{\sigma(z)^2}{2} S_{zz}(z,n) \\
S(z,n) &\geq \pi_{Exit} \qquad \text{(Exit)} \qquad \underbrace{S_z(z,n) = 0 \quad , \quad S_n(z,n) = 0}_{\text{Smooth pasting}} \\
S_n(z,n) &\geq 0 \qquad \text{(Separation)}
\end{aligned}$$

Characterization - Firm dynamics

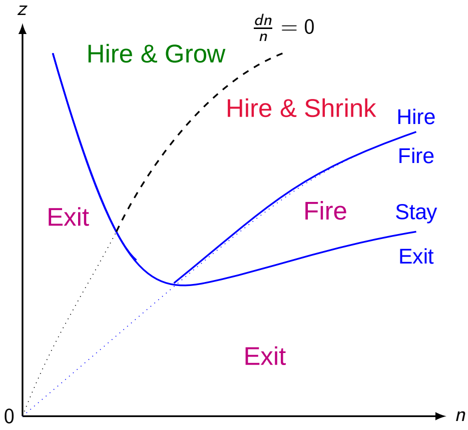


Characterization - Firm dynamics



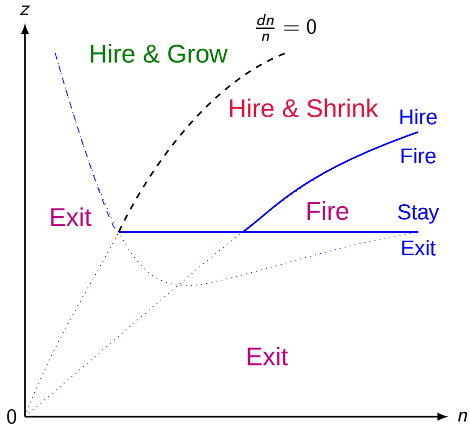
$$\frac{dn}{n} = \underbrace{\frac{v(S_n)}{n} q(\theta)\left[\phi + (1-\phi) G_n(S_n)\right]}_{UE_{Hires} + EE_{Hires}} - \underbrace{\left[\delta + \xi f(\theta) \tilde{G}_v(S_n)\right]}_{EU_{Layoffs} + EE_{Quits}}$$

Characterization - Firm dynamics



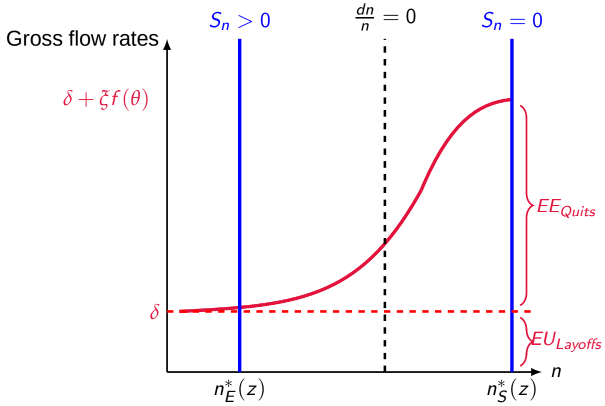
$$\frac{dn}{n} = \underbrace{\frac{v(S_n)}{n} q(\theta)\left[\phi + (1-\phi) G_n(S_n)\right]}_{UE_{Hires} + EE_{Hires}} - \underbrace{\left[\delta + \xi f(\theta) \tilde{G}_v(S_n)\right]}_{EU_{Layoffs} + EE_{Quits}}$$

Characterization - Firm dynamics



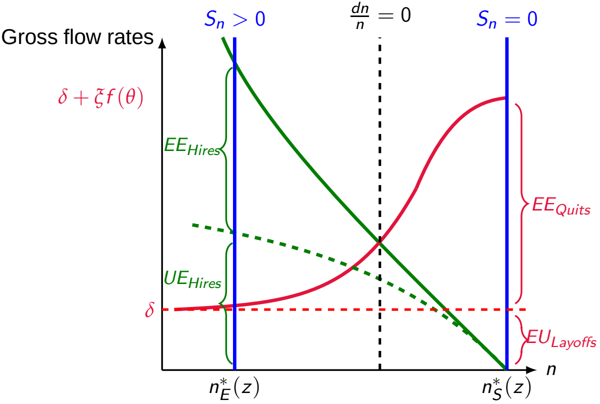
$$\frac{dn}{n} = \underbrace{\frac{v(S_n)}{n} q(\theta)\left[\phi + (1-\phi)G_n(S_n)\right]}_{UE_{Hires} + EE_{Hires}} - \underbrace{\left[\delta + \xi f(\theta)\tilde{G}_v(S_n)\right]}_{EU_{Layoffs} + EE_{Quits}}$$

Characterization - Firm level worker flows



$$\frac{dn}{n} = \underbrace{\frac{v(S_n)}{n} q(\theta)\left[\phi + (1-\phi) G_n(S_n)\right]}_{UE_{Hires} + EE_{Hires}} - \underbrace{\left[\delta + \xi f(\theta) \tilde{G}_v(S_n)\right]}_{EU_{Layoffs} + EE_{Quits}}$$

Characterization - Firm level worker flows



$$\frac{dn}{n} = \underbrace{\frac{v(S_n)}{n} q(\theta) \left[\phi + (1-\phi) G_n(S_n)\right]}_{UE_{Hires} + EE_{Hires}} - \underbrace{\left[\delta + \xi f(\theta) \tilde{G}_v(S_n)\right]}_{EU_{Layoffs} + EE_{Quits}}$$

Calibration - Parameters

Parameter		Value	Target	Data	Model
μ	Drift of productivity	-0.000	Exit rate, unweighted$^{(a)}$	0.076	0.077
σ	Std. dev. of productivity shocks	0.013	Std. dev. of TFP	0.40	0.493
ζ	Shape of entry distribution	2.240	Std. dev. of TFP (entrants)	0.35	0.451
d	Exogenous death rate of firms	0.001	Exit rate, weighted$^{(a)}$	0.020	0.039
α	Curvature of production	0.723	Employment share $n \geq 500$	0.518	0.298
χ	Matching efficiency	0.508	Job creation$^{(a)}$	0.136	0.106
γ	Curvature of hiring cost	9.807	UE rate$^{(m)}$	0.35	0.348
b	Flow value of leisure	0.388	EU rate$^{(m)}$	0.035	0.048
ϕ	Relative search efficiency	0.405	EE rate$^{(m)}$	0.018	0.005
δ	Exogenous separation rate	0.006	Layoff at high growth firms$^{(m)}$	0.025	0.025

Notes: (a) =Annual rate, (m) =Monthly rate. Moments regarding productivity are taken from Decker et al (2018). Moments regarding firm dynamics are averages of HP-filtered Census BDS data between 2013–2016. Moments regarding worker dynamics are averages of HP-filtered CPS data between 2013–2016, where matched samples are used and workers are over 16 years of age.

- Weekly solution. $c(v) = v^{\gamma}$. $z_0 \sim Pareto(\zeta)$.
- UE, EU, EE rates are weekly hazards $\times 4$ (need to time aggregate)

Calibration - Gross and net worker flows

	A. Employment		B. Productivity	
	Small $n < 50$	Large $n \geq 500$	High $Q5$	Low $Q1$
Data				
Gross *EE* hires - EE_{Hires}	8.9	7.7	6.5	7.2
Gross *EE* quits - EE_{Quits}	8.7	7.8	5.7	8.0
Net poaching	**0.2**	**-0.1**	**0.8**	**-0.8**
Model				
Gross *EE* hires - EE_{Hires}	4.1	3.5	3.9	3.2
Gross *EE* quits - EE_{Quits}	3.4	3.7	3.5	4.0
Net poaching	**0.7**	**-0.2**	**0.8**	**-0.4**

Notes: All data are quarterly. Authors' calculations from public data disclosed by Haltiwanger, Hyatt, McEntarfer (2017) (productivity), and Haltiwanger, Hyatt, McEntarfer Kahn (2018) (size). *Net poaching* is equal to *Gross EE hires* minus *Gross EE quits*. *Gross-Net ratio* is the mean of *Gross EE hires* and *Gross EE quits* divided by the absolute value of *Net poaching*.

Result Qualitatively generates correct size and productivity 'ladders'

Conclusion

Tractable framework that speaks to both firm and worker dynamics

- Equilibrium allocation characterized by surplus
- Life-cycle of firms and marginal surplus job ladder
- Frictions governing firm-dynamics founded in search and matching

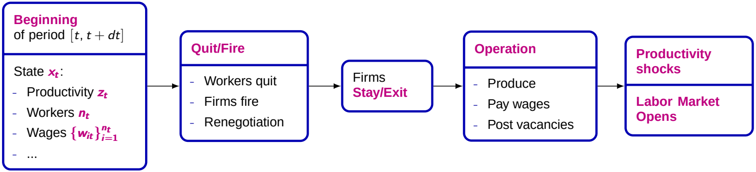
Next

- Theory — Wage determination
- Calibration — Resolve worker vs. job flow tensions
- Tests — Size-Age distribution, Growth-Age relationship
- Exercises — Importance of G.E. search frictions
  - vs. Multi-worker firm model with reduced form convex cost
  - vs. Multi-worker firm model with no OJS
  - vs. Single-worker firm with OJS

THANK YOU!

APPENDIX

Timeline


Beginning
of period $[t, t + dt]$
State x_t:
- Productivity z_t
- Workers n_t
- Wages $\{w_{it}\}_{i=1}^{n_t}$
- ...
Quit/Fire
- Workers quit
- Firms fire
- Renegotiation
Firms
Stay/Exit
Operation
- Produce
- Pay wages
- Post vacancies
Productivity shocks
Labor Market Opens

Timeline - Labor market