Firm and Worker Dynamics in a Frictional Labor Market

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Job-to-job flows

Account for a lot of reallocation

- 50% of hires are from another job, 50% of quits are to another job

Characterize firm dynamics

... in cross-section

- Firms with higher growth rates have lower quit rates

  Davis Faberman Haltiwanger Rucker (2010), Faberman Nagypal (2008)

- Small, young & high productivity firms poach on net $\left(EE^{+} > EE^{-}\right)$

  Haltiwanger Hyatt McEntarfer (2017), Haltiwanger Hyatt McEntarfer Kahn (2018)

... in response to shocks

- 72% of net-growth accounted for by increasing $\uparrow EE^{+}$, reducing $\downarrow EE^{-}$

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What role does job-to-job mobility play in determining $F(n, z)$?

This paper

Model

- Multi-worker firms $+$ Random search on and off-the-job
- Decreasing returns to scale $+$ Convex vacancy costs
- Entry, Exit, Age, Size, Growth $+$ Unemployment, Job-to-job flows

Entry, Exit, Age, Size, Growth: Firm dynamics / misallocation literature e.g. Hopenhayn Rogerson (1993)

Unemployment, Job-to-job flows: 1 worker firm OJS literature e.g. Postel-Vinay Robin (2002)

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Questions

✓ Calibrated to other data, does the model match new *EE* facts?

- Endogeneity of hiring costs important for allocation $F(n, z)$?

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New tractable framework that can handle lots of (firm) heterogeneity

Today

- Model
- Characterize firm dynamics
- Calibration
- Validate against two sets of empirical facts
    1. Cross-section
    2. Response of firms to a shock

Model - Continuous time environment

Workers

- Measure $\overline{\mathtt{N}}$ of infinitely lived, homogeneous workers
- Risk neutral. Discount rate ρ. Exogenous separation rate δ
- Supply one unit of labor. Produce b when unemployed.

Firms

- Produce $y(n, z) = zn^{\alpha}$, where $d \log z \sim GBM(\mu, \sigma)$. Scrap value π.
- $H_v(z, n) = v$-weighted distribution. $H_n(z, n) = n$-weighted distribution
- M_e entrants pay ψ_e for a draw of $z_e \sim Pareto(\zeta)$ and n_e workers

Search and matching

- Workers search for free on- and off- the job
- Firms search by posting costly vacancies $c(v, z, n)$
- Aggregate matching function $\mathtt{M} = \chi \mathtt{V}^{\beta} (\mathtt{U} + \xi \mathtt{E})^{1-\beta}$

Approach

Challenge

- Value sharing between workers and firms with DRS

Proposed solution

- Allocation characterized by joint value of firm + all employees
- Joint value depends only on firm productivity + size

✗ Complete contracts + full commitment: Coase theorem

✓ Incomplete contracts + limited commitment

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- ✓ Incomplete contracts + limited commitment

Why random search?

- Directed search environments with on the job search - Schaal (2014)
- ✓ Predictions for the composition of flows *out* of a firm: EE^- vs EU^-
- Hiring firms indifferent over the markets in which they post vacancies
- ✗ No predictions for the composition of flows *into* a firm: EE^+ vs UE^+

Assumptions

1. Two-sided limited commitment

- Firms can fire workers, workers can quit at will. Both at any time.

2. Bargaining

- Firms make take-it-or-leave-it offers
- Poaching firm makes first offer, competitor responds, worker decides
- Internal renegotiation - Value not destroyed $\rightarrow$ Care only about allocations

3. Vacancy posting

- Internal negotiation with wage cuts to post ‘coalition’-optimal vacancies

4. Mutual consent

- Wages modified only when an outside option gives a party a credible threat
  - $\pi(z, w, \cdot)$ meets unemployed worker, makes offer of b.
  - Credible threat to fire if $\pi(z, \cdot, b) > \pi(z, w, b)$ so cut $w' \in [b, w]$
  - No credible threat to fire if $\pi(z, w, b) > \pi(z, \cdot, b)$ so keep w

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$$EE^{+} \quad + v(n,z,...)q(\theta)(1-\phi)\int_{(n,z,...)\in\mathcal{Q}^E(n',z',\Omega'))} \Big[\boldsymbol{\Omega}(n+1,z,...) - \Omega(n,z, ...$$

$$- \left[\Omega(n',z',...) - \boldsymbol{\Omega}(n'-1,z',...)\right]\Big] dH_n(n', z',$$

$$\text{Shocks} \quad + \mathbb{E}_\eta\left[\boldsymbol{\Omega}(n, g(z,\eta), ...) - \Omega(n,z,...)\right]$$

Model - $\Omega(x) = \Omega(n, z, ...)$

$$\rho\Omega(n,z,...) = y(n,z,...) - c(v(n,z,...), n, z)$$

$$EU^{-}_{Layoff} \quad + \sum_{i=1}^{n(x)} \delta \left[\mathbf{\Omega}(n-1,z,...) - \Omega(n,z,...) + U \right]$$

$$UE^{+} \quad + v(n,z,...)q(\theta)\phi \left[\mathbf{\Omega}(n+1,z,...) - \Omega(n,z,...) - U \right] \cdot \mathbb{I}_{\{x \in \mathcal{A}\}}$$

$$EE^{+} \quad + v(n,z,...)q(\theta)(1-\phi) \int_{(n,z,...)\in\mathcal{Q}^E(n',z',\Omega'))} \Big[\mathbf{\Omega}(n+1,z,...) - \Omega(n,z,...$$

$$- \left[\Omega(n',z',...) - \mathbf{\Omega}(n'-1,z',...) \right] \Big] dH_n(n',z',$$

$$\text{Shocks} \quad + \mathbb{E}_\eta \left[\mathbf{\Omega}(n, g(z,\eta), ...) - \Omega(n,z,...) \right]$$

$$\mathbf{\Omega}(n,z,...) = \max \left\{ \pi + nU \, , \max_{s\in\{1,2,...,n-1\}} \Omega(n-s,z,...) + sU \, , \Omega(n,z,...) \right\}$$

Model - $\Omega(x) = \Omega(n, z)$

$$\rho\Omega(n,z) = y(n,z) - c(v(n,z), n, z)$$

$$EU^{-}_{Layoff} \quad + \sum_{i=1}^{n(x)} \delta \left[\boldsymbol{\Omega}(n-1,z) - \Omega(n,z) + U \right]$$

$$UE^{+} \quad + v(n,z)q(\theta)\phi \left[\boldsymbol{\Omega}(n+1,z) - \Omega(n,z) - U \right] \cdot \mathbb{I}_{\{x \in \mathcal{A}\}}$$

$$EE^{+} \quad + v(n,z)q(\theta)(1-\phi) \int_{(n,z)\in \mathcal{Q}^E(n',z')} \Big[\boldsymbol{\Omega}(n+1,z) - \Omega(n,z)$$

$$- \left[\Omega(n',z') - \boldsymbol{\Omega}(n'-1,z') \right] \Big] dH_n(n',z')$$

$$\text{Shocks} \quad + \mathbb{E}_\eta \left[\boldsymbol{\Omega}(n, g(z,\eta)) - \Omega(n,z) \right]$$

$$\boldsymbol{\Omega}(n,z) = \max \left\{ \pi + nU \, , \max_{s\in\{1,2,\ldots,n-1\}} \Omega(n-s,z) + sU \, , \Omega(n,z) \right\}$$

Model - $\Omega(x) = \Omega(n, z)$

$$\rho\Omega(n,z) = y(n,z) - c(v(n,z), n, z)$$

EU^{-}_{Layoff}
$$+ n\delta\Big[\boldsymbol{\Omega}(n-1,z) - \Omega(n,z) + U\Big]$$

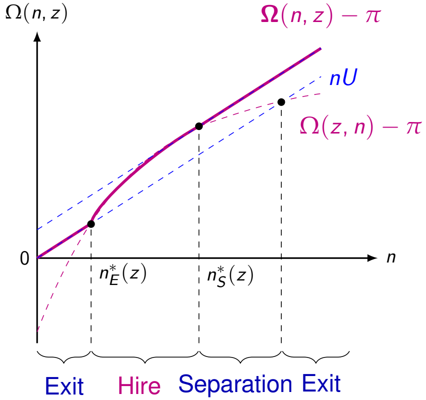
UE^{+}
$$+ v(n,z)q(\theta)\phi\Big[\boldsymbol{\Omega}(n+1,z) - \Omega(n,z) - U\Big] \cdot \mathbb{I}_{\{x\in\mathcal{A}\}}$$

EE^{+}
$$+ v(n,z)q(\theta)(1-\phi)\int_{(n,z)\in\mathcal{Q}^E(n',z')}\Big[\boldsymbol{\Omega}(n+1,z) - \Omega(n,z) - \big[\Omega(n',z') - \boldsymbol{\Omega}(n'-1,z')\big]\Big] dH_n(n',z')$$

Shocks
$$+ \mathbb{E}_\eta\Big[\boldsymbol{\Omega}(n, g(z,\eta)) - \Omega(n,z)\Big]$$

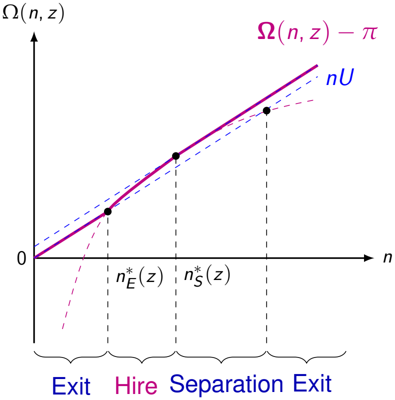
$$\boldsymbol{\Omega}(n,z) = \max\Big\{\pi + nU\,,\ \max_{s\in\{1,2,\ldots,n-1\}}\Omega(n-s,z) + sU\,,\ \Omega(n,z)\Big\}$$

Action regions



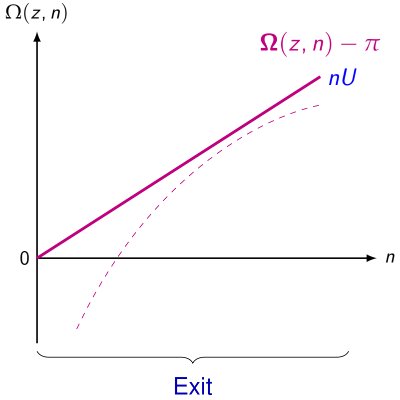
$$\boldsymbol{\Omega}(n,z)-\pi = \max\left\{nU\,,\ \Omega(n,z)-\pi\,,\ \max_{s\in\{1,2,\ldots\}}\ \Omega(n-s,z)-\pi+sU\right\}$$

Action regions



$$\boldsymbol{\Omega}(n,z)-\pi = \max\left\{ nU\,,\ \Omega(n,z)-\pi\,,\ \max_{s\in\{1,2,\dots\}} \Omega(n-s,z)-\pi+sU \right\}$$

Action regions



$$\boldsymbol{\Omega}(n, z) - \pi = \max \left\{ nU \, , \, \Omega(n, z) - \pi \, , \, \max_{s \in \{1,2,\dots\}} \Omega(n - s, z) - \pi + sU \right\}$$

Model - $S(n,z) = \Omega(n,z) - nU$

$$
\begin{aligned}
\rho\Omega(n,z) \quad &= \quad y(n,z) - c(v(n,z), n, z) \\
EU^{-}_{Layoff} \quad & + n\delta\Big[\mathbf{\Omega}(n-1,z) - \Omega(n,z) + U\Big] \\
UE^{+} \quad & + v(n,z)q(\theta)\phi\Big[\mathbf{\Omega}(n+1,z) - \Omega(n,z) - U\Big] \cdot \mathbb{I}_{\{x \in \mathcal{A}\}} \\
EE^{+} \quad & + v(n,z)q(\theta)(1-\phi)\int_{(n,z)\in\mathcal{Q}^E(n',z')} \Big[\mathbf{\Omega}(n+1,z) - \Omega(n,z) \\
& \qquad - \left[\Omega(n',z') - \mathbf{\Omega}(n'-1,z')\right]\Big] dH_n(n',z') \\
\text{Shocks} \quad & + \mathbb{E}_\eta\Big[\mathbf{\Omega}(n, g(z,\eta)) - \Omega(n,z)\Big] \\
\mathbf{\Omega}(n,z) \quad &= \quad \max\Big\{\pi + nU\,,\ \max_{s\in\{1,2,\dots,n-1\}} \Omega(n-s,z) + sU\,,\ \Omega(n,z)\Big\}
\end{aligned}
$$

Model - $S(n,z) = \Omega(n,z) - nU$

$$
\begin{aligned}
\rho S(n,z) &= \max_{v \geq 0} \quad y(n,z) - nb \quad - \quad \delta n \mathbf{S}_n(n,z) \quad - \quad c(v,n,z) \\
&+ \quad vq(\theta) \underbrace{\left[\phi \mathbf{S}_n(n,z) + (1-\phi) \int_0^{\mathbf{S}_n(n,z)} \mathbf{S}_n(z,n) - S'_n \, dH_n\left(S'_n\right) \right]}_{\text{Return on a vacancy } R(\mathbf{S}_n)} \\
&+ \quad \mathbb{E}_\eta \left[\mathbf{S}\left(g_z(z,\eta), n\right) - S(n,z) \right]
\end{aligned}
$$

$$
\mathbf{S}(n,z) = \max \left\{ \pi \, , \, \max_{n' \leq n} S(n',z) \, , \, S(n,z) \right\}
$$

Model - $S(n,z) = \Omega(n,z) - nU$

$$\rho S(n,z) = \max_{v \geq 0} \quad y(n,z) - nb \quad - \quad \delta n S_n(n,z) \quad - \quad c(v,n,z)$$

$$+ \quad vq(\theta) \underbrace{\left[\phi S_n(n,z) + (1-\phi) \int_0^{S_n(n,z)} S_n(z,n) - S_n' \, dH_n(S_n')\right]}_{\text{Return on a vacancy } R(S_n)}$$

$$+ \quad \mu(z) S_z(n,z) + \frac{\sigma(z)^2}{2} S_{zz}(n,z)$$

$$S(n,z) \geq \pi \qquad \text{(Exit)} \qquad \underbrace{S_z(n_E^*(z), z) = 0 \quad , \quad S_n(n_E^*(z), z) = 0}_{\text{Smooth pasting}}$$

$$S_n(n,z) \geq 0 \qquad \text{(Separation)}$$

Proposition - Comparative statics

Assumptions

- GBM: $\mu(z) = \mu \times z$, $\sigma(z) = \sigma \times z$
- $c(v, n, z) = cv^{1+\gamma}$, $\gamma > 0$
- $y_z > 0$, $y_n > 0$, $y_{nn} < 0$, $y_{nz} > 0$

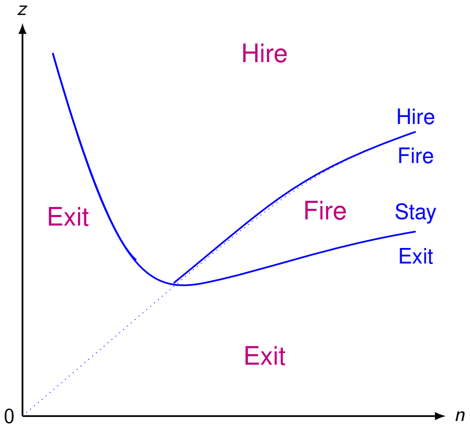
Surplus

- Increasing and concave in employment: $S_n > 0$, $S_{nn} < 0$
- Increasing in productivity: $S_z > 0$
- Higher z raises the marginal surplus of labor: $S_{zn} > 0$

Hiring

- Net employment growth increases with z, decreases with n
- Net poaching $(EE^+ - EE^-)$ increases with z, decreases with n

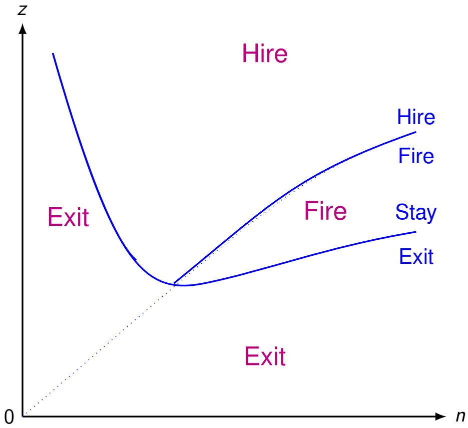
Characterization - Firm dynamics



$$Stay\,/\,Exit: \qquad S(n,z) \;=\; \pi \quad , \quad dz/dn = -S_n/S_z^+$$

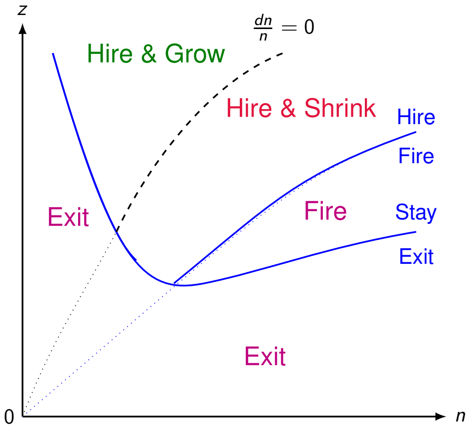
$$Hire\,/\,Fire: \qquad S_n(n,z) \;=\; 0 \quad , \quad dz/dn = -S_{nn}^-/S_{nz}^+ > 0$$

Characterization - Firm dynamics



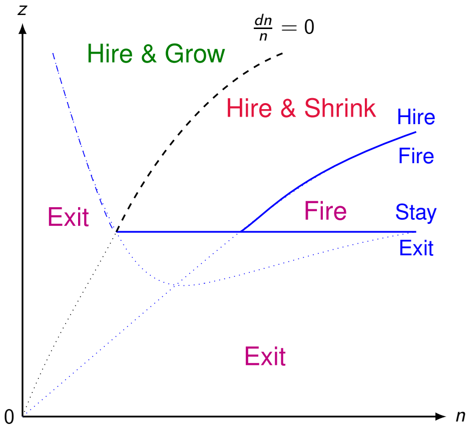
$$\frac{dn}{n} = \underbrace{\frac{v(S_n)}{n} q(\theta)\Big[\phi + (1-\phi) H_n(S_n)\Big]}_{UE^+ + EE^+} - \underbrace{\Big[\delta + \xi f(\theta) \widetilde{H}_v(S_n)\Big]}_{EU^- + EE^-}$$

Characterization - Firm dynamics



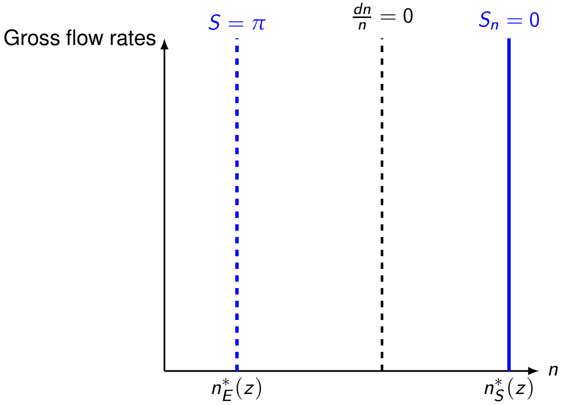
$$\frac{dn}{n} = \underbrace{\frac{v(S_n)}{n} q(\theta)\Big[\phi + (1-\phi) H_n(S_n)\Big]}_{UE^+ + EE^+} - \underbrace{\Big[\delta + \xi f(\theta) \widetilde{H}_v(S_n)\Big]}_{EU^- + EE^-}$$

Characterization - Firm dynamics



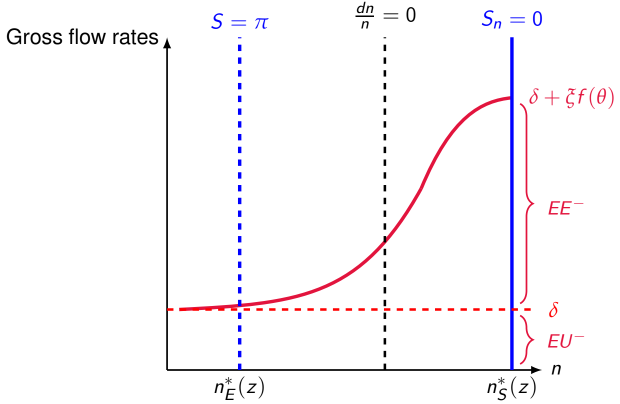
$$\frac{dn}{n} = \underbrace{\frac{v(S_n)}{n} q(\theta) \Big[\phi + (1-\phi) H_n(S_n) \Big]}_{UE^{+} + EE^{+}} - \underbrace{\Big[\delta + \xi f(\theta) \widetilde{H}_v(S_n) \Big]}_{EU^{-} + EE^{-}}$$

Characterization - Firm level worker flows



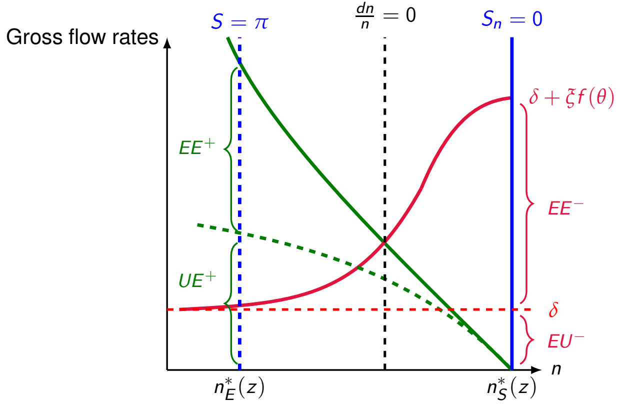
$$\frac{dn}{n} = \underbrace{\frac{v(S_n)}{n} q(\theta)\Big[\phi + (1-\phi)H_n(S_n)\Big]}_{UE^+ + EE^+} - \underbrace{\Big[\delta + \xi f(\theta)\widetilde{H}_v(S_n)\Big]}_{EU^- + EE^-}$$

Characterization - Firm level worker flows



$$\frac{dn}{n} = \underbrace{\frac{v(S_n)}{n} q(\theta)\left[\phi + (1-\phi) H_n(S_n)\right]}_{UE^+ + EE^+} - \underbrace{\left[\delta + \xi f(\theta) \widetilde{H}_v(S_n)\right]}_{EU^- + EE^-}$$

Characterization - Firm level worker flows



$$\frac{dn}{n} = \underbrace{\frac{v(S_n)}{n} q(\theta)\Big[\phi + (1-\phi) H_n(S_n)\Big]}_{UE^+ + EE^+} - \underbrace{\Big[\delta + \xi f(\theta) \widetilde{H}_v(S_n)\Big]}_{EU^- + EE^-}$$

Calibration - Strategy

Parameter		Value	Target
ρ	Discount rate	0.004	5% annual real interest rate
β	Elasticity of matches w.r.t. V	0.5	Petrongolo and Pissarides (2001)
c_f	Fixed cost of production	1	Normalization
$\mathtt{M}$	Number of active firms	0.95/22	Average firm size (BDS)

- Following the same strategy as GMV (2018)
- Vacancy costs: $c(v, n) = c(v/n)^{\gamma+1}v$
- Add exogenous death at rate d
- Additional moments on *EE* rates
- To do - Add heterogeneity in DRS $\alpha \in \{\alpha_1, \alpha_2, \alpha_3\}$

Calibration - Parameters

Parameter		Value	Target	Data	Model
μ	Drift of prod.	-0.004	Annual exit rate, unweigh.	0.076	0.078
σ	SD of log prod. shocks	0.025	SD of TFP	0.400	0.405
ζ	Pareto tail of Π_0	2.857	Prod. entrants / incumb.	-0.30	—
d	Exog. firm death rate	0.001	Annual exit rate, weighted	0.020	0.020
α	Curvature of production	0.624	Employment share $n \geq 500$	0.518	0.547
c	Scalar in hiring cost	14.485	Cost per hire / wage	1.0	1.780
γ	Curvature of hiring cost	2.892	SD of growth rates	0.420	0.535
A	Matching efficiency	0.582	UE rate	0.242	0.271
ϕ	Rel. search efficiency	0.122	EE/UE	0.076	0.086
δ	Exog. separation rate	0.010	EU rate	0.011	0.010
b	Flow value of leisure	0.217	SD labor productivity	0.500	0.427

Notes: (a) =Annual rate, (m) =Monthly rate. Moments regarding productivity are taken from Decker et al (2018). Moments regarding firm dynamics are averages of HP-filtered Census BDS data between 2013–2016. Moments regarding worker dynamics are averages of HP-filtered CPS data between 2013–2016, where matched samples are used and workers are over 16 years of age.

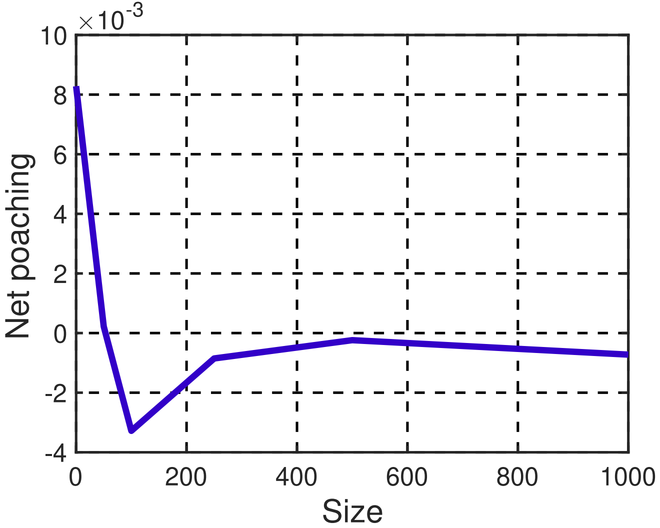
Gross and net worker flows

	A. Employment		B. Productivity	
	Small $n < 50$	Large $n \geq 500$	High $Q5$	Low $Q1$
Data				
Gross *EE* hires - EE^{+}	8.9	7.7	6.5	7.2
Gross *EE* quits - EE^{-}	8.7	7.8	5.7	8.0
Net poaching	**0.2**	**-0.1**	**0.8**	**-0.8**
Model				
Gross *EE* hires - EE^{+}	4.1	3.5	3.9	3.2
Gross *EE* quits - EE^{-}	3.4	3.7	3.5	4.0
Net poaching	**0.7**	**-0.2**	**0.8**	**-0.4**

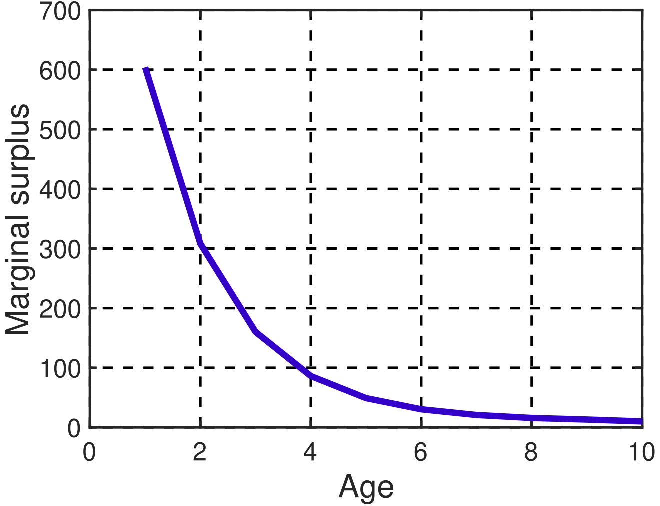
Notes: All data are quarterly. Authors' calculations from public data disclosed by Haltiwanger, Hyatt, McEntarfer (2017) (productivity), and Haltiwanger, Hyatt, McEntarfer Kahn (2018) (size). *Net poaching* is equal to *Gross EE hires* minus *Gross EE quits*. *Gross-Net ratio* is the mean of *Gross EE hires* and *Gross EE quits* divided by the absolute value of *Net poaching*.

Result Qualitatively generates correct size and productivity 'ladders'

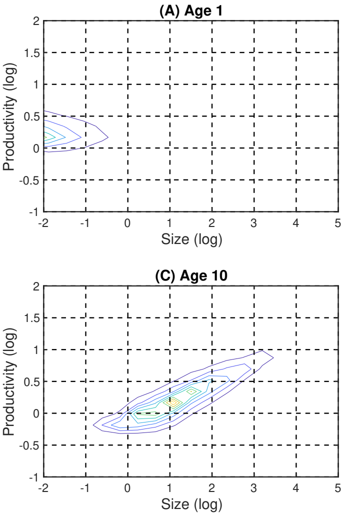
Net poaching rate by size



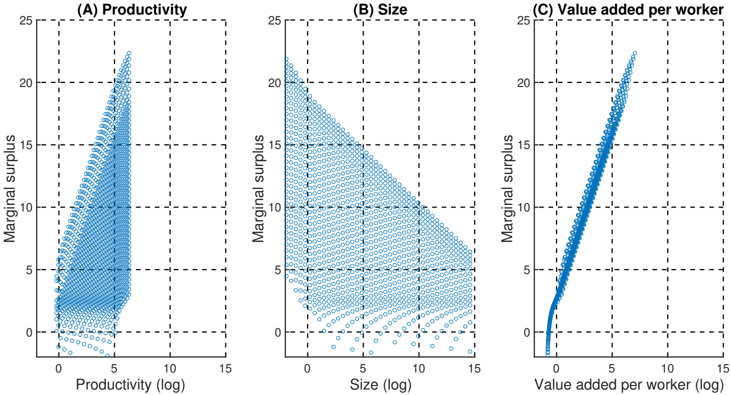
Marginal surplus by age



Life-cycle dynamics of firm distribution over (z, n)

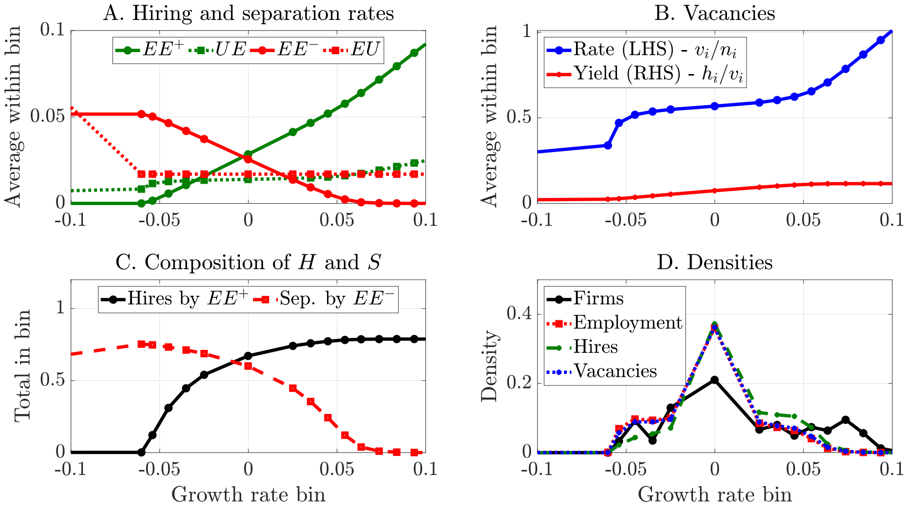


Marginal surplus by productivity, size and VA



- Quantitatively the model would map empirical measures of value added per worker to marginal surplus
- Useful for interpreting and replicating Bagger et. al. (2019)

Hiring by firm growth



Conclusion

Tractable framework that speaks to both firm and worker dynamics

- Equilibrium allocation characterized by surplus
- Life-cycle of firms and marginal surplus job ladder
- Frictions governing firm-dynamics founded in search and matching

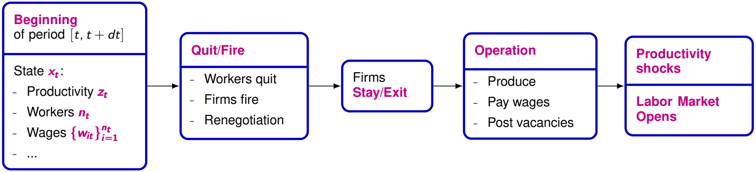
Next

- Calibration — Add heterogeneity in DRS
- Tests — Validate against Bagger et. al. (2019)
- Exercises — Comparative static of output w/r/t search efficiency ξ
- Exercises — Shock to entry costs

THANK YOU!

APPENDIX

Timeline



Beginning
of period $[t, t + dt]$
State x_t:
- Productivity z_t
- Workers n_t
- Wages $\{w_{it}\}_{i=1}^{n_t}$
- ...
Quit/Fire
- Workers quit
- Firms fire
- Renegotiation
Firms
Stay/Exit
Operation
- Produce
- Pay wages
- Post vacancies
Productivity shocks
Labor Market Opens

Timeline - Labor market